Spektraltheorie der unendlichen Matrizen. Einführung in den analytischen Apparat der Quantenmechanik. By Aurel Wintner. Leipzig, S. Hirzel, 1929. xii+280 pp.

In the Introduction the author defines the main purpose of the book (not quite in accord with the subtitle) to serve as an introduction to the general theory of linear analysis of infinitely many variables. It is stated that, according to the desire of the publishers, the book is designed primarily for beginners. This desire led the author to some compromises as to the choice of material and the character of exposition, of which the most regrettable, although perhaps not entirely unavoidable, is the exclusion of the notion of Lebesgue's integral.

Chapter I is devoted to a rapid survey of fundamental facts of the theory of matrices and bilinear forms in a finite number of variables. We find here a condensed treatment of reduction of matrices to various canonical forms with applications to the theory of hermitian, unitary and normal matrices, of Jacobi's transformation of matrices and its application to hermitian matrices with a simple spectrum. As the author expresses himself, the material is presented here, not always in the simplest and most natural way, but rather in such a way as to permit extension almost without modifications to the theory of infinite matrices. While much might be said in favor of such a method of presentation, it can be hardly considered as the best one for the beginners. Only an experienced and well informed ("kundiger") reader can appreciate many a subtle detail which would puzzle a beginner.

In Chapter II the author gives a discussion of indispensable analytic tools: Stieltjes integrals, properties of sequences of functions of bounded variation (theorems of Helly and Helly-Bray), inversion formulas of Stieltjes and Hilbert, theorems of Grommer and Hamburger. At the end of the chapter the author attempts to give a "gemeinverständlich" report on Hellinger's integrals. No adequate idea of this theory can be given without using the notion of measure, which is being carefully avoided by the author. Hence, in the reviewer's opinion, the book would only gain if the corresponding pages, 106–120, had been omitted.

Chapter III deals with general properties of bounded matrices and of their resolvents: "Faltung" theorems of Hilbert, criteria and "formal" theorems of Toeplitz, theorems of Hellinger and Toeplitz, C. Neumann's series for the resolvent, characterization of the resolvent as an analytic function of the parameter. The treatment is elegant and presents several novel points of interest.

Chapter IV gives a rather condensed and somewhat incomplete discussion of the spectral matrices. A "beginner" will not readily understand the "Hauptsatz über Einzelmatrizen" on page 159 and the subsequent discussion, even if the formula (258) on page 160 did not contain a disturbing misprint (compare with (76) on page 53).

Chapter V is devoted to the existence proof of spectral matrices for various classes of bounded matrices. This chapter contains material of considerable interest and importance. Several results of this chapter are due to the author, among them the elegant treatment of the unitary matrices and their spectral matrices on the basis of the trigonometric moment problem. The reviewer was not able, however, to follow the proof on page 174.

In Chapter VI we find various extensions of previous results to certain hermitian non-bounded matrices. Some of Carleman's results are interpreted from the point of view of matrices. A relationship with the Stieltjes-Hamburger moment problem is briefly indicated.

In the Appendix the author gives a rather brief sketch of his own investigations in the spectrum theory of the almost-periodic functions of H. Bohr.

From the above enumeration, which is necessarily rather incomplete, it is seen that the book in question contains interesting and important material which is partly new or is treated from an original point of view. When reading the book one can not help feeling, however, that the author endeavored to put too much material in too restricted a space. The result is that the author has not completely succeeded in writing an "introduction" to a great theory, which could be used to advantage by a "beginner." As to a well informed and experienced reader, it might happen that the latter will feel better off when he turns to the original memoirs, including some by the author himself (not to speak of papers of J. v. Neumann, which have appeared simultaneously with the publication of the book). One point deserves to be mentioned separately. There are found in the book about three dozen new terms introduced by the author; here are some of the most striking ones: "Hellysche Fortpflanzungssatz," "quadratically convergent vectors," "infinitesimale und integrale Integrabilitätsbedingungen," "statistisch sinnvolle Matrizen," "wasserstoffähnliche Spektra," "Carleman's Feldtheorie," "Stabilität des reducierten Spektrums," "Hellinger function-pairs of the first and second kind," etc. Not all, and even not most, of these new terms correspond to actually new notions, and many of them do not serve to describe the situation in the best way. For instance a Hellinger function-pair of the second kind simply means a pair of functions $\rho(\mu)$, $\sigma(\mu)$, continuous, bounded and not decreasing on $-\infty<\mu<\infty$ and approaching 0 as $\mu\rightarrow-\infty$. If, however, the introduction of a new term is unavoidable, it seems desirable that its definition should be stated in precise expressions, which in many cases has not been done in this book. The fact that not one of the theorems or definitions is underscored or separated from the body of the text will also contribute somewhat to the discomfort of the reader, particularly of a beginner. The bibliography of the subject is given "en bloc" at the end of the book, without explicit references in the text; this is not always convenient. Finally, misprints and slips of the pen are not infrequent.

J. TAMARKIN

A Course of Geometrical Analysis. By Haridas Bagchi. Calcutta, Chuckervertty, Chatterji and Company, 1926. iv+562 pp.

This rather long book on elementary differential geometry is written by a Premchand Roychand Scholar and Lecturer in Mathematics in Calcutta University. It gives evidence of wide reading and of much thought and study. While for the most part the topics and the treatment follow classic lines, there are many discussions evidently original with the writer. It is apparently intended as a textbook, for there are frequent references to the student and suggestions offered to him. It is, however, curious in arrangement, widely discursive in treatment and remarkably uneven in difficulty; it does not seem to us so well adapted for one beginning the subject as the familiar French, German and