order of $f(z)$ on $|z|=R$ is $\omega'>\omega$, there either exist at least two points of order ω' on this circle or the singularity of order ω' is non-Fuchsian. By means of an extension of Mandelbrojt's method for finding the singularities of an analytic function on its circle of convergence, the theorem gives a formula for the order of every pole lying outside of the convex hull of non-polar singularities of $f(z)$, and for the order of every Fuchsian singularity on the boundary V of the convex hull, provided the singularity is not an interior point of a straight-line segment of V. (Received April 12, 1943.)

190. Harry Pollard: *A new criterion for completely monotonic functions.*

The Bernstein criterion for completely monotonic functions states that if (i) $f(0+)$ exists and (ii) $(-1)^k\Delta_\delta^k f(x)\geqq 0$ for $k\geqq 0$, $\delta>0$, $x>0$, then $f(x)$ is completely monotonic in $0\leqq x<\infty$. It is established in this paper that (ii) can be weakened to $(-1)^k\Delta_{\delta_k}^k f(x)\geqq 0$ for a suitable sequence $\{\delta_k\}$. (Received April 10, 1943.)

191. W. J. Thron: *A general theorem on convergence regions for continued fractions $b_0+K(1/b_n)$.*

Let the regions B_0 and B_1 be defined by: $r\cdot e^{i\theta}\in B_0$ if $r>(1+\epsilon)\cdot f(\theta)$, $r\cdot e^{i\theta}\in B_1$ if $r>(1+\epsilon)g(\theta)$, where ϵ is an arbitrary small positive number and the functions $f(\theta)$ and $g(\theta)$ are positive in the interval $[0, 2\pi]$. If it is required that the complements of the regions B_0 and B_1 be both convex and if $f(\theta)\cdot g(\pi-\theta)\geqq 4$, then the continued fraction $b_0+K(1/b_n)$ converges if $b_{2n}\in B_0$ and $b_{2n+1}\in B_1$, that is B_0 and B_1 are twin convergence regions for the continued fraction. The condition $f(\theta)g(\pi-\theta)\geqq 4$ is a necessary condition for two regions to be twin convergence regions. (Received April 19, 1943.)

192. W. J. Thron: *Convergence regions for the general continued fraction.*

It is shown that the continued fraction $K(a_n/b_n)$ converges if all $a_n=r\cdot e^{i\theta}$ lie in a bounded part of the parabola $r\leqq a^2/2(1-\cos(\theta-2\gamma))$, and if all b_n lie in the half-plane $R(b_ne^{i\gamma})\geqq a+\epsilon$. Here $a>0$ and ϵ is an arbitrary small positive number. (Received April 24, 1943.)

193. Hassler Whitney: *On the extension of differentiable functions.*

Let A be a bounded closed set in Euclidean space E. Suppose that for some number ω any two points of A are joined by an arc in A of length not more than ω times their distance apart. Then any function of class C^m in A which, with derivatives through the mth order, is sufficiently small in A, may be extended throughout E so as to be small, with its derivatives. (Received May 11, 1943.)

Geometry

194. T. C. Doyle: *Tensor theory of invariants for the projective differential geometry of a curved surface.*

This paper completes the explicit determination of differential invariants of all orders for a curved two dimensional surface begun by E. J. Wilczynski, *Projective differential geometry of curved surfaces (fourth memoir)*, Trans. Amer. Math. Soc. vol. 10 (1909) pp. 176–200. The Lie theory of groups serves to determine the number of existing invariants of any specified order and these are then exhibited by contraction of fundamental surface tensors and their covariant derivatives appearing as coefficients of a system of partial differential equations defining the surface. The projective normal is attained by the formal elimination procedure of tensor analysis. (Received April 10, 1943.)

195. H. T. Muhly: *Independent integral bases and a characterization of regular surfaces.*

Let $\mathfrak{o}^*$ denote the ring of homogeneous coordinates, $\xi_0^*, \xi_1^*, \cdots, \xi_n^*$ associated with a normal, nonsingular model F, of a field Σ of algebraic functions of two variables, and assume that $\xi_0^*, \xi_1^*, \xi_2^*$ are selected so that they are algebraically independent and so that each element of $\mathfrak{o}^*$ depends integrally upon these three elements. Let the relative degree $[K(\xi_0^*, \xi_1^*, \cdots, \xi_n^*):K(\xi_0^*, \xi_1^*, \xi_2^*)]$ be denoted by ν. It is shown in this paper that if there exists a set of ν elements $\lambda_1^*, \lambda_2^*, \cdots, \lambda_\nu^*$ in $\mathfrak{o}^*$ which form an independent modular base for $\mathfrak{o}^*$ over the ring $K[\xi_0^*, \xi_1^*, \xi_2^*]$, then the field Σ is regular, that is, its arithmetic genus p_a coincides with its geometric genus p_g. Furthermore, it is shown that if the field Σ is regular then there exist projective models of Σ which are normal and nonsingular, and which are such that the associated ring of homogeneous coordinates has a linearly independent modular base over a suitably chosen ring of independent variables. In fact if F is any normal, nonsingular model of a regular field Σ, and if F_h is the derived normal model belonging to the character of homogeneity h, then F_h has these properties for all sufficiently large values of h. (Received May 15, 1943.)

Numerical Computation

196. A. N. Lowan and H. E. Salzer: *Coefficients in the expansion of derivatives in terms of central differences.*

The coefficients in the expansion of the first 52 derivatives in terms of mean central and central differences (in most cases up to the 42nd difference) were computed by the Mathematical Tables Project. For the first 30 derivatives the exact values of the coefficients are given in the form of ordinary fractions for the first 30 differences and for some differences beyond the 30th. For all derivatives beyond the 30th exact fractional values are given for the coefficients of differences up to orders varying between the 41st and 52nd. Finally for most of the coefficients of differences of orders varying from the 31st to the 42nd, 18 significant figures are given. All fractions are believed to be in lowest terms. The tabulated coefficients are valuable in the evaluation of analytic functions of a complex variable when known along a straight line within the region. The coefficients were checked by means of two recursion formulas, which are not mentioned in the literature. (Received April 8, 1943.)

Statistics and Probability

197. Jacob Wolfowitz: *Asymptotic distributions of ascending and descending runs.*

Let $a_1, a_2, \cdots, a_N$ be any permutation of N unequal numbers. Let there be assigned to each permutation the same probability. An element a_i $(1 < i < N)$ is called a turning point if a_i is greater than or less than both a_{i-1} and a_{i+1}. Let a_j and a_{j+k} be consecutive turning points; they are said to determine a "run" of length k. The author