SHORTER NOTICES

Vorlesungen über die Grundzüge der mathematischen Statistik. 2d edition. By C. V. L. Charlier. Lund, Verlag Scientia. 125 pp.

During the past two decades there has been a considerable advancement in the mathematics of statistics by workers in the countries of northern Europe. The results of their activity have been rather slow in coming to the attention of Americans because most of the work has been published in one of the Scandinavian languages or at least in Scandinavian journals. Of these workers, Charlier is one of the best known. In his published memoirs he goes back to pre-Gaussian times and builds up a logical science of statistics from a few principles from Laplace, making much use of the theory of the superposition of small errors.

The little book under review sums up the results of his work. It is not a treatise or a text-book but simply a book of directions for applying his methods to statistical data with many problems worked out in detail. For mathematical details the reader is referred to the original articles. The book begins with the usual discussion of the arithmetic mean, measures of dispersion and probable error, using Charlier's own self-checking plan of computation. Then follows a very clear discussion, well illustrated by examples, of the series of Bernoulli, Poisson and Lexis leading to the notions of "übernormal" and "unternormal" dispersion of Lexis and to Charlier's "coefficient of disturbancy," a measure of the effect of causes which cannot be explained by the theory of probability.

To American readers the most interesting part of the book is that dealing with Charlier's two representations of frequency curves. Type I curves are represented by

$$f(x) = \beta_0\varphi_0(x) + \beta_3\varphi_0'''(x) + \beta_4\varphi_0^{\mathrm{IV}}(x) + \cdots,$$

where

$$\varphi_0(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

and the prime marks denote differentiation. Type II curves are given by

$$F(x) = N[\psi(x) + \gamma_2\Delta^2\psi + \gamma_3\Delta^3\psi + \cdots],$$

where

$$\psi(x) = \frac{e^{-\lambda}\lambda^x}{x!} \qquad \text{and} \qquad \Delta\psi = \psi(x) - \psi(x-1).$$

In the text Charlier outlines plans of the numerical work for calculating the constants β, λ, and γ, and carries out the computations for two frequency distributions. One of the most interesting questions in mathematical statistical circles at present is the analytical representation of frequency distributions. In this country and in England Pearson's methods have long held sway, and now comes a rival, Charlier. It seems logical to think that Charlier's methods should give a better fit than the more empirical and pragmatic methods of Pearson, but actual tests do not always bear this out. Experience alone will tell us which method is best in actual service.

The last three chapters in the book are devoted to the theory of correlation, first the general theory and then that for four-fold tables. Chapter XIII is a tantalizing chapter. It begins with the statement that two phenomena are correlated, when in whole or in part they are the resultants of the same elementary causes, and then simply gives the ordinary product-moment formula for the correlation coefficient with plans for calculating the various constants.

Interested students will find Charlier's methods discussed in considerable detail in Arne Fisher's recent work, *The Mathematical Theory of Probability.*

A. R. CRATHORNE

Théorie Mathématique des Phénomènes Thermiques produits par la Radiation Solaire. By M. Milankovitch. Paris, Gauthier-Villars, 1920. xvi + 340 pp.

This book is of great interest in several respects. It is written by a Serb and was interrupted by war when the author was taken prisoner by Austria-Hungary. Granted the freedom to pursue his work the book was completed owing to the courtesy of the Hungarian Academy of Sciences. But its greater title to interest is that, apparently, it is the first complete treatise on the thermal effects of solar radiation treated from a systematic point of view.

At the first stage the problem is considered of the amount of heat received per unit area on a rotating planet devoid of atmosphere with a sun fixed in distance and direction. Account is then taken of the elliptic orbit and obliquity of the ecliptic to determine the "radiation-constant" day by day. In particular, the amount of heat received in each season of the year is expressed by formula.

Up to this point the work is practically a branch of celestial kinematics. At the next step the loss of radiation due to absorption by an atmosphere etc. is considered, especially as it tends to diminish the intensity of oblique rays. This factor has a large influence on the temperature of points near the arctic circle. It is due to this factor that a variation in the obliquity of the ecliptic may produce large changes in the climate of higher latitudes.

Passing over an excursion relative to the effect of conduction of heat in the solid crust, the next topic is the determination of the loss caused by the atmosphere. This loss consists partly of scattering in the air, and reflection by clouds and by the surface of the earth, partly by true absorption with re-emission at a much longer wave-length. One outstanding fact is that the atmosphere absorbs far more of the outwardly emitted long-wave radiation than of the incoming visible light.

To give a mathematical theory it is necessary to consider the thermal mechanics of gases. Two simplifying assumptions may be made. Of these, adiabatic equilibrium is shown to lead to results quite out of keeping with observation. On the other hand, the assumption of temperature equilibrium under absorption and emission of radiation appears to fit the facts well, and when there are two gases with distinct properties a reversal of temperature gradient is shown theoretically to be possible. A particular example is given by a mixture of oxygen and carbon dioxide such as that actually in the atmosphere.