THE APRIL MEETING IN BERKELEY

The four hundred thirty-sixth meeting of the American Mathematical Society was held at the University of California, Berkeley, California, on Saturday, April 17, 1948. The attendance was approximately 125, including the following 86 members of the Society:

M. T. Aissen, H. L. Alder, T. M. Apostol, H. M. Bacon, E. W. Barankin, H. W. Becker, E. M. Beesley, B. A. Bernstein, M. T. Bird, Gertrude Blanch, H. F. Bohnenblust, J. L. Botsford, J. L. Brenner, H. D. Brunk, Albert Cahn, L. H. Chin, F. G. Creese, P. H. Daus, Harold Davenport, A. C. Davis, E. A. Davis, D. B. Dekker, C. R. DePrima, A. H. Diamond, H. R. C. Dieckmann, L. K. Durst, Arthur Erdélyi, E. A. Fay, A. L. Foster, Marianne Freundlich, J. N. Goodier, Arthur Grad, J. W. Green, Leonard Greenstone, John Gurland, S. M. Hallam, Frank Harary, W. L. Hart, J. G. Herriot, J. L. Hodges, Alfred Horn, Rufus Isaacs, R. C. James, T. A. Jeeves, B. W. Jones, Samuel Karlin, J. L. Kelley, E. L. Lehmann, D. H. Lehmer, R. B. Leipnik, Robert Mann, A. V. Martin, Burnet Meyer, E. D. Miller, C. B. Morrey, A. P. Morse, J. D. Newburgh, Ivan Nivan, A. B. Novikoff, C. D. Olds, Edmund Pinney, R. I. Piper, George Pólya, R. R. Putz, W. C. Randels, J. B. Robinson, R. M. Robinson, R. A. Rosenbaum, Rafael Sánchez-Díaz, S. A. Schaaf, M. M. Schiffer, Abraham Seidenberg, Edward Silverman, Ernst Snapper, D. C. Spencer, Pauline Sperry, R. G. Stoneham, Otto Szász, Gabor Szegö, Alfred Tarski, Olga Taussky-Todd, John Todd, R. K. Wakerling, Morgan Ward, J. G. Wendel, P. A. White, A. R. Williams.

In the morning there was a short session for research papers, followed by the hour address, *On van der Pol's equation*, by Professor H. F. Bohnenblust of the California Institute of Technology. Professor Gabor Szegö presided. In the afternoon, additional papers were presented in two sections, at which Professors B. A. Bernstein and C. B. Morrey, respectively, presided.

Following the meetings, those attending were guests of the department of mathematics of the University of California at a tea at Stevens Union.

Abstracts of all papers presented at the meeting are given below. Papers read by title are indicated by the letter "*t*". Paper number 340 was presented by Mr. Shapley, number 354 by Professor Szegö, and number 361 by Dr. Horn. Mr. Blaney was introduced by Professor Harold Davenport, and Mr. Shapley and Mr. Snow by Professor J. C. C. McKinsey,

ALGEBRA AND THEORY OF NUMBERS

332*t*. Hugh Blaney: *Indefinite quadratic forms in n variables.*

It has been shown recently by Davenport (C. R. Acad. Sci. Paris vol. 224 (1947) 990–991) that Minkowski's theorem on nonhomogeneous binary quadratic forms can be extended to ternary forms. The author has obtained the following results for indefinite forms in n variables. Let $Q(x)=Q(x_1, \cdots, x_n)$ be such a form, with real coefficients and determinant $D\neq 0$. (1) For any $c\geqq 0$ there exists $C=C(c, n)$ such that the inequality $c|D|^{1/n}<Q(x)<C|D|^{1/n}$ has a solution in integers $x_1, \cdots, x_n$ whose highest common factor is 1. (2) For any $\gamma\geqq 0$ there exists $\Gamma=\Gamma(\gamma, n)$ such that for any real $x_1', \cdots, x_n'$ the inequality $\gamma|D|^{1/n}<Q(x+x')<\Gamma|D|^{1/n}$ has an integral solution. In particular, a result of Minkowski's type is valid for indefinite quadratic forms in n variables. (Received February 16, 1948.)

333. J. L. Brenner: *Right multiplication by a unitary matrix.*

A canonical form is obtained for a matrix A under right multiplication by a unitary matrix U. The relation $AU=B$ is an equivalence relation. (Received April 16, 1948.)

334. Harold Davenport: *Indefinite binary quadratic forms and Euclid's algorithm in real quadratic fields.*

Let $f(x, y)=ax^2+bxy+cy^2$ have real coefficients and discriminant $d=b^2-4ac\neq 0$, and suppose $f(x, y)\neq 0$ for integral x, y other than 0, 0. A classical theorem of Minkowski asserts that for any real p, q there are an infinity of integers x, y for which $|f(x+p, y+q)|<(1/4)d^{1/2}$. I have now proved the following result in the opposite direction: there exist real p, q such that $|f(x+p, y+q)|>d^{1/2}/128$ for all integral x, y. This may be regarded as a generalization of a theorem of Khintchine on Diophantine approximation. The construction of p, q is such as to ensure that if a, b, c are integers, then p, q are *rational*. It follows that Euclid's algorithm cannot hold in any field $k(m^{1/2})$ for which $m>(128)^2$. (Received February 16, 1948.)

335. B. W. Jones: *Composition of binary forms.*

An ideal class in a quadratic field is called ambiguous if it is equal to its conjugate class and a binary quadratic form is ambiguous if it is improperly equivalent to itself. Along the lines of Dedekind's theory, it is shown that there is a 1-1 correspondence between ambiguous ideal classes over a given field and ambiguous classes of forms of corresponding determinant, while the correspondence between non-ambiguous ideal classes and form classes is 2-2. This result is used to prove Gauss's theorem on the duplication of binary classes and other classical results. (Received March 8, 1948.)

336. B. W. Jones: *Representations by quadratic forms.*

If A is a symmetric n by n matrix with integral elements and $T=X$ and $T=Y$ are two solutions of $T^TAT=c$ for some integer c, then X and Y are called essentially equal if there is an automorph P of A with integer elements and determinant 1 such that $X=PY$. $G(A, C)$ is the number of essentially distinct solutions of $T^TA_kT=c$ summed over all classes A_k in the genus of A. A formula is obtained for $G(A, C)$ which for the case c prime to $2|A|$ reduces to $\rho(c)\sum s(E)$ summed over all classes of forms E in $n-1$ variables and of a certain genus or two genera; $\rho(c)$ depends solely on the number of odd prime factors of c and $s(E)$ is the reciprocal of the number of automorphs of E whose first rows are distinct (mod c). When $n=3$ this sum may be expressed in terms of the binary class number. (Received March 8, 1948.)

337. D. H. Lehmer: *On a conjecture of Krishnaswami.*

Let $T(N)$ denote the number of right triangles whose perimeters do not exceed

$2N$ and whose sides are relatively prime integers. On the basis of tabular evidence A. A. Krishnaswami has conjectured that $T(N) \sim N/7$. In this paper it is shown that this is not quite the case. In fact it is proved that $T(N) = \pi^{-2}N \log 4 + O(N^{1/2} \log N)$. The constant $\pi^2/\log 4 = 7.1194 \cdots$. Hence Krishnaswami's conjecture is not far wrong. The fact that $T(N)/N$ tends to this constant follows from the theory of "totient points" as developed by D. N. Lehmer in 1900. The above error term is obtained from elementary considerations of Dirichlet series. (Received March 13, 1948.)

338t. D. H. Lehmer: *On the vanishing of Ramanujan's function.* II.

In a previous note (Bull. Amer. Math. Soc. vol. 53 (1947) p. 478) it was shown that Ramanujan's function $\tau(n)$, defined as the coefficient of x^{n-1} in the 24th power of the lacunary power series $(1-x)(1-x^2)(1-x^3) \cdots$, does not vanish for $n < 3316799$. This result has now been extended to $n < 214928639999$. The function $\tau(n)$ vanishes at most 7 times for $n < 10^{12}$. The present results are a consequence of new congruence relations for $\tau(n)$ with respect to the moduli 2^{10}, 3^6, and 5^3, given by Lahiri. (Received March 13, 1948.)

339. Ivan Niven: *Fermat's theorem for matrices*

Let A be any nonsingular matrix of order n with elements in $GF(p^m)$. A minimum value q is specifically determined as a function of p, m, and n such that $A^q = I$ for every A. (Received March 6, 1948.)

340. L. S. Shapley and R. N. Snow: *The solutions of the general two-person zero-sum game with a finite number of strategies.*

It is shown that all solutions of the general two-person zero-sum game can be represented by means of a finite number of "basic solutions," which may be visualized as pairs of vertices from two polyhedra in the spaces of all mixed strategies for the two players. Each solution has associated with it a certain submatrix of the whole game-matrix, called the "kernel" of the solution. The kernels of the basic solutions are nonsingular. Once a basic kernel has been located in the game-matrix, the associated basic solution may be computed directly by a formula. (Received April 4, 1948.)

341. Ernst Snapper: *Completely indecomposable modules.*

Let A be a commutative ring with unit element 1. Let $\mathfrak{B}$ be an abelian group with A as operator domain, where 1 is unit operator and where $\mathfrak{B}$ has a composition series of finite length. $\mathfrak{B}$ is called a *completely indecomposable module* if $\mathfrak{B}$ and all its submodules are indecomposable; that is, if $\mathfrak{B}$ has a unique minimal submodule. It is proved that two completely indecomposable modules are A-isomorphic if and only if they have the same annihilating ideal. In other words, two faithful representations of A whose representation spaces have unique minimal submodules are equivalent. It follows that a completely indecomposable module is cyclic if and only if its annihilating ideal is intersection-irreducible. It is shown how the classical elementary divisor theory can be derived from the theory of completely indecomposable modules. (Received January 29, 1948.)

342. Morgan Ward: *Divisibility sequences associated with the real multiplications of the Jacobian elliptic functions.* Preliminary report.

This paper extends the results of the author on divisibility sequences associated with the real multiplication of the Weirstrass elliptic functions (Amer. J. Math. vol. 70 (1948) pp. 31–74) to the corresponding sequences associated with the Jacobian elliptic functions, obtaining elliptic function analogues of Lucas' function V_n as well as the already treated analogue of U_n. (Received March 13, 1948.)

ANALYSIS

343. H. D. Brunk: *A consistency theorem.*

The concept of asymptotic representation in a strip region of a function by Dirichlet series with a certain logarithmic precision has been introduced by Mandelbrojt (see S. Mandelbrojt, *Sur une inegalite fondamentale*, Ann. École Norm. (3) vol. 63, pp. 351–378). He has proved that if a Dirichlet series represents a function $F(s)$ in a strip region with sufficient logarithmic precision the series converges. The present paper completes the theorem by showing that under the same conditions the series necessarily converges to the function $F(s)$. (Received March 2, 1948.)

344*t*. S. P. Diliberto: *On special properties of measure preserving transformations.*

The author proves: The only point transformations of class C^3 of Euclidean 3-space into itself which preserve area are rigid motions or reflections. A partial generalization is established: Let $T(t)$ be a transformation of Euclidean n-space into itself defined by $dx_i/dt = X_i(x_1, \cdots, x_n)$, $i=1, \cdots, n$; $X_i \in C^2$ (that is, each point x "moves" for fixed time t along the trajectory through it; thus $T(0)$ is the identity map). A necessary and sufficient condition that $T(t)$ preserve p measure, p fixed and not greater than $n-1$, for all t is that $\partial X_i/\partial x_j = -\partial X_j/\partial x_i$ for $i, j = 1, 2, \cdots, n$. This implies that $dx/dt = Ax + a$ where A is skew-symmetric. This can be integrated and shows the transformation is rigid. The first theorem is proved by deriving from the images of the planes $x_i + x_j = 0$ $(i, j = 1, 2, 3)$ conditions which imply the map sends triply orthogonal surfaces into triply orthogonal surfaces. By Liouville's theorem the only possibilities are rigid motions, reflections, and inversions—of which the last is ruled out. The second theorem is established by (a) applying the invariance on the planes $x_i + x_j = 0$, $i, j = 1, 2, \cdots, n$, and their images, (b) requiring conditions (a) to hold for all t. (Received March 12, 1948.)

345. S. P. Diliberto: *On systems of ordinary differential equations.*

For $dx/dt = Ax$ (x a column vector, A an n square matrix with $a_{ij}(t)$ continuous for all t) the author proves there exists an orthogonal matrix B (defined for all t), with $b_{ij}(t) \in C^1$, such that if $By = x$ then $dy/dt = Cy$ with $c_{ij} = 0$ if $i > j$ (c_{ij} bounded if a_{ij} bounded). The proof generalizes the author's result that the variational equations of the system $dx_i/dt = X_i(x_1, x_2)$, $i = 1, 2$, are integrable by quadratures. This quadrature leads to the following (the first results on the 2nd half of Hilbert's 16th problem): If div $X \neq 0$ ($X = (X_1, X_2)$) on any periodic solution (closed trajectory) of $dx_i/dt = X_i(x_1, x_2)$, $i = 1, 2$, where X_i are polynomials of degree not greater than n then the number of periodic solutions is not greater than $(1/2)(n-2)(n-3)+1$; if the periodic solutions are nested the upper bound is sharpened to $[(n-1)/2]$—which is best possible. From the above quadrature it follows that if K, the curvature of the orthogonal trajectories, does not vanish on a periodic solution then that solution is stable if $K < 0$ and unstable if $K > 0$. The author's results are used to give a simplified treatment of

Liapounoff's "characteristic exponents." One difficulty in generalizing the Poincaré-Bendixson theorem is established by proving that there exist flows into a torus which have periodic solutions (inside the torus) which are arbitrarily knotted and linked. (Received March 12, 1948.)

346t. G. E. Forsythe: *An error of Hayashi and Izumi on summability.*

Let $p_0 \neq 0$, p_1, p_2, $\cdots$ be a given sequence of non-negative numbers, and set $P_n = p_0 + p_1 + \cdots + p_n$. For a sequence $\{s_n\}$ let $\sigma_n = (p_n s_0 + p_{n-1} s_1 + \cdots + p_0 s_n)/P_n$ and let $\rho_n = (p_0 s_0 + p_1 s_1 + \cdots + p_n s_n)/P_n$. If $\sigma_n \to \sigma$ $[\rho_n \to \sigma]$ as $n \to \infty$, $\{s_n\}$ is said to be summable-N_p [summable-R_p] to σ. Hayashi and Izumi (Tôhoku Math. J. vol. 47 (1940) pp. 69–73, Theorem 3, p. 73) stated an incorrect sufficient condition for equivalence of (the Nörlund mean) N_p and (the Riesz typical mean) R_p. For the special weights $p_n^{(r)} = n^{r-1}$ it would follow from Hayashi and Izumi that, for each $r = 1, 2, \cdots$, $R_p^{(r)}$ is equivalent to the Cesàro mean C_r. But Hardy (Quarterly Journal of Pure and Applied Mathematics vol. 38 (1907) pp. 269–288) proved that for each $r \geqq 1$ the Riesz mean $R_p^{(r)}$ is equivalent to the Cesàro mean C_1. Hayashi and Izumi erred in applying the Bloch-Pólya inequality on page 72. (Received March 3, 1948.)

347. Arthur Grad: *The distortion of schlicht functions.*

Let $w = z + a_2 z^2 + a_3 z^3 + \cdots$ be regular and schlicht in the unit circle. The problem of distortion consists in determining all possible values of $w'(z_1)$, where z_1 is any fixed point of $|z| < 1$. In 1909 Koebe proved the existence of bounds for $w'(z_1)$, these bounds depending on $|z_1|$ only. In 1916 Faber, Bieberbach and others found the precise bounds for $|w'(z_1)|$, and in 1936 Golusin found the precise bounds for $|\arg w'(z_1)|$. The distortion problem is now solved completely; the exact region of variability of $w'(z_1)$, z_1 fixed, as $w(z)$ ranges over the whole family of schlicht functions, is determined. The region is more conveniently represented in the logarithmic plane where, for $|z_1| \leqq 1/2^{1/2}$, the boundary of the domain corresponds to functions each of which maps the unit circle on the exterior of an analytic slit extending to infinity. For $1/2^{1/2} < |z_1| < 1$, the boundary consists of four alternating analytic arcs. Two opposite arcs correspond to functions mapping the unit circle on the exterior of slits as above. The other two arcs are straight lines, and correspond to functions for which $\arg w'(z_1)$ attains its maximum and minimum respectively. These functions map the unit circle on the exterior of slits as above, having one finite fork. (Received March 27, 1948.)

348t. Rufus Isaacs: *Linear monodiffric difference equations.*

In the classical sense, the general solutions of many difference equations are obtained by replacing their arbitrary constants in the general solutions of their differential analogues by arbitrary span-periodic functions. If a linear monodiffric difference equation with polynomial coefficients be constructed by utilizing monodiffric multiplication, a set of solutions is obtained similarly by now replacing the arbitrary constants by doubly period functions with a "span square" as period. But aside from these, many such difference equations possess additional solutions which are defined only on the span lattice or on a sub-region of the plane. They are of a more complicated functional nature than the usual solutions and appear to have no classical analogues. (Received February 27, 1948.)

349. Rufus Isaacs: *Monodiffric multiplication.*

The theory of monodiffric functions (here of the first kind only; that is, those polygenic functions which satisfy forward difference equations of the Cauchy-Riemann type) has been handicapped in that the class of them is not closed under multiplication. The following operation appears to be the logical replacement. If $f(z)$ is monodiffric, $z \cdot f(z)$ means $xf(z-1)+iyf(z-i)$ and is monodiffric. "Multiplication" of f by any pseudo-power of z now follows by induction. By requiring distributivity, "multiplication" of f by any monodiffric polynomial has a meaning. The difference quotients of such a "product" is in accordance with the Leibniz rule. For zero span, the "product" becomes the ordinary product. When both factors are pseudo-powers the additive law of exponents is obeyed. Thus the totality of monodiffric polynomials forms a ring which is isomorphic to that of monogenic polynomials. Difference quotients and derivatives correspond under the same isomorphism. Light is thrown on the peculiar role of wedge functions by the fact that they are zero multipliers for this ring. (Received February 27, 1948.)

350. R. C. James: *Unconditional convergence in Banach spaces.*

A series $\sum_{n=1}^{\infty} x_n$ of elements of a Banach space is said to converge unconditionally if it converges for every rearrangement of terms. It is absolutely convergent if $\sum_{n=1}^{\infty}\|x_n\|$ converges. Most known examples of infinite-dimensional Banach spaces are known to have unconditionally convergent sequences of elements which are not absolutely convergent. This class of spaces is extended to include all infinite-dimensional Banach spaces which are uniformly convex, whose adjoint is uniformly convex, or which contain subspaces having a Schauder basis which is unconditionally convergent. (Received March 16, 1948.)

351. Samuel Karlin: *Bases in L^p spaces.* Preliminary report.

A basis in a Banach space E is a set of elements x_n such that each x is uniquely representable in terms of x_n ($x=\sum a_n x_n$). Nina Bary (C. R. Akad. Sci. (Doklody) URSS. vol. 54 (1946)) has introduced the following definitions: A system of elements x_n in L^2 is a Bessel system if there exists a biorthogonal system f_n to x_n with $\sum f_n^2(x)$ convergent for every x. Furthermore, whenever $\sum c_n^2<\infty$ implies that there exists an x with $f_n(x)=c_n$, then x_n is said to be a Hilbert system. A complete system x_n which is both Hilbert and Bessel constitutes a Riesz basis. A system (x_n, f_n) is said to be bounded if $\|x_n\|\leqq c$ and $\|f_n\|\leqq c$. It is shown in this investigation that a bounded system $(x_n f_n)$ is a Riesz basis if and only if x_n is an absolute basis. (If $\sum a_n x_n$ converges, then $\sum a_{n(q)}x_{n(q)}$ converges for every rearrangement.) In addition, if x_n is a basis and (x_n, f_n) is bounded, then x_n is Bessel if and only if x_n is Hilbert. Let $y_n=\sum a_{nk}x_k$ denote a transformation of a Riesz basis. General conditions are given when y_n remains Reisz. In particular, if (a_{nk}) is a Koch determinant then y_n is Riesz if and only if x_n is Riesz. The above definitions are generalized to L^p spaces. A system x_n in L^p is said to be a Y system if $\sum|f(x_n)|^{p'}$ converges. If $\sum|c_n|^{p'}<\infty$ implies that there exists an x such that $f_n(x)=c_n$, then x_n is said to constitute an (H) system. The cases $p=1, \infty$ have the usual interpretations. Similar results as for the case L^2 are obtained for L^p. (Received March 13, 1948.)

352. H. L. Meyer: *Multiple integral problems in the calculus of variations.* II.

In this paper, general sufficiency theorems established by the author for

$\mathcal{P}_{m,n}$ ($m>1$, $n>1$) problems in the calculus of variations are specialized to the $\mathcal{P}_{2,n}$ and $\mathcal{P}_{m,2}$ cases. Weak relative minimum theorems are established under weaker hypotheses: it is only required that the usual Legendre inequality $Q(\pi)>0$ hold for matrices π of rank 1. This hypothesis is the natural one in light of the necessary condition $Q(\pi)\geqq 0$, π of rank 1, for a minimizing surface. The proofs are made by eliminating the rank 1 condition on π through the adjunction of appropriately constructed invariant integrals (cf. Hestenes and McShane, Trans. Amer. Math. Soc. vol. 47 (1940) pp. 501–512), and then applying the previous general theorems. (Received March 12, 1948.)

353. R. A. Rosenbaum: *Sub-additive functions.*

This paper contains a generalization to Euclidean space of n dimensions of the results of Hille (Amer. Math. Soc. Colloquium Publications, vol. 31) on sub-additive functions in 1 dimension. A real-valued function $f(p)$ of a point in E_n is termed "sub-additive" if $f(p+p')\leqq f(p)+f(p')$ for all points p, p' in the domain of definition. Among the topics considered are: the construction of sub-additive functions and their relationships to other easily defined classes of functions, among which the convex and concave functions play a large role; infinitary sub-additive functions; boundedness and rate of growth, continuity and differentiability of sub-additive functions; generalizations of the notion of sub-additivity. (Received March 13, 1948.)

354. M. M. Schiffer and Gabor Szegö: *Virtual mass and polarization.*

Let A be a given solid, λ a unit vector. There are defined two harmonic functions g and h both of order r^{-2} at infinity and satisfying the conditions $-\partial g/\partial n=n\lambda$, $h=r\lambda$ on the boundary of A. Their gradient integrals over the exterior of A are two quadratic forms W and P in the projections of λ which represent the *virtual mass* and the *polarization* in the direction λ. The present investigation deals with obtaining bounds for the quantities W and P. Among other inequalities the following is proved: $WP\geqq V^2$ where V is the volume of the given solid A. Moreover it is shown that the quantities $W+V$ and $P+V$ are monotonically increasing set-functions. (Received April 6, 1948.)

355t. Gabor Szegö: *On the virtual mass of nearly spherical solids.*

Let $W=\sum W_{ik}h_ih_k$ (i, $k=1$, 2, 3) be the virtual mass of a solid immersed in an ideal fluid of density 1 which is at rest at infinity. The solid is moving with uniform speed in the direction h_1, h_2, h_3. In a recent paper (Proc. Nat. Acad. Sci. U.S.A. vol. 33 (1947) pp. 218–221) Professor Pólya dealt with the "average virtual mass W_m defined by $3W_m=W_{11}+W_{22}+W_{33}$ in case of a two-dimensional flow. (The quantity W_m is independent of the coordinate system.) There are numerous evidences that for a wide class of solids $W_m\geqq W_m^0$ holds where W_m^0 is the virtual mass of a *sphere* which has the same volume as the given solid. The purpose of the present investigation is to confirm this inequality for "nearly spherical" solids. The principal result can be formulated as follows. Let $r=1+\rho(\theta,\phi)=1+\sum_{n=0}^{\infty}X_n(\theta,\phi)$ be the equation of the boundary of the solid in polar coordinates, X_n a surface harmonic of degree n with coefficients of order δ. Neglecting terms higher than the second order in δ we have $W_m=2\pi R^3/3$ where $R=1+X_0+\sum_{n=1}^{\infty}\{1+3(n-1)(2n-1)[2(2n+1)]^{-1}\}(4\pi)^{-1}\cdot\int\int X_n^2 d\omega$. (Received March 27, 1948.)

Applied Mathematics

356. H. W. Becker: *Circuit algebraic efficiencies.*

A measure of the efficiency of an algebraic derivation is $F=T/(KT+C)$, where T is the number of terms remaining in the formula, after all cancellations. So a result written directly by inspection, or "mental algebra," would be obtained at 100% F. In formulating a complicated circuit function by (1) a chain of $Y\Delta$ transformations, or (2) a mesh determinant, most terms cancel by (1) division, or (2) subtraction. As the number of meshes increases, the F of these methods therefore tends to zero. For example, in the diamond-cross or Shannon bridge $Z=(a+{}_1A)\|(s+{}_{12}S)\|(b+{}_2B)\perp\alpha\perp\beta=D/N$, $D=\sum 30$ terms obtained at (1) $30/270=11\%$ F, (2) $30/330=9\%$ F. In contrast, the method of fracto components operates at 100% F, even with mutual inductances present, whenever the components are as above self-evident to the operator; thus it is the natural mechanism for amassing an inventory of all possible circuit functions of a given number of branch impedances. The advantage is in transferring jurisdiction of the problem from the theory of equations to the theory of partitions. (Received March 13, 1948.)

357*t*. H. W. Becker: *Semicircuits and semidentities.* Preliminary report.

The most compact storage of all passive circuit functions $Y=N/D$, in loose leaves, punched cards, and so on, is in tabulation of all semicircuit functions N and D, coded for reference from the drawn Y atlas. The economy is in the numerous semidentities $Y==Y'=N'/D'$, where $N=N'$ or $D=D'$, or $N=D'$. The switching functions $Y\rightarrow Y'$ are called semidentical transformations, s.t. The N s.t. switches one or both source leads to other junctions. The D s.t. repartitions the branches at the source terminals, or switches the source leads between any bi-partition of branches at any other junction. The ND and DN s.t. effect various combinations of N and D s.t. The four s.t. offer the readiest explanation of the cross reciprocities in $Y_s=(rN_\infty+N_0)/(rD_\infty+D_0)\rightarrow=(rN_\infty+D_\infty)/(rN_0+D_0)\leftarrow$, the terms of the latter quotient pertaining to admittances of the circuits looking back from a receiver r, with source s opened and shorted. The semicircuits may be drawn, or put in the notation of graphs, or of a logic with four conjunctions: X, $+$, $\|$, and $\perp$. (Received February 26, 1948.)

358*t*. H. W. Becker: *The duals of nonplanar networks.*

The dual of a pc (passive circuit) $Z=\sum z_1\cdots z_n/\sum z_1\cdots z_{n-1}=\theta_0/\theta_\infty$ is ${}^{-}Z=\sum(z_1\cdots z_{n-1})^{-1}/\sum(z_1\cdots z_n)^{-1}={}^{-}\theta_\infty/{}^{-}\theta_0$, where θ_0 and θ_∞ are combinatorially the characteristic functions of the semicircuits formed by shorting and opening the Z terminals, and topologically the semicircuits themselves. If Z is nonplanar, ${}^{-}Z$ is an imaginary circuit, which nevertheless has real fracto component circuits and physical $Y\Delta$ and ideal transformer equivalent circuits. Also, if Z is (1) planar, (2) global, (3) infraglobal, (4) five to infinite dimensional, then the semiduals ${}^{-}\theta_\infty$ and ${}^{-}\theta_0$ of Z are (1) planar and mutually realizable, (2) one or both planar but source terminal indeterminates, (3) one or both imaginary, (4) both imaginary. The active circuit θ of Z, and its dual ${}^{-}\theta$, are computed from ${}^{\pm}\theta=s\cdot{}^{\pm}\theta_\infty+{}^{\pm}\theta_0$, s the source impedance. The proportion of nonplanar to all pc is 1/717 for 8 branches, about 1/30 for 12 branches, and tends ultimately to a majority. All nonplanar pc are reducible to the abstract multivibrator or Bloch bridge $(a+{}_1f+{}_2b)\|(A+{}_2F+{}_1B)\perp\phi\perp\Phi$, whose imaginary dual has an equivalent circuit at the terminals and in 5 branches of the

Kelvin bridge $(a+{}_1A)\|\{\phi'+(f+{}_1\Phi)\|b+F'\}\perp B'$, with $F'=FB/S$, $B'=F\phi/S$, $\phi'=\phi B/S$, $S=F+B+\phi$. (Received February 27, 1948.)

359. Edmund Pinney: *Nonlinear external resonance.*

A simple nonlinear oscillatory system subjected to an external harmonic force is studied in its resonance states. Typical systems are the electrical system consisting of a linear inductance, nonlinear resistance, and nonlinear capacitance in series with a sinusoidal generator, and the mechanical system consisting of a mass suspended by a nonlinear spring, acted upon by a nonlinear frictional resistance, and subjected to a nonlinear exciting force. Both systems lead to a differential equation of the form $\ddot{x}+f(\dot{x})+g(x)=a+b \sin(\alpha t)$. In addition to the fundamental oscillation, the nonlinearity of the system induces harmonics and subharmonics—oscillations whose frequencies are rational fractions of the fundamental frequency. The present paper develops the general machinery for studying these oscillations, with their attendant phenomena of hard and soft frequency entrainment, asynchronous excitation and quenching. The particular cases $f(\dot{x})=0$ and $g(x)=0$ are treated in greater detail. (Received March 12, 1948.)

360. S. A. Schaaf: *The problem of nozzle design for low pressure supersonic wind tunnels.*

It is shown that the effect of very low pressures in a gas flow is to make viscosity effects of equal importance to compressibility effects, thus invalidating the characteristics method of nozzle design. A generalized Polhausen-Tsien approximate method is developed for calculation of the two-dimensional viscous compressible isothermal flow, and its application to nozzle design is discussed. This work was done under an Office of Naval Research contract. (Received March 6, 1948.)

Geometry

361. Alfred Horn and F. A. Valentine: *Some properties of L sets in the plane.*

A set S is an L set if any two points of S can be joined by an at most two-sided polygon lying in S. If S is a bounded closed L set in the plane, every bounded component of its complement $C(S)$ is an L set. Furthermore through each point of the unbounded component T of $C(S)$ there is an infinite ray lying in T. Hence any two points of T can be joined by a three-sided polygon contained in T. If $C(S)$ has no bounded components, S may be expressed as a sum of convex sets every two of which have a point in common. This statement is proved using the theorem: Given any set of closed intervals on the boundary of a circle such that any two have a common point, then there exists a pair of diametrically opposite points p, q such that every interval of the family contains at least one of the points p, q. Other properties of L sets are investigated. An example of an L set is the set of points of a plane convex body through which there pass two or more lines bisecting the area. (Received March 12, 1948.)

Logic and Foundations

362t. J. D. Swift: *On the structure of n-valued logics.*

The set of $n^{(n^m)}$ possible m-ary closed operations on n elements (m-ary n-valued propositional functions) is considered, particular attention being given to the case

$m=2$. The result of Zylinski on the uniqueness of the generating stroke operations for $n=2$ is not true for $n>2$. General classes of such stroke functions are obtained. The operations under which the set is a given algebraic variety are considered by the direct sum technique, and theorems on the number and type of rings, groups, and lattices which are formed by the members and selected operations of the set are obtained. The modular theory of Bernstein is generalized for the case $n=p^t$. (Received March 13, 1948.)

Topology

363. P. A. White: *On a certain class of set theoretic properties.*

A study is made of those properties of sets that satisfy the following theorem: If $P\subset S$, both have a property p, and $S-P=X\cup Y$, separate, then $X\cup P$ and $Y\cup P$ have the property p. For example, connectedness satisfies this theorem. A study is also made of the conditions under which a local property satisfies this theorem if the corresponding property in the large satisfies the theorem (for example, local connectedness satisfies it since connectedness does). A total of 53 properties are shown to satisfy the theorem including arcwise and cyclic connectedness, simple-i-connectedness, and the property of being an open generalized-n-manifold. (Received October 30, 1947.)

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