irreducible representations for the compact groups. As an illustration of the use of characters, the author determines the irreducible components of the Kronecker product of irreducible representations.

In the first part of Chapter VIII the general definition of an invariant is specialized to the classical case. Here the representation space is determined by the coefficients of a homogeneous polynomial. The symbolic method as well as certain irrational methods are discussed as tools to obtain the first fundamental theorem. Hilbert's basis theorem for polynomial ideals settles the question of the finiteness of relations among the basic invariants. The symbolic method is then used to extend these results to general orthogonal and symplectic invariants. The ideas in the last half of the chapter cluster around the concept of a Lie group and its Lie algebra. The local properties of these groups are all expressible in terms of the algebras and if the group is simply connected one may readily pass to its properties in the large. On the other hand Lie algebras may be studied independently of the group theory and questions analogous to those regarding representations, invariants, and so on, may be raised. For compact Lie groups the powerful integration method finds another application here in a very simple derivation of the first fundamental theorem for general invariants. The last chapter resumes the discussion of algebras. A number of interesting results concerning automorphisms and direct products are obtained by matrix methods.

We have confined our description to the broad outlines of the subject. Space does not permit a detailed account of the interesting bypaths indicated by the author. We may mention, for example, a new formulation of Klein's Erlanger program, the close connection between representation theory and the theory of almost periodic functions and Fourier series, the topology of the classical groups. There are ample indications in the notes and bibliography to enable the reader to pursue these questions further.

In short, the book is heartily recommended to the present generation of algebraists as an introduction to a rich and rather neglected field and to those educated in the classical tradition for an insight into important recent algebraic ideas and their applicability to familiar problems.

N. JACOBSON

Aspects of the Calculus of Variations. Notes by J. W. Green after lectures by Hans Lewy. Berkeley, University of California Press, 1939. 6+96 pp.

As indicated in the introduction, the goal of the lecturer was to acquaint the reader with a skeleton of methods such as he is apt to encounter in the calculus of variations, rather than to formulate results as generally as possible. The first chapter is devoted to a discussion of various problems, and the derivation of the first order necessary conditions for an extremum. Chapter II treats quadratic problems; in particular, the theory of integral equations with real symmetric kernel and boundary value problems associated with a second order linear differential equation. Chapter III, which deals with sufficient conditions, is extremely brief; nowhere in the lectures is the Weierstrass $\mathcal{E}$-function mentioned. In Chapter IV the work of Lewy (Mathematische Annalen, vol. 98 (1928), pp. 107–124) on the absolute minimum for nonparametric problems in the plane is presented. Chapter V is concerned with harmonic functions and associated boundary value problems; this chapter is preliminary to the discussion of the problem of Plateau and conformal mapping given in Chapter VI. In this last chapter the problem of Plateau is solved for a single contour in three dimensions; the method of proof is that of Courant (Annals of Mathematics, (2), vol. 38 (1937), pp. 676–724).

W. T. Reid

Die Zylinderfunktionen und ihre Anwendungen. By R. Weyrich. Leipzig, Teubner, 1937. 5+137 pp.

A cylinder function may be defined as any solution of Bessel's differential equation. They include functions of the first kind, $J_p(z)$, also called Bessel functions, which are regular at $z=0$, and functions of the second kind which are not regular at $z=0$. There has been considerable confusion in the notation and canonical form of the cylinder functions of the second kind. They have been denoted by Y, G, K, and so on, by various authors and often the same notation is used with different meanings. Following the tables of Jahnke and Emde, our author uses the notation $N_p(z)$ for functions of the second kind and calls them Neumann functions. They are the same as those denoted by $Y_p(z)$ by Nielsen and also by Watson. (See Watson, *Theory of Bessel functions*, p. 57, for a further discussion.) In addition there are the Hankel cylinder functions, which are really functions of the second kind, defined by

$$H_p^{(m)}(z) = J_p(z)+i(-1)^{m+1}N_p(z), \qquad m=1,\ 2,\ i=(-1)^{1/2}.$$

The importance of the Hankel functions arises from the fact that alone among the cylinder functions $H_p^{(m)}(re^{i\theta})\rightarrow 0$ as $r\rightarrow\infty$, provided that $m=1$, $0\leqq\theta\leqq\pi$, and that $m=2$, $\pi\leqq\theta\leqq 2\pi$.

One way to introduce cylinder functions is to define them, as above,