

tive model. In view of all this, one wonders whether Chap. VII is absolutely necessary for the *general* theory of algebraic varieties, or even for the theory of Abelian varieties.

Weil's book is the first purely arithmetic exposition of an important sector of algebraic geometry, and is therefore a landmark in the literature of this field. This, and the competence of the author, give the book added significance.

In the remainder of the book we wish to recommend especially the interesting chapter entitled *Comments and discussions*, where various unsolved problems are discussed and possible directions of future research are indicated. The book has an excellent list of definitions and table of notations.

OSCAR ZARISKI

*Transformations on lattices and structures of logic.* By S. A. Kiss. New York, 1947. 10+322 pp. \$5.00.

This volume by Dr. Kiss is an instance of a rare phenomenon—a significant contribution to mathematics by one who is not a professional mathematician. Dr. Kiss is a patent lawyer by profession, with an advanced degree in chemistry, who has here enriched the algebra of logic by a new idea, and developed the idea in full detail. Moreover enough elementary material has been adapted from standard sources so that the book is self-contained as regards both algebra and logic. The book was published by the author, and can be obtained from him at 11 E. 92nd Street, New York City.

Boolean algebra may be described as the algebra of true and false. Numerous equivalent postulate systems for it are known, involving from one to three undefined operations or relations, in terms of which all  $2^{2^n}$  possible  $n$ -ary operations on a two-element system can be defined. Thus *every finitary operation on a two-element system is a Boolean operation*.

But the corresponding result for the four-element Boolean algebra  $B^2$  does not hold. Only  $2^{2^2}$  of the  $4^{4^2}$  possible binary operations on  $B^2$  are "Boolean," in the usual sense. Dr. Kiss proposes the following ingenious extension. Consider the self-dual ternary operation  $(x, y, z) = (x \cup y) \cap (y \cup z) \cap (z \cup x)$ . In terms of this, joins and meets can be defined by fixing  $y$  as one of the two logical constants 0 and  $I$ :  $x \cup z = (x, I, z)$  and  $x \cap z = (x, 0, z)$ . If we use the other two constants  $e$  and  $e'$  of  $B^2$  in place of 0 and  $I$ , we get two further binary Boolean-like operations  $(x, e, z)$  and  $(x, e', z)$ . And in terms of these, all binary operations can be defined.

It is still too early to appraise the ultimate importance of these and

related algebraic operations discussed by Dr. Kiss. In the reviewer's opinion, they meet various esthetic requirements of symmetry and harmony, which it is hard to formulate exactly. Thus all operations are mutually distributive; in the case of the Boolean algebra  $B^n$  of order  $2^n$ , Dr. Kiss's algebra admits  $2^n(n!)$  "link"-automorphisms, in place of the  $2(n!)$  automorphisms and dual automorphisms usually admitted. Mr. Kiss suggests applications to the algebra of classes of classes which, if they can be developed more precisely and fully, may be of great value.

But for the present, the book's chief merit consists in its orderly and detailed development of an intriguing and highly symmetrical calculus, which contains ordinary Boolean algebra as a special case. The author deserves the sincere thanks of persons interested in this field, for his work.

GARRETT BIRKHOFF