case of the method of elimination; (2) the characteristic vectors of a companion matrix with simple roots are given by a Vandermonde matrix and its easily derived inverse; (3) the characteristic vectors of any matrix can therefore be derived with about  $2n^3$  multiplications all told, by applying the Danielewsky transformations to the vectors of the corresponding companion matrix. (Received February 1, 1946.)

## 86. P. A. Samuelson: Generalization of the Laplace transform for difference equations.

The Laplace transformation has standardized operational methods in the field of ordinary differential equations. Its efficacy hinges on the fundamental relation  $L(s; Df)_D = sL(s; f)_D - f(0)$  where  $L(s; f)_D = \int_0^\infty \exp{(-st)} f(t) dt$ . The Laplace transform has been applied to difference equations, but it is a clumsy tool there by virtue of the fact that it does *not* satisfy a similar fundamental relation with respect to the shifting operator E. One can easily verify that the linear functional  $L(s; f)_E = \sum_0^\infty f(i)s^{-i-1}$  does have the fundamental property  $L(s; Ef)_E = sL(s; f)_E - f(0)$ . This generalized transform can also be easily inverted by the calculus of residues and extended by suitably defined "convolution." Consequently, after a table of "generalized" transform pairs has been drawn up, the solution of ordinary difference equations can be derived by operational methods exactly like those of differential equations. The most important of these transform pairs is  $y(t) = t(t-1), \cdots, (t-n+1)a^{t-n}$  and  $\bar{y}(s) = (s-a)^{-n}(n-1)!, |s| > |a|$ . (Received February 1, 1946.)

### 87. C. A. Truesdell: On Sokolovsky's "momentless shells."

V. V. Sokolovsky (Applied Mathematics and Mechanics n.s. vol. 1 (1937) pp. 291–306) has given expressions for the membrane stress resultant Fourier coefficients for surfaces of revolution whose meridians may be expressed in Cartesian coordinates in the forms:  $f = kz^{\mu}$ ;  $f = a \sin^{\rho} \phi$ ,  $z = -ca \int \sin^{\rho} \phi d\phi$ ;  $f = a \sec^{\rho} \phi$ ,  $z = -ca \int \sec^{\rho} \phi \tan^{\rho} \phi d\phi$ . The first family has already been treated and generalized by the author. In the present note the author shows that a slight modification of his previous treatment of Nemenyi's stress functions enables us quickly to find solutions in terms of hypergeometric functions for the family of surfaces whose meridian is  $f = a \sin^{\rho} \xi$ ,  $z = -pb \int \sin^{\rho} \xi \tan^{\rho} \xi d\xi$ , including Sokolovsky's second and third families of surfaces as special cases. Surfaces having meridians given by an error integral curve,  $z = ap! \int_{0}^{k_f} \exp(-t^{\rho}) dt$ , are shown by the same means to have solutions in terms of Whittaker functions. (Received January 29, 1946.)

## 88. Alexander Weinstein: On Stokes' stream function and Weber's discontinuous integral.

It is shown that the stream function  $\psi$  corresponding to sources distributed with the density one over a circumference C is a many-valued function with the period  $4\pi a$ , where a denotes the radius of C. This fact, combined with the divergence theorem, yields a new proof for Weber's formula (J. Reine Angew. Math. vol. 75 (1873) p. 80) for the discontinuous integral  $\int_0^\infty J_0(as) J_1(bs) ds$ , which is equal to 1/b for b>a, and to 0 for a>b. (Received January 17, 1946.)

#### GEOMETRY

### 89. Reinhold Baer: Polarities in finite projective planes.

It is shown that every polarity in a finite projective plane possesses at least as

many absolute points (points lying on their polars) as there are points on a line. This minimum may be attained. If there are more than the minimum number of absolute points, then the number of points on a line reduced by one is a square. More detailed information is available for regular polarities which have the property that any two lines which are not absolute, but carry absolute points, carry the same number of absolute points. These results are applied to prove a generalization of a theorem due to Topel which asserts that every geometry of Bolyai-Lobachevskii is infinite. (January 18, 1946.)

# 90. T. C. Doyle: Tensor theory of invariants for the projective differential geometry of a ruled surface.

The differential equations of Wilczynski defining a ruled surface to within a projective transformation are expressed in the tensor form  $y_i ... = (U^2 U_i r + \rho \delta_i r) y_r$ , and from the tensor coefficients and arguments of these equations there is derived by formal tensor processes the same complete system of invariants and covariants of a ruled surface as first derived by Wilczynski, using integrational methods, in his *Projective differential geometry of curves and surfaces*, Leipzig, Teubner, 1906. One arrives at a systematic procedure for the transition from canonical to general forms and in this way the invariant equations of many of the covariant loci heretofore known only in their canonical forms are displayed. (Received December 17, 1945.)

### 91. H. W. Eves: Arc chains and arc necklaces. Preliminary report.

An arc chain is a sequence of circular arcs (called links), of arbitrary central angles and radii, trailed end to end. If the end points of the chain coincide one has an arc necklace. This paper deals with the elementary geometry of plane and spherical arc chains and necklaces. In addition to a number of new theorems, several well known results of elementary geometry are generalized to hold for arc chains and necklaces. Of particular interest are those arc chains and necklaces in which all the links lie on the circumferences of a two-parameter family of circles, for example, on the circumferences of a family of concurrent circles. Some attention is paid to arc necklaces whose vertices are concyclic. (Received January 27, 1946.)

### 92. D. P. Ling: Geodesics on surfaces of revolution.

The author investigates the number and the distribution of double points of the geodesics on the members of a broad class of surfaces of revolution. A "zoning" of these surfaces is established in a manner dictated by this distribution. It is shown that each surface falls into one or another of three subclasses according as each geodesic has infinitely many double points, a finite number of double points bounded from above for the whole set of geodesics, or a finite number which by a proper choice of the geodesic can be made arbitrarily large. Analytic means of distinguishing between these subclasses is set up, and a particular class of surfaces is given to serve as illustration and counter example. (Received December 3, 1945.)

#### LOGIC AND FOUNDATIONS

# 93. G. D. Birkhoff and Garrett Birkhoff: Distributive postulates for systems like Boolean algebras.

By slightly strengthening Newman's postulates for direct sums of Boolean algebras and Boolean rings, a simpler proof of sufficiency is obtained. A related set of postulates for distributive lattices is given, together with a discussion of alternative