Mathematics of Relativity; *Lecture Notes*. By G. Y. Rainich. Ann Arbor, Edwards Brothers, 1932. i+67 pp.

As those who have had the pleasure of hearing Professor Rainich lecture would expect, this set of mimeographed lecture notes is very stimulating, original, and interesting. Vectors and tensors are first introduced in terms of cartesian coordinates in euclidean space of three dimensions, and the tensor form of Euler's hydrodynamical equations and of Maxwell's equations is quickly arrived at. Then follows a chapter on four-dimensional euclidean geometry and an account of an axiomatic, non-coordinate, basis for tensor analysis. This basis, which is largely due to the author, is very important and furnishes a method for the introduction of coordinates. The theory of special relativity with its most important implications is given in Chapter 3. Chapter 4 is devoted to curved space and general (Gaussian) coordinates and an adequate account of the curvature tensor, normal coordinates, etc. is given. Chapter 5 furnishes the best concise account we have seen of general relativity and the three fundamental tests—motion of a planet, bending of a ray of light, and shift of spectral lines. We hope very much that the author will be encouraged to amplify (and supply with adequate references) these notes in a printed volume.

F. D. MURNAGHAN

Lehrbuch der Funktionentheorie. By Ludwig Bieberbach. Volume II, *Moderne Funktionentheorie*. 2d edition. Berlin and Leipzig, B. G. Teubner, 1931. vi+370 pp.

It suffices to compare this treatise with any one published ten years ago or earlier to conceive the tremendous change and progress achieved by the theory of functions of a complex variable during recent years. The questions whose treatment occupied chapters in earlier publications, are given here but few pages, or even are entirely eliminated to be replaced by scores of new problems, methods, and results. The following list of contents hardly can give an adequate idea of the variety of material contained in the *Lehrbuch* of Bieberbach. Chapter 1 (83 pp.) is devoted to the discussion of the classical problem of conformal mapping of simply-connected domains; the interior problem is treated as well as the problem of the correspondence of the boundaries. Brief indications concerning the problem of conformal mapping of multiply-connected domains are given. At the end of the chapter fundamental properties of univalent ("schlichte") functions are exposed, including "Flächensatz," "Verzerrungssatz," and Littlewood's estimates for the coefficients of the power series expansion of a univalent function. Chapter 2 (20 pp.) contains a discussion of indispensable properties of the modular function. Chapter 3 (56 pp.) gives an exposition of fundamental results of the theory of analytic functions bounded in the unit circle, and of its various generalizations. Here we find Schwarz's lemma and its generalizations and modifications (Lindelöf's principle, theorems of Julia and Löwner), Jensen-Nevanlinna's formula and its applications, Schur's method of the determination of the power series expansion of a bounded function, Fatou's theorem, generalizations of Vitali's theorem (theorems of F. and M. Riesz, Khintchine, Ostrowski). Chapter 4 (56 pp.) treats of the uniformisation problem. Problems connected with Picard's theorem are discussed in Chapter 5. We mention particularly the proof of Bloch, the idea of which begins to play an important role in the most recent investigations. Theory of entire functions is discussed in Chapter 6. In the exposition of the general theory the author uses mainly ideas of R. Nevanlinna. The end of the chapter is devoted to the beautiful recent proof by Ahlfors of the theorem of Denjoy-Ahlfors concerning the asymptotic values of entire functions. Chapter 7 (67 pp.) treats of various problems of analytic continuation and of related questions. We mention theorems of Hadamard, Fabry, Wiegert-Faber, Pólya-Carlson, and many others. The book ends with the very appropriate application of the general theory to the theory of zeta-functions (Chapter 8, 33 pp.).

Several remarks might have been made concerning various details of exposition, a few lapses and mistakes might have been mentioned. In particular it is the reviewer's opinion that the reader will find many difficulties in reading and understanding Chapter 4, on uniformisation, and that this important and difficult branch of function theory is still lacking an adequate exposition. All these remarks will be of comparatively minor importance, however. Most, if not all of the defects will be probably eliminated in the third edition of this extremely useful and suggestive book, which undoubtedly will appear before long.

J. D. TAMARKIN

Vorlesungen über Geometrie. By A. Clebsch and F. Lindemann. Volume I, Part I, Number 3, second augmented edition. Leipzig and Berlin, Teubner, 1932. xvi+101+ii pp.

The portion of the second edition which is at last published in full covers, in 869 pages, the same topics which were treated in 284 pages of the first edition. The principal divisions are:—1. Introductory Considerations, Ranges and Pencils, 2. Curves of the Second Order and Second Class, 3. Introduction to the Theory of Algebraic Forms. The three numbers were issued in an inconvenient manner. Number 1, published in 1906, had pp. 1–480, and broke off in the middle of a chapter on collineations in the ternary domain. Four years later, the publishers put out pp. 481–768, and again stopped *in medias res*, this time in a consideration of sundry problems from the theory of binary forms of higher order. Now, after the lapse of twenty-two years, we have the final pp. 769–869 in a pamphlet which also contains a table of contents and an index for the whole of Part I. It seems unlikely that more of the second edition will ever be published. Since the first, or 1876, edition had 1700 pages, it appears that the second edition covers only about a sixth of the original work.

In the issue now under review Lindemann completes his introduction to the theory of algebraic forms by discussing individual problems which show the connection between binary forms of higher order and the theory of linear differential equations, spherical and Lamé functions. The treatment is by means of extensions of certain theories due to Hilbert. The last two chapters use invariant-theoretic ideas about binary cubic forms, in conjunction with the representation of binary forms in the complex plane, to furnish a general principle which unifies a part of the immense group of theorems about noteworthy points and circles of the triangle. The discussion here is particularly elegant, but it presupposes a considerable knowledge of the geometry of the triangle.

C. A. RUPP