ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

ALGEBRA

1. R. H. Bruck: *The structure of the rational representations of a wide class of linear groups.*

Let θ, $\phi=\theta^{-1}$ be fixed automorphisms of the underlying field (of characteristic 0 or p). Denote by A^* the transposed of the matrix obtained from A by subjecting its elements to the automorphism ϕ. Attention is restricted to groups $\mathfrak{G}$ of $n\times n$ matrices with the property that A in $\mathfrak{G}$ implies A^* in $\mathfrak{G}$; by choice of ϕ a large number of groups may be brought under review. $\mathfrak{G}$ being taken as a group of transformations of a contravariant vector x^i, the fth Kronecker product of $\mathfrak{G}$ may be regarded as a group of transformations of the linear vector space S of all forms $F(T)=\sum c_{i_1i_2\cdots i_f}T^{i_1i_2\cdots i_f}$, where $T^{i_1i_2\cdots i_f}$ is a contravariant tensor. The fundamental idea of the paper is to analyse the Kronecker product by means of a scalar product defined on S. The scalar product of F and $G=\sum d_{i_1i_2\cdots i_f}T^{i_1i_2\cdots i_f}$ is taken as $F\circ G\equiv\sum \overset{\theta}{c}_{i_1i_2\cdots i_f}d_{i_1i_2\cdots i_f}$, and has properties (1) linearity, (2) $(bF)\circ G=F\circ(b^\theta G)=b^\theta(F\circ G)$, (3) $F\circ G=F_{(A^*)}\circ G_{(A^{-1})}$. (Here $F_{(A)}$ is defined by $F(T)\equiv F_{(A)}(T')$ where $x=Ax'$.) The following special result indicates the nature of the paper: If $\mathfrak{G}$ (over the field of rationals) contains the transposed of each group matrix, the Kronecker product representations are completely reducible. (Received November 12, 1940.)

2. C. C. Camp: *A root cubing method of solving equations.*

Algebraic equations were solved by Dandelin and Graeffe by the well known root squaring process. Instead of squaring the roots the present paper gives a method for cubing the original roots. No Encke roots are needed and the sign of a real root is preserved on successive cubings. The general rule is derived by two independent methods and in the case of the cubic by three. The cases of complex and repeated roots are treated in the applications together with that of distinct real roots. (Received November 23, 1940.)

3. W. D. Duthie: *Segments in ordered sets.*

The notion of "segment" in a linearly ordered set is extended to (partially) ordered sets. Segments in ordered sets are self-dual, and can be used to characterize various special properties of ordered sets such as being a lattice, modular, distributive, or complemented lattice, giving a completely self-dual set of postulates for these lattices. Convex subsets of a lattice are defined in the natural way, and various relations among a lattice and its lattices of segments and convex subsets are discussed, leading to results on imbedding of lattices in complete lattices. The lattices of segments of modular and distributive lattices are special cases of pseudo-modular and pseudo-distributive lattices, respectively; the latter property is of particular interest, since it is applicable to modular lattices, though the application is somewhat restricted by the fact that the only complete complemented pseudo-distributive modular lattices which are not distributive are one-dimensional projective geometries. However in such cases the existence of a dimension function permits replacing pseudo-distributivity by a more general notion of "m-distributivity" (m having the range of the dimension function). Application of the general theory of segments to Boolean algebras yields some known results by new and shorter methods. (Received November 25, 1940.)

4. C. J. Everett: *An extension theory for rings.*

A ring theoretic analogue of the O. Schreier "Erweiterungstheorie" for groups is developed about the central problem of determining all rings R containing a given ring N as ideal and such that R/N is isomorphic to a given ring F, employing factor systems and their equivalence. The Artin "splitting theorem" is proved: If R contains ideal N, R has a splitting ring over N. Constructive proof is given of the existence of rings whose chains of left and right annihilators for successive powers of the ring have first repetitions at arbitrary positions. "Closed rings" are shown to be direct summands of over-rings containing them as ideals. Equivalence classes of suitably restricted factor systems form an abelian group under addition. (Received November 23, 1940.)

5. Irving Kaplansky: *Maximal fields with valuations.*

A field K with a valuation is said to be maximal if no proper extension of K has the same value-group and same residue-class field as K. It was shown by Krull that any field with a valuation could be embedded in a maximal field, and he proposed the problem of determining whether this maximal extension is unique. This question is examined in the present paper, and the following result is obtained. The maximal extension is always unique if the residue-class field has characteristic zero; but if the latter has characteristic p, it must first be required that the value-group have no extensions of degree p, and second that the residue-class field satisfy a certain condition somewhat stronger than algebraic perfection. Under the same hypotheses it is shown that in the equal characteristic case every maximal field is a generalized type of power series field. The chief tool employed is a generalization of Ostrowski's notion of pseudo-convergence, in terms of which it is possible to give new criteria for maximality and completeness. (Received December 2, 1940.)

6. Rufus Oldenburger: *The minimal number problem.*

It has been proved that, for a field with at least $n+1$ elements, each binary form F of degree n with a symmetric tensor A of coefficients can be written as a linear combination of nth powers of linear forms. The least number of terms for which F can be so written is called the minimal number m of F. The elements of A can be arranged in ordinary matrix displays $B_0, \cdots, B_n$ when B_i is the display obtained from A by using i indices of A as column indices. The minimal number problem is the problem of determining the precise relation between the ranks of $\{B_i\}$ and m. The ranks of the matrices $\{B_i\}$ are determined by one of them, denoted by γ. In the present paper the minimal number problem is solved for algebraically closed fields by proving that $m=\gamma$ or $m=n-\gamma+2$. A modification of this result holds for arbitrary fields. The proof is accomplished in part by the solution of the problem of construction of all representations of F. This latter problem is the same as the problem of constructing all forms apolar to F. Solutions for several major classical problems of algebra follow. (Received November 23, 1940.)

7. G. Y. Rainich: *Postulates for fields.* Preliminary report.

Instead of binary operations or functions of two variables, namely, the sum $x+y$ and the product xy, in terms of which a field is usually defined, corresponding functions of one variable or transformations, namely, addition $x'=y+a$ and multiplication $x'=xb$, are considered. The problem is to characterize the field by studying the formal properties of these or, rather, of the group $x'=xb+a$ generated by them. This group is postulationally defined in two ways; once as a two-fold transitive group of transformations and then again as an abstract non-abelian group. From the first point of view the additions and multiplications are introduced as transformations without fixed point and with a given fixed point respectively. This point of view is closely related to projective geometry. From the other point of view the subgroups of additions and multiplications are introduced as the commutator subgroup and its factor group respectively. (Received October 25, 1940.)

8. N. E. Rutt: *Rectangular arrays of combinations.*

Let C be the collection of different combinations, two at a time, of the objects in a set D of $2n$ different symbols. From the elements of C construct a rectangular arrangement, henceforth to be called an array, such that no element of D occurs twice in the same row of the arrangement, and no element of C occurs twice in the same arrangement. Two arrays are the same if and only if a reordering of their rows or a reordering of the members of C in their rows will make them formally identical. An investigation of the properties of arrays is undertaken in this paper. It is proved that, for each even natural number $2n$, there is an array whose rows contain n elements of C (the maximum possible), and whose rows are in number $2n-1$ (the maximum possible). Features of the internal structure of arrays are considered, among them the existence and inter-relations of the proper sub-arrays of arrays. The behavior of arrays under permutations of the elements of D receives some attention. An attempt is also made to determine the number of different arrays of certain specified types. (Received November 18, 1940.)

9. M. F. Smiley: *Measurability and distributivity in the theory of lattices.*

G. Birkhoff's criterion that a metric lattice be distributive (American Mathematical Society Colloquium Publications, vol. 25, p. 81) is quantified so as to provide a second generalization (this Bulletin, vol. 46 (1940), pp. 239–241) of the measurability of Carathéodory (*Vorlesungen über Reelle Funktionen*, 2d edition, p. 246). Closure properties of the set of "measurable" elements are derived as before. The influence of distributivity on measurability and on the measurability of complements of measurable elements is discussed. (Received November 20, 1940.)

10. T. L. Wade: *Tensor algebra and Young's symmetry operators.*

A. Young's symmetry operators (Proceedings of the London Mathematical Society, vol. 33 (1900), pp. 97–146) have recently been used by H. Weyl (*The Classical Groups*, Princeton, 1939, pp. 119–131) in the decomposition of tensor space into irreducible subspaces. This paper is concerned with a study of the law of combination satisfied by the indices of the tensor associated with each diagram or partition of the indices of the arbitrary tensor. It is noted that corresponding to each partition there can be associated a specialization of C. M. Cramlet's general invariant tensor (Tôhoku Mathematical Journal, vol. 28 (1927), pp. 242–250), and that an arbitrary tensor can be decomposed by multiplying an appropriate numerical tensor identity by it, and then contracting. (Received November 25, 1940.)

11. R. W. Wagner: *The differentials of analytic matrix functions.*

The notions of Hausdorff and Fréchet concerning differentials are applied to analytic matrix functions as defined by the author in a previous paper. The differential of an analytic matrix function is shown to have a simple form when it is written in terms of the characteristic roots and partial idempotent and partial nilpotent elements of the argument. The function enters the differential through divided difference quotients. This simple form of the differential is used to deduce local properties of the mapping defined by the function. (Received November 23, 1940.)

12. Morgan Ward: *The fundamental theorem of arithmetic.*

A set of necessary and sufficient conditions for the fundamental theorem of arithmetic to hold in a semi-group is obtained from the theory of residuated lattices and a new inductive proof of the theorem is given for the lattice of positive integers. (Received October 30, 1940.)

13. Hermann Weyl: *Theory of reduction for arithmetical equivalence.* II.

Instead of the arithmetically refined method of reduction used in the first paper, the author now resorts to a rougher method which also goes back to Minkowski, and works without the assumption that the class number of ideals is 1. Its generalization to algebraic number fields F with several infinite prime spots is due to Siegel and P. Humbert (Commentarii Mathematici Helvetici, vol. 12 (1939–1940), pp. 263–306). The author follows the same geometric approach as before, including all classes of lattices over F and adding to the case of a field F that of a field quaternion algebra with totally positive norm over a totally real field. (Received November 23, 1940.)

Analysis

14. Warren Ambrose: *Representation of ergodic flows.* Preliminary report.

A flow is a one-parameter group T_t $(-\infty < t < \infty)$ of measure preserving transformations of a space Ω into itself. It is measurable if the function T_tP is a measurable function on $T \times \Omega$, where T denotes the real line taken with Lebesgue measure. For a measurable ergodic flow it is shown that a necessary and sufficient condition that a group of unitary operators U_t defined by $U_tf(P) = f(T_tP)$ have an eigenvalue (other than the trivial eigenvalue 1) is that the flow be isomorphic (with respect to measure properties) to a flow built on a measure preserving transformation (for definition of such a flow see abstract 46-11-446), thus showing that if an ergodic flow has an eigenvalue the measure on Ω must be the direct product measure of a cross section measure with Lebesgue measure along the trajectories of the flow. It is also shown that if the flow has no eigenvalues but satisfies certain conditions which are stronger than