$[(1+y \sec \phi_p)\ (1+y \sec \phi_{p+1})]$, $y>0$, $-(\pi/2)<\phi_p<+(\pi/2)$, $0\leqq g_p\leqq 1$. Also, (4) converges if $\liminf |c_p|<\infty$ and $\sum[|c_p|-\Re(c_p)]<2$. (Received July 2, 1943.)

## 217. F. T. Wang: *On Riesz summability of Fourier series.* III.

Denote by $\phi_\alpha(t)$ the fractional integral of order $\alpha$ of an even integrable function $\phi(t)$, periodic, with period $2\pi$. The following two results are proved: (i) If $\phi_\alpha(t)=o(t^\gamma)$, $\gamma>\alpha>0$, then the Fourier series of $\phi(t)$ is summable $(R, e^{n^{1-\alpha/\gamma}}, \gamma+\delta)$, $\delta>0$, at $t=0$. (ii) If $\phi_\alpha(t)=o(t^\alpha(\log t)^{-1})$, $\alpha>0$, then the Fourier series of $\phi(t)$ is summable $(R, n^{(\log n)^{1/\alpha}}, \alpha+1)$ at $t=0$. (Received June 11, 1943.)

# Applied Mathematics

## 218. Stefan Bergman: *Solutions of linear partial differential equations of fourth order.*

Generalizing the known formula $u=\mathrm{Re}[f_1(z)+\bar{z}f_2(z)]$, $z=x+iy$, $\bar{z}=x-iy$, for the biharmonic functions (that is, functions $u$ for which $\Delta\Delta u=16u_{zz\bar{z}\bar{z}}=0$), the author proves that for every equation $L(u)=U_{zz\bar{z}\bar{z}}+a_1u_{zz}+a_2u_{z\bar{z}}+a_3u_{\bar{z}\bar{z}}+a_4u_z+a_5u_{\bar{z}}+a_6u=0$ where $a_\nu=a_\nu(z,\bar{z})$ are analytic functions of $z$ and $\bar{z}$ there exist two functions of $z$, $\bar{z}$ and a real variable $t$, $\mathrm{E}_k(z,\bar{z},t)$, $k=1, 2$, such that every solution of $L(u)=0$ which is regular in a star domain $\mathfrak{D}$ can be represented in $\mathfrak{D}$ in the form $u(\bar{z})=\mathrm{Re}\{\int_{-1}^{+1}[\sum_{k=1}^{2}\mathrm{E}_k(z,\bar{z},t)f_k(z(1-t^2)/2)dt/(1-t^2)^{1/2}]\}$. The methods and results of the paper *Linear operators in the theory of partial differential equations* (Trans. Amer. Math. Soc. vol. 53 pp. 130–155) can be applied to the functions $u$, satisfying $L(u)=0$. (Received July 30, 1943.)

## 219. G. E. Forsythe: *Note on equivalent potential temperature.*

Let a parcel of moist air be saturated with $w_s=w_s(p, T)$ tons of water vapor per ton of dry air. Let the following quantities be measured in meter-ton-second-absolute degree mechanical units: $T$=temperature; $p$=total air pressure; $L=L(T)$=latent heat of evaporation of water; $c_p=c_p(T)$ $[c_v=c_v(T)]$=specific heat of dry air at constant pressure [volume]; $e_s=e_s(T)$=vapor pressure of saturated water vapor; $p_d=p-e_s$; $k=(c_p-c_v)/c_p$; $\theta_d=T(100/p_d)^k$. Let $\lambda=\lambda(p, T)=Lw_s/(c_pT)$. Rossby (Massachusetts Institute of Technology Meteorological Papers, vol. I, no. 3 (1932)) defines the equivalent potential temperature $\theta_e$ by the relation $\theta_e=\theta_d\exp\lambda$. He asserts without proof that, as $p\rightarrow 0$ in a process for which $\theta_e$ is constant, one has $\theta_d\rightarrow\theta_e$. The present note uses elementary estimates to prove Rossby's assertion: (i) under the oversimplifying assumption that $L$ and $c_p$ are bounded away from 0, as $T\rightarrow 0$; (ii) under weaker but physically artificial assumptions about $L$ and $c_p$. The note includes a further discussion of the important meteorological quantity $\lambda$. (Received July 26, 1943.)

## 220. S. H. Gould: *The Rayleigh-Ritz method for higher eigenvalues.*

An elementary proof is given of the following theorem, fundamental in applications of the Rayleigh-Ritz method for calculating the eigenvalues of a given variational eigenvalue problem: the $n$ roots, arranged in order of magnitude, of the determinantal equation obtained by using $n$ coordinate functions are upper bounds respectively for the first $n$ eigenvalues of the original problem. (Received July 14, 1943.)

221. G. E. Hay and Willy Prager: *On plane rigid frames containing curved members.*

In a recent paper (Quarterly of Applied Mathematics vol. 1 (1943) pp. 49–60), the authors have extended the method of the conjugate beam to the case of plane rigid frames consisting of straight members and loaded perpendicularly to the plane of the frame. In the present paper, the method is again extended to include frames containing curved members, with loading both in the plane of the frame and perpendicular to it. There is introduced a conjugate frame similar in shape to the original frame, but with different supports in general. This conjugate frame is subjected to a load which is a certain function of the distribution in the original frame of bending moment, Young's modulus, and the moment of inertia of the cross section. The equilibrium conditions of this conjugate frame afford a simple determination of redundant reactions in the original frame. Also, the distribution of bending moment and shear in the conjugate frame are equal respectively to the distribution of displacement and rotation in the original frame, which fact affords a simple determination of the distribution of displacement and rotation in the original frame. (Received July 31, 1943.)

222. F. B. Hildebrand: *On the stress distribution in cantilever beams.*

The problem of determining the stress distribution in a flanged cantilever beam of narrow rectangular cross section, supported in such a way that no distortion of the cross section takes place at one end of the beam, is treated as a problem in the theory of plane stress. A limiting case of anisotropy is considered, in which the material of the beam is perfectly rigid in the direction of the prescribed transverse loading which acts in the plane of the web plate. The exact solution is found in terms of a rapidly convergent series of almost periodic functions and a corresponding approximate solution is obtained in closed form by a direct application of the calculus of variations. The results are evaluated numerically in representative cases and, on the basis of comparisons with earlier solutions of related problems (F. B. Hildebrand and E. Reissner, Journal of Applied Mechanics vol. 9 (1942) pp. A-108–A-116), conclusions regarding the validity of certain assumptions and approximations are drawn. (Received July 27, 1943.)

223. E. G. Kogbetliantz: *Detailed quantitative interpretation of maps of gravitational anomalies with the aid of mathematical analysis.* Preliminary report.

The importance of gravific methods in geophysical surveying, especially in the prospecting for new oil fields, is very well known. The interpretation of gravity and other gravific maps is today limited to only qualitative conclusions. All attempts made for their qualitative interpretation, that is, for the numerical determination of important geological parameters such as depth below the surface, thickness of the disturbing layer, slope of an anticlinal flank, were unsuccessful. The reason for these systematic failures is the fact that the observed values, mapped after all usual corrections—correction for the regional anomaly included—reflect not only the effect due to the studied underground structure or body but also the gravific action of the immediate neighborhood under the station. This punctual anomaly due to the nonhomogeneity of the ground and affecting only one point-station cannot be evaluated and corrected. To eliminate from the interpretation the errors due to these unknown punctual anomalies, it is necessary to use in the computations of geological parameters only the average values. In these average values are melted all observed values and the effect of punctual anomalies is eliminated because their average values are negligible. The importance of this new interpretation-method is illustrated by an anomaly map due to an anticlinal for which the numerical values of all its geological parameters are expressed by definite integrals. These integrals are easily computed with the aid of a planimeter if the gravific map is given. (Received August 2, 1943.)

### 224. A. N. Lowan and H. E. Salzer: *Table of coefficients for inverse interpolation.*

Although many tables have been published to facilitate direct interpolation, there has been to date no table of coefficients to handle the more cumbersome task of inverse interpolation. To meet this need, the Mathematical Tables Project has computed the polynomial coefficients of the ratios of differences of various orders in the formula obtained by the inversion of the Everett formula for direct interpolation (H. T. Davis, *Tables of the higher mathematical functions*, vol. 1, pp. 82–83). The coefficients of the five fourth order terms were calculated to ten decimal places at intervals of 0.001 of the argument $m=(u-u_0)/(u_1-u_0)$. Also a short table gives the values of the coefficients of the ten sixth order terms. The values of the coefficients were computed at intervals of 0.1. It was not necessary to compute the coefficients of the second order terms, since these coefficients are tabulated at intervals of 0.0001 in the table of *Lagrangian interpolation coefficients* to be published by the Columbia University Press. The table here described will be of particular value whenever inverse interpolation is to be carried out in a table with a fairly large tabular interval. It is proposed to designate the coefficients here discussed as "inversolants." (Received June 11, 1943.)

### 225. Morris Marden: *On the stream function of axially symmetric flows.*

In his papers (Math. Zeit. vol. 27 (1926) p. 641 and Math. Ann. vol. 99 (1928) p. 629) Bergman studies the behavior of harmonic functions in the neighborhood of point and line singularities. Using the fact that the functions $\Phi=\Phi_1+i\Phi_2=(1/2\pi)\cdot\int_0^2 f(u)dt$ and $\Psi=\Psi_1+i\Psi_2=(1/2\pi)\int_0^{2\pi}f(u)(x-u)dt$, where $u=x+iy\cos t+iz\sin t$, are the potential and stream function of two three-dimensional axial-symmetric flows of an incompressible fluid, the author investigates the behavior of the stream function in the neighborhood of the above mentioned singularities. The author constructs certain functions possessing infinitely many line singularities (periodic stream functions) and proves certain properties of these functions. (Received July 30, 1943.)

### 226. Seymour Sherman: *Finite aspect ratio effect in the Glauert-Prandtl regime.*

The usual simplified development of the Prandtl integral equation for the circulation about a wing of finite aspect ratio is not readily extended to the case of compressible flow because the expression for the downwash due to a vortex line becomes more complicated. By first making the usual assumptions of thin airfoil theory and linearising the equations of compressible flow and then appealing to the work of E. Reissner (*The equations of lifting strip theory for tapered wings*, Curtiss Research Laboratory Report SB-76-S-1) on the flutter of finite aspect ratio it is shown that the compressibility effect can be included in the Prandtl equation by substituting for $\partial C_{L\ \mathrm{inc}}(y)/\partial\alpha$ the quantity $(1/(1-\beta^2)^{1/2})\partial C_{L\ \mathrm{inc}}(y)/\partial\alpha$. This is done by setting up the problem as a boundary value problem for the appropriate elliptic partial differential equation. (Received July 26, 1943.)

## 227. Alexander Weinstein: *On the bending of a clamped plate.*

Let $\{p_k(x, y)\}$ denote a complete, but not necessarily orthogonal, sequence of harmonic functions of integrable square in a domain $S$ with the boundary $C$. Consider the equation $\Delta\Delta w=f$ with the boundary conditions $w=dw/dn=0$. Let $w_m=GGf+\sum_{i=1}^{m}a_{mi}Gp_i$, where $Gf$ is the Green's potential of $f$, and the constants $a_{mi}$ are determined by the equations $\sum_{i=1}^{m}a_{mi}(p_i, p_k)=-(f, Gp_k)$, $k=1, 2, \cdots, m$. It is proved in this paper that $w_m$ converges uniformly to $w$ in $S+C$ and that $\partial w_m/\partial x$, $\partial w_m/\partial y$ converge uniformly to $\partial w/\partial x$, $\partial w/\partial y$ in every domain interior to $S$. The approximating functions $w_m$ can be computed explicitly in the case when $S$ is a rectangle. The proofs are based on the results of a paper by N. Aronszajn and A. Weinstein, Amer. J. Math. vol. 44 (1942) p. 625. (Received July 10, 1943.)

# Geometry

## 228. Herbert Busemann: *On spaces in which two points determine a geodesic.*

Let $R$ be a finitely compact, convex, and locally strictly externally convex, metric space. A geodesic in $R$ is defined as a continuous curve which is locally isometric with the real axis. (A symmetric variational problem in parametric form will satisfy these conditions, when the extremals are considered as geodesics). There is at least one geodesic through any two distinct points of $R$. If this geodesic is unique, then $R$ is either simply connected and all geodesics are isometric with a euclidean straight line, or $R$ has a two-sheeted universal covering space and all geodesics of $R$ are congruent to one euclidean circle. (Received July 26, 1943.)

## 229. John DeCicco: *Kasner's pseudo-angle.*

The theory of functions of a single complex variable is identical with plane conformal geometry. This is not the case in the theory of functions of two or more complex variables. A set of $n$ functions of $n$ complex variables induces a correspondence between the points of a real $2n$-dimensional space $R_{2n}$. The infinite group $G$ of these correspondences has been termed the pseudo-conformal group by Kasner. In 1908, Kasner showed that for $n=2$ a transformation of $R_4$ is pseudo-conformal if and only if it preserves the pseudo-angle between a curve and a three-dimensional variety. This pseudo-angle theorem can be carried over to $2n$ dimensions almost without change. A system of $(2n-1)$-dimensional hypersurfaces is said to be bi-isothermal if it is pseudo-conformally equivalent to $\infty^1$-parallel $(2n-1)$-dimensional flats. Any system of $(2n-1)$-dimensional hypersurfaces is bi-isothermal if and only if the pseudo-angle between the given hypersurfaces and any system of parallel lines is a biharmonic