ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

Algebra and Theory of Numbers

1. A. A. Albert: *A structure theory for Jordan algebras.*

This paper gives the first general structure theory for Jordan algebras, that is, commutative algebras in which the identity $x(yx^2) = (xy)x^2$ holds. Using identities on right multiplications it is shown that every solvable algebra over a field of characteristic not two is nilpotent in the sense that there exists an integer k such that every product of k elements is zero. Also an algebra whose quantities are all nilpotent is necessarily solvable. For algebras over a nonmodular field a trace criterion is developed yielding a decomposition relative to an idempotent, and it is shown that if N is the maximal solvable ideal of A then $A - N$ is a direct sum of simple algebras. Finally, it is shown that every simple Jordan algebra is isomorphic to a Jordan algebra of linear transformations except for a certain class of algebras of order 27 over the center. (Received October 10, 1946.)

2. R. F. Arens: *Representation of rings in which $x^p = x$.*

McCoy and Montgomery, Duke Math. J. vol. 3 (1937), have partially generalized the representation of Boolean algebras of Stone, Trans. Amer. Math. Soc. vol. 40 (1936), by showing that a ring A in which $x^p = x$ and $px = 0$ is a subdirect sum of the corresponding Galois field. In the present paper, the author follows Stone in making this representation sharper and unique, in a certain sense, by considering the locally compact 0-dimensional structure space of A. The case of rings in which $x^{p^n} = x$ is also considered. (Received November 19, 1946.)

3. Reinhold Baer: *Direct decompositions.*

In this paper a comprehensive refinement theorem for direct decompositions of operator loops is proved; and this theorem is shown to contain as special cases the theorems of W. Krull, O. Schmidt, V. Kořínek and A. Kurosh. (Received October 14, 1946.)

4. Reinhold Baer: *Endomorphism rings of operator loops.*

If L is an operator loop, and if A is a commutative and associative admissible subloop of L, then the set θ of all the endomorphisms of L which map L into A is a ring with respect to the customary operations of addition and multiplications. In this note the structure of θ is investigated, mainly under the hypothesis that all the endomorphisms in θ split L. (Received October 7, 1946.)

5. Reinhold Baer: *Splitting endomorphisms.*

If t is an endomorphism of the operator loop L, then its radical $R(t)$ consists of all the elements x in L such that $0=xt^n$ for some integer $n=n(x)$. The radical is a normal and admissible subloop. A complement of the endomorphism t is an admissible subloop C such that every coset of $L/R(t)$ contains one and only one element in C and such that $C=Ct$. The object of this note is to find criteria for the existence of complements. (Received October 7, 1946.)

6. Reinhold Baer: *The double chain condition in cyclic operator groups.*

If A is a cyclic abelian operator group, then it is possible that the descending, but not the ascending, chain condition is satisfied by the admissible subgroups of A. It is therefore desirable to obtain criteria for the validity of the double chain condition in a cyclic abelian operator group; and in the present note there are offered several criteria of this type. (Received November 16, 1946.)

7. J. D. Bankier: *Extended regular continued fractions.*

An *extended* regular continued fraction (e.r.c.f.) is defined recursively by the relations $y_n=\epsilon_n/(y_{n-1}-b_{n-1})$ $(n=1, 2, 3, \cdots)$, where $\epsilon_n=\pm 1$, $\epsilon_n(y_{n-1}-b_{n-1})>0$ and b_n is the integer further from y_n of the integers $[y_n]$, $[y_n]+1$, providing such an integer exists. If $y_n=I+1/2$, where I is an integer, $b_n=I+1$, if $y_n=2$, $b_n=1$, and, if $y_n=1$, $b_n=1$. A necessary and sufficient condition that an e.r.c.f. terminate is that y_0 be rational. A nonterminating e.r.c.f. converges to the generating value y_0. Necessary and sufficient conditions that a nonterminating continued fraction be an e.r.c.f. are that its elements be integers such that $b_n=1$ or 2, if $b_n=1$, $\epsilon_{n+1}=1$, if $b_n=2$, $\epsilon_{n+1}=-1$ $(n=1, 2, 3, \cdots)$, and that, corresponding to every positive integer n, there exists an integer m such that $n<m$ and $b_m=1$. For a continued fraction terminating with b_k, the additional condition is required that $b_{k-1}=b_k=-\epsilon_{k-1}=\epsilon_k=1$. An e.r.c.f. is periodic if and only if it converges to a quadratic irrational. A method is given for obtaining the e.r.c.f. expansion of a real number directly from the corresponding r.c.f. The convergents of an e.r.c.f. are shown to be an arrangement of the primary and secondary convergents of the corresponding r.c.f. The e.r.c.f. enables one to find solutions for a larger class of Pell equations than the r.c.f. (Received November 19, 1946.)

8. B. A. Bernstein: *Weak definitions of field.*

The author obtains from a typical addition-multiplication definition of field two weaker definitions. In one of these definitions the distributive law is replaced by a law involving only two variables. In the other definition each of the additive laws of the type definition is replaced by a law involving fewer variables, and the distributive law is entirely dispensed with. The definitions are postulational, with independence among the postulates established. (Received October 23, 1946.)

9. Garrett Birkhoff: *Notes on lattices.*

The congruence relations on any abstract algebra A with permutable congruence relations form a modular lattice; hence the Kurosch-Ore theorem applies. If A also contains a one-element subalgebra, then the unique factorization theorem of Wedderburn-Remak, and all Jordan-Hölder-Schreier-Zassenhaus theorems hold. An abstract lattice L is isomorphic with the lattice of all subalgebras of a suitable abstract algebra if and only if (i) L is complete, (ii) in L, the meet operation is continuous in the order topology, (iii) every element a of L is the join of elements a_α, no one of which is the topological limit of smaller elements. Any complete metric lattice is a topological lattice. (Received October 25, 1946.)

10. Garrett Birkhoff: *Subdirect representation of certain algebras.*

Let A be any algebra with finitary operations, which has a finite number of non-isomorphic subdirectly irreducible homomorphic images $F_1, \cdots, F_n$. Suppose also that each F_i is finite. Then A can be represented as a totally disconnected topological algebra of continuous functions from a totally disconnected bicompact "space" (of homomorphisms) onto the union F of the F_i. (Received October 31, 1946.)

11. R. H. Bruck: *An extension theory for Moufang loops.*

For terminology, see the author's recent paper (Trans. Amer. Math. Soc. vol. 60 (1946) pp. 245–354). In the present note he treats the following problem: Given a Moufang loop M and a group G, to determine all Moufang loops E homomorphic to M with kernel G contained in the associator of E. In sharp contrast with many other extension problems for loops, there emerges a theory of factor sets closely akin to that for groups. If M has finite order m and f is a central Moufang factor set, $f^n \sim 1$ where $n = 2m^2$ (*not* m, as in the group case). The powers f^m are nontrivial factor sets of a particularly simple form, and these are thoroughly investigated. When M is abelian, each f^{2m} is a group factor set. When M is a noncommutative group the situation is more complex, but a variety of relations is obtained between central group factor sets and central Moufang factor sets. (Received November 18, 1946.)

12. R. P. Dilworth: *Decomposition of relatively complemented lattices.*

The following theorem is proved: *Every indecomposable, relatively complemented lattice of finite dimensions is simple.* (Received October 17, 1946.)

13. J. S. Frame: *Group decomposition by double coset matrices.*

A formula $n_i = g/\sum h_\alpha^{st} |\rho_{\alpha i}^{st}|^2$ is obtained for the degrees n_i of the common irreducible components of two isomorphic (but not necessarily distinct) representations $R^s(\gamma)$ and $R^t(\gamma)$ of a finite group G of order g by transitive groups of permutation matrices of degree n^s and n^t respectively. Matrices V_α^{ts} consisting of 0's and 1's, and such that $R^t(\gamma) V_\alpha^{ts} = V_\alpha^{ts} R^s(\gamma)$, correspond to the double cosets $H_\alpha^{st} = H^s \gamma_\alpha H^t / h_\alpha^{st}$. The orders of $\gamma_\alpha^{-1} H^s \gamma_\alpha$, H^t, and their intersection are respectively h^s, h^t, and h_α^{st}. The quantities $\rho_{\alpha i}^{st}$ appear as coefficients in the transforms of V_α^{ts} by the same matrix U which reduces the direct sum of R^s and R^t. In general $\rho_{\alpha 1}^{st} = (h^s h^t)^{1/2} / h_\alpha^{st}$, and it seems that for a suitable reducing transformation U all the $\rho_{\alpha i}^{st}$ will be algebraic integers. In certain important cases the $\rho_{\alpha i}^{tt}$ are integral factors of h^t / h_α^{tt}, or zero, and the degrees n_i are easily found. The irreducible group characters χ_λ^i for the class C_λ are $\chi_\lambda^i = n_i (\sum \gamma h_\alpha^{st} \rho_{\alpha i}^{st} k_{\alpha\lambda}^{st}) / (h_1^{st} \rho_{1i}^{st} g_\lambda)$, where $k_{\alpha\lambda}^{st}$ is the number of the g_λ elements of the class C_λ which are in the double coset H_α^{st}. (Received November 25, 1946.)

14. Wallace Givens: *Parametric solution of linear homogeneous Diophantine equations.*

A system of r linearly independent linear homogeneous equations in n unknowns with rational integers as coefficients is considered. Every solution of such a system is proved to be obtainable by giving integral values to the parameters $p_{\rho j}$ ($\rho=1, \cdots, n-r-1; j=1, \cdots, n$) and k (which may be set equal to one if $n>r+1$) in the formulas $x_i=kF_i(p_{\rho j})$, where the F_i are homogeneous polynomials with integral coefficients which are given explicitly in terms of the coefficients of the given equations. With k allowed to be rational instead of integral, this theorem was recently proved by L. W. Griffiths (Bull. Amer. Math. Soc. vol. 52 (1946) pp. 734–736). A new proof of Miss Griffiths' result is first given and then the stronger theorem is reduced to the special case $r=0$ where it is equivalent to a theorem due to Hermite. (Received November 22, 1946.)

15. L. K. Hua: *Geometries of matrices.* II. *Study of involutions in the geometry of symmetric matrices.*

In the first part of the paper, the explicit normal forms of involutions and anti-involutions and the manifold of fixed points of all sorts of involutions are determined. Secondly, the author enumerates the geometries keeping an involution or an anti-involution as absolute. Those obtained from anti-involutions are generalizations of non-Euclidean geometries, among which the geometry of the hyperbolic type is Siegel's symplectic geometry. Those obtained from involutions give several new types of geometries, one whose real analog is a generalization of the Möbius geometry of circles. The author shows that every symplectic transformation is a product of *two* involutions and *four* anti-involutions and that in the space of symmetric matrices of order $n=2^\sigma\tau$, τ being odd, are at most $\sigma+3$ pairs of points of which any two pairs separate each other harmonically. As a by-product, the author proved also that if $S_1, \cdots, S_s$ are n-rowed symmetric matrices satisfying $s_i^2=-I$, $s_is_j=-s_js_i$, then $s\leqq\sigma+1$. (Received October 23, 1946.)

16. L. K. Hua: *On the automorphisms of the symplectic group over any field.* Preliminary report.

Let Φ be any field with characteristic not equal to 2 and containing more than three elements. Let F be a $2n$-rowed nonsingular skew-symmetric matrix, say one with first row 0, I and second row $-I$, 0. Let Γ be the group of $2n$-rowed symplectic matrices, that is, the group is constituted by all those elements P satisfying $PFP'=F$, where P' denotes the transposed matrix of P. In this paper, all automorphisms of the group Γ are determined. More precisely, all the automorphisms $A(X)$ on Γ are of the form $R^{-1}P^{-1}X^\sigma PR$, where P runs over all symplectic matrices, σ runs over all the automorphisms of the field, R is a matrix with first row I, 0 and second row 0, aI, and a runs over a complete residue system of the factor group Φ/Φ_2. Φ is considered as a multiplicative group and Φ_2 is the multiplicative group of all the square elements of Φ. (Received November 20, 1946.)

17. Ralph Hull: *Generators of the rational integral symplectic groups.*

Elementary methods are employed to determine a finite set of generating elements of the group of $2n$ by $2n$ symplectic matrices with rational integral elements. The methods may also be applied to the case of symplectic groups over an algebraic number field in which there is a division algorithm. (Received October 24, 1946.)

18. Irving Kaplansky: *Semi-simple algebraic algebras.*

Representation theorems for a commutative semi-simple algebraic algebra A over a field F are given, analogous to Stone's topological representation of Boolean rings. If F is algebraically closed, A consists of all continuous functions from a compact totally disconnected space to F. In the general case the functions may take values in the various subfields of the algebraic closure of F. If F is complete in a valuation, and A is normed and complete, then the elements of A have bounded degrees. Here A can be any algebraic algebra whose radical is the set of all nilpotent elements. This generalizes a theorem announced by Mazur. (Received November 21, 1946.)

19. B. E. Meserve: *Division sequences by cross-multiplication.*

A synthetic method is given for obtaining the division sequences of any two polynomials in one unknown with real coefficients. It is a generalization of the process of cross multiplication used by E. J. Routh (*Advanced rigid dynamics*, 6th ed., 1905, p. 226) to obtain conditions for the stability of a linear dynamical system. Special applications of the method give the Sturm sequences of a single polynomial in one unknown with real coefficients and a highest common factor of any two such polynomials. (Received October 28, 1946.)

20. L. J. Paige: *A note on finite abelian groups.*

Let $[X_\rho, A_\rho, B_\rho]$ be an ordered triple of ordered subsets $X_\rho=(x_1, \cdots, x_\rho)$, $A_\rho=(a_1, \cdots, a_\rho)$, $B_\rho=(b_1, \cdots, b_\rho)$ of an abelian group G of order N. Defining a triple to be admissible if and only if X_ρ, A_ρ, B_ρ each contain ρ distinct elements and $x_i a_i = b_i$ $(i=1, 2, \cdots, \rho)$, the author proves that an admissible triple $[X_N, A_N, B_N]$ exists if and only if G does not contain a unique element of order 2. For all G there exist admissible triples $[X_{N-1}, A_{N-1}, B_{N-1}]$. (Received October 11, 1946.)

21. H. J. Ryser: *Rational vector spaces.* Preliminary report.

Let V_n be a rational vector space of dimension n admitting rational inner product (α, β) (Bull. Amer. Math. Soc. Abstract 52-9-267). V_n is called an I-space provided it has basis $\delta_1, \delta_2, \cdots, \delta_n$ such that $(\delta_i, \delta_j)=\delta_{ij}$. If $\xi_1, \xi_2, \cdots, \xi_n$ are n linearly independent elements of V_n, the square free integer part d of the determinant of the matrix $[(\xi_i, \xi_j)]$ is an invariant of the space. If V_n has inner product (α, β) and invariant d and V_m inner product (γ, δ) and invariant D, then a direct product space $V_n \otimes V_m$ of dimension nm is defined in which $(\alpha, \beta)(\gamma, \delta)$ acts as inner product. If $n=2r$ and $m=2s$, a necessary and sufficient condition that $V_n \otimes V_m$ be an I-space relative to $(\alpha, \beta)(\gamma, \delta)$ is that the product of Hilbert symbols $(d, d)_p^s (D, D)_p^r (d, D)_p = +1$ for $p=2$ and for each of the odd prime divisors of dD. Similar results hold for n odd, and both n and m odd. Algebraic number fields are studied as rational vector spaces, and an explicit determination is made of all cyclotomic I-fields, all cyclic I-fields of even degree, and all I-fields defined by the e periods of the pth roots of unity. (Received November 4, 1946.)

22. R. M. Thrall: *On ahdir algebras.* Preliminary report.

If an algebra A over a field k possesses indecomposable representations of arbitrarily high degree it will be called an *ahdir* algebra. Assume k algebraically closed. R. Brauer (*On the indecomposable representation of algebras*, Bull. Amer. Math. Soc. Abstract 47-9-334) has given three conditions each sufficient to ensure that A is ahdir. These conditions are stated in terms of the Cartan invariants of A, A/N, $A/N^2, \cdots$ where N is the radical of A. In the present work a fourth sufficient condition of the same general character is given and conversely it is proved that if $N^2=0$ and none of the four sufficient conditions hold, then A is not ahdir. In the case $N^2=0$ the four sufficient conditions can be stated in the following manner. Suppose that A/N has t simple ideals. Then the Cartan invariants are the elements of a t by t matrix C with integer elements. Set $D=C-I$. Associate a graph G of $2t$ vertices $P_1, \cdots, P_t \cdot Q_1, \cdots, Q_t$ with D by joining P_i and Q_j if and only if $d_{ij}\neq 0$. The four sufficient conditions are then (1) some $d_{ij}>1$; (2) G is not a tree; (3) some vertex of G is of order greater than 3; (4) some connected subgraph of G has more than one vertex of order 3. (Received November 21, 1946.)

23. Bernard Vinograde. *Radicals associated with equivalent semi-simple residue systems.*

This paper investigates rings wherein the radical is a homomorphic additive image of the semi-simple part and satisfies $f(xy)=xf(y)+f(x)y+f(x)f(y)$, where f is the homomorphism. $f(xy)$ affords a trioperational approach. This is an aspect of the distribution of residue systems in a semi-primary ring. (Received October 24, 1946.)

24. Daniel Zelinsky: *Nonassociative valuations.*

An ordered quasigroup G is a quasigroup, written additively, which is linearly ordered by a transitive, binary relation $>$, having the property that $x>y$ implies $x+z>y+z$ and $z+x>z+y$ for all x, y, z of G. A valuation, V, of a (nonassociative) ring R is a function on R to an ordered quasigroup with ∞ adjoined such that for all a, b of R, $V(a+b)\geqq \min [V(a), V(b)]$, $V(ab)=V(a)+V(b)$, $V(a)=\infty$ if and only if $a=0$. The principal theorem of this paper is the following: If R is an algebra of finite order over a field F, if R has a unity quantity and if $V(F)$ is an archimedean-ordered group, then $V(R)$ is an archimedean-ordered abelian group in which $V(F)$ has finite index. Examples of nonassociative ordered loops are obtained by simple loop extensions. The existence of a ring with arbitrary prescribed value loop and residue-class ring (without zero divisors) is proved. From these two facts follow examples showing that the hypothesis "$V(F)$ is archimedean-ordered" cannot be omitted in the theorem above. This is in strong contrast with the associative, noncommutative case. (See O. F. G. Schilling, *Noncommutative valuations*, Bull. Amer. Math. Soc. vol. 51 (1945) pp. 297–304.) (Received November 13, 1946.)

ANALYSIS

25. H. W. Becker: *Generalizations of the Epstein-Fourier series.*

These series are combinations of exponential and Fourier series (Leo F. Epstein, Journal of Mathematics and Physics vol. 18 (1939) p. 60, (19)). Where $K_0=1$, $x=r\cos\theta$, $y=r\sin\theta$, and "soc" means "sine or cosine," some generalizations are: (1) $\exp[X+r \operatorname{soc}(\ )\theta\cdot(KX+Z)]=[\operatorname{soc}(Zy+e^{Xx}\sin Xy)]\cdot[\exp(Zx+e^{Xx}\cos Xy)]$, the $(KX+Z)$ except for change of sign of X being the polynomials of Steffensen (*Some recent researches in the theory of statistics and actuarial science*, Cambridge Press, 1930, p. 24); (2) $\exp[e\{1+r\operatorname{soc}(\ )\theta\cdot K^{(2)}\}]=[\operatorname{soc}\{e^{e^x\cos y}\sin(e^x\sin y)\}]\cdot[\exp\{e^{e^x\cos y}\cos(e^x\sin y)\}]$, the $K^{(2)}$ being Bell numbers (Ann. of Math. vol. 39 (1938) p. 539). Under the substitutions $r\to rX$, $X\to X^{-1}$, $Z=0$, (1) becomes (3) $\exp[X^{-1}+r\operatorname{soc}(\ )\theta\cdot T]=[\operatorname{soc}(e^x\sin y)]\cdot[\exp(e^x\cos y)]$, where $T=XKX^{-1}$ is an umbral transform of Riordan and Kaplansky (*The problem of the rooks and its applications*). It is noteworthy that the right side of (3) is free of X, endowing the left side with a kind of invariance. (Received October 1, 1946.)