

65. Harry Pollard: *A theory of the Laplace integral*. Preliminary report.

It is easily verified that under simple conditions on $f(x)$ the formula $f(x) = \int_0^{k/t} (1 - xt/k)^{k-1} L_{k,t}(f) dt$ is an identity, where $L_{k,t}(f) = \left(\frac{-1}{k}\right)^k k^{k+1} [k' t^{k+1}]^{-1} \cdot f^{(k)}(k/t)$ is the Post-Widder operator. If $k \rightarrow \infty$, $f(x) = \int_0^\infty e^{-xt} g(t) dt$, where $g(t) = \lim_{k \rightarrow \infty} L_{k,t}(f)$. Obtaining conditions for which this passage to the limit is valid is tantamount to establishing a representation theory for the Laplace integral. The final results coincide with Widder's theory, but new light is shed on the central role played by the operator $L_{k,t}(f)$. (Received January 20, 1945.)

66. P. R. Rider: *A new use for tables of the incomplete beta function*.

The solution of the problem of minimizing a definite integral having an integrand of the form $(1 + y'^2)^m / y''$ has been given. This paper points out how tables of the incomplete beta function can be used to advantage in carrying out the solution in a numerical case. (Received January 22, 1945.)

APPLIED MATHEMATICS

67. C. A. Truesdell: *The membrane theory of shells of revolution*.

The differential equations of the bending theory, and hence of the membrane theory of shells of revolution, are derived as consequences of the equations of three-dimensional elasticity and certain additional assumptions. Less restrictive than customary boundary conditions at the apex of closed domes are proposed. By means of stress functions satisfying a simple ordinary differential equation, solutions of the stress equations are surveyed, and it is shown that the new boundary conditions can be satisfied for a large class of surfaces for which the old could not. Displacement functions are introduced which reduce the integration of the displacement equations to the integration of a fairly simple ordinary differential equation. Numerous exact solutions of the differential equations are given explicitly, with the aid of the stress and displacement functions, and some numerical examples are given. (Received January 10, 1945.)

GEOMETRY

68. S. B. Jackson: *The four-vertex theorem for surfaces of constant curvature*.

In an earlier paper by the writer (S. B. Jackson, *The four-vertex theorem for spherical curves*, Amer. J. Math. vol. 62 (1940) pp. 795-812) it was shown that on any simple closed spherical curve of class C''' , not a circle, there are at least four geodesic vertices, that is, extrema of the geodesic curvature. The aim of the present paper is to extend this result to any surface of constant curvature. Specifically, it is shown that every simple closed curve of class C'' , not a geodesic circle, in a simply connected region of a surface of constant curvature has at least four geodesic vertices. The technique of the paper is to map the given region of the surface onto the plane in such a way that the geodesic vertices of the given curve go into the vertices of the corresponding plane curve. The theorem then follows from the known facts about the vertices of plane curves. It is shown by example that the restriction that the curve be in a simply connected region of the surface is essential. Finally, it is proved that the theorem

cannot be extended to any surfaces of variable curvature. (Received January 30, 1945.)

69. Edward Kasner and John DeCicco: *Some properties of minimal surfaces.*

It is known that for any minimal surface and also for a sphere, any parallel system of plane sections is an isothermal family on the surface. The authors prove that this property is characteristic for the minimal surfaces and the spherical surfaces. Moreover the following fundamental theorem is obtained. If a surface possesses more than four distinct isothermal systems of parallel plane sections, all of which are parallel to a given rectilinear direction L , then every parallel system of plane sections is isothermal, and the surface is either minimal or spherical. Some further results are the following. Any surface which has more than two simply-infinite families of plane geodesics of nonzero curvature is either a sphere or a plane. If a surface has a single isothermal system of parallel plane sections such that the mean curvature is the same at all points of any one of these sections, then it is a surface of revolution. (See for related theorems paper in Proc. Nat. Acad. Sci. U.S.A. vol. 31 (1945) (Received January 24, 1945.)

STATISTICS AND PROBABILITY

70. E. J. Gumbel: *Expected values of a statistical variate.*

Let x be a continuous unlimited variate; let $F(x)$ be the probability of a value equal to, or less than, x . What values may be expected in a sample of size n ? The n expected values \bar{x}_m ($m=1, 2, \dots, n$) and the n most probable values \hat{x}_m are, respectively, the means and the modes of the distributions of the m th values. As the calculation of these means is in general extremely tiresome, and the calculation of the n modes laborious, it is proposed to use as approximations the values \hat{x}_m which correspond to the probabilities F_m obtained from linear interpolation between F_1 and F_n . Here, F_1 and F_n are the probabilities of the most probable largest and of the most probable smallest value in a sample of size n . The probabilities F_m are subject to the inequalities $(m-1)/n < F_m < m/n$. The distribution of the values x_m converges, with increasing sample size, toward the theoretical distribution. For a symmetrical distribution the mean of the values \hat{x}_m coincides with the mean of the population. (Received January 22, 1945.)

TOPOLOGY

71. R. H. Sorgenfrey: *A condition that a continuum be irreducible between two points.*

It is proved that in order that a (compact) continuum M be irreducible between two of its points, it is necessary and sufficient that of every three continua whose sum is M , either two fail to intersect or the sum of two contains the third. (Received December 4, 1944.)

72. P. A. White: *On new types of regular convergence.*

New types of convergence are introduced making use of other local properties in the same way that local connectedness was used in the definition of r -regular con-