THE FEBRUARY MEETING OF THE AMERICAN MATHEMATICAL SOCIETY

The two hundred twenty-first regular meeting of the Society was held at Columbia University on Saturday, February 25, extending through the usual morning and afternoon sessions. The attendance included the following seventy-five members:

Alexander, Beal, Birkhoff, Borden, Bowden, E. W. Brown, T. H. Brown, E. T. Browne, B. H. Camp, G. A. Campbell, Cole, Coolidge, Cowley, Cronin, Dantzig, Douglas, Eisenhart, Fine, Fischer, Fite, Philip Franklin, Fry, Gafafer, Gilman, W. C. Graustein, Gronwall, Grove, Haskins, Hausle, Hebbert, Robert Henderson, Hitchcock, Huntington, Dunham Jackson, Joffe, Kellogg, Kline, Lamson, Leavens, J. L. Love, McDonnell, McMackin, MacDuffee, J. B. Maclean, MacNeish, Mathews, H. H. Mitchell, Moir, Frank Morley, Mullins, Northcott, Oglesby, Osgood, Pfeiffer, Post, Press, Reddick, Reid, R. G. D. Richardson, Ritt, Ruger, Safford, Seely, P. F. Smith, Sosnow, Strong, Stuerm, Tanoue, J. S. Thompson, Wedderburn, Weisner, M. E. Wells, H. S. White, R. G. Wood, J. W. Young.

Thirty-five persons were elected to membership by the Council at this meeting as follows:

Dr. Anne Dale Biddle Andrews, University of California;
Professor Lloyd Collier Bagby, Ottawa University;
Professor Apolinario Baltazar, University of the Philippines;
Dr. John Douglas Barter, University of California;
Mr. John Biggerstaff, Reed College;
Professor Augustus Bogard, College of St. Teresa;
Mr. Garland Baird Briggs, University of Virginia;
Professor Frank E. Burcham, Central College;
Professor William Russell Burwell, University of Tennessee;
Mr. José Caminero, Havana;
Mr. Wendell Morris Coates, University of Michigan;
Professor Harold Athelstane Fales, Columbia University;
Professor Maurice Fréchet, University of Strasbourg;
Mr. Carl Arshag Garabedian, Harvard University;
Mrs. Eleanor Growe, University of California;
Professor Napoleon Bonaparte Heller, Temple University;
Professor Frank Kelso Hyatt, Pennsylvania Military College;
Professor Jesse Bythwood Jackson, University of South Carolina;
Professor Miles Abdel Keasey, Drexel Institute;
Professor Joseph Patrick Kelly, Boston College;
Mr. Arthur Louis Klein, California Institute of Technology;
Dr. Sophia Hazel Levy, University of California;

Mr. David A. Lunden, Cooper Union;
Professor Daniel Joseph Lynch, Boston College;
Professor James Thomas McCormick, Boston College;
Professor Patrick Joseph McHugh, Boston College;
Professor Pius Stephen Pretz, St. Benedict's College;
Miss Constance Rummons, University of Nebraska;
Professor Paul Gerhard Schmidt, St. Olaf College;
Professor William Henry Schuerman, Vanderbilt University;
Dr. Charles Donald Shane, University of California;
Professor George Waddel Snedecor, Iowa State College;
Miss Elizabeth Webb Wilson, Washington, D. C.;
Mr. Bing Chin Wong, University of California;
Professor Frank George Wren, Tufts College.

The Secretary also announced the election to membership, on January 16 (by special provision of the Council, under the suspension of By-Law I), of the following twenty-seven persons:

Mr. Rodney Whittemore Babcock, University of Wisconsin;
Mr. Raymond Walter Barnard, University of Michigan;
Professor Lawrence Henry Bowen, Furman University;
Mr. Martin Allen Brumbaugh, University of Pennsylvania;
Miss Elizabeth Carlson, University of Minnesota;
Professor Henry Clyde Carver, University of Michigan;
Miss Margaret Francis Chapman, Milwaukee State Normal School;
Professor Earl Church, Pennsylvania Military College;
Professor Charles Neblett Dickinson, Hollins College;
President Frances Augustine Driscoll, Villanova College;
Dean Robert Ryland Fleet, William Jewell College;
Mr. William Rogers Herod, General Electric Company, Schenectady;
Mr. Hemphill Hosford, Southern Methodist University;
Professor George Harold Jamison, Missouri State Teachers College, Kirksville;
Mr. Bradford Fisher Kimball, Harvard University;
Professor Paul Victor Levain, Holy Cross College;
Mr. James Lee Love, Gastonia Cotton Manufacturing Company;
Professor Ross W. Marriott, Swarthmore College;
Professor Ernest Alanson Pattengill, Iowa State College;
Professor William Richard Ransom, Tufts College;
Miss Margaret A. Shelley, College of St. Catherine;
Professor Hugo Ferdinand Sloctemyer, St. Louis University;
Mr. Sinclair Smith, California Institute of Technology;
Professor Arthur Dodd Snyder, Union College;
Professor Frederick Joseph Taylor, College of St. Thomas;
Mr. Wilmer Nichols Thompson, Drury College;
Professor Dudley Weldon Woodard, Howard University.

At the meeting of the Council, a letter was read from Professor E. V. Huntington reporting that an anonymous donor had offered to give the sum of $4,000 to print an additional volume of the TRANSACTIONS in 1922; the Council accepted the offer, with thanks to the donor and the intermediary. The Society also adopted a resolution of thanks for this very generous gift when it was announced by the Secretary at the morning session.

The Council appointed a committee consisting of Professors P. F. Smith (chairman), Eisenhart, and Hedrick, with power to arrange for the printing of the Society's publications; this committee was in particular authorized to proceed with the immediate publication of the additional volume of the TRANSACTIONS.

Professor C. N. Haskins was elected to succeed Professor L. E. Dickson as representative of the Society on the National Research Council for a period of three years beginning July 1, 1922.

On recommendation of the Council, the Society voted to extend the suspension of By-Law I until the end of April, and also to adopt the following substitute for Section 3 of By-Law II:

"Any member not in arrears of dues may become a life member on the payment of a sum to be determined in accordance with the principle that the life membership be regarded as the present worth of a life annuity due of a yearly payment equal to the net value of the annual dues, the annuity to be based on McClintock's 4 per cent *Tables for Annuitants* (*Male Lives*)."

Professor H. B. Fine presided at the morning session, relieved in the afternoon by Professors C. N. Haskins and H. S. White. The afternoon session was especially marked by a paper read, at the request of the programme committee, by Professor J. L. Coolidge, on *The basis of mathematical probability*. A number of members of the Actuarial Society were present, by invitation, to hear this paper.

Titles and abstracts of the papers read at this meeting follow below. Mr. Langer's paper was read by Professor Birkhoff, and Professor Schwatt's two papers by Professor

H. H. Mitchell. The papers of Professor Kellogg, Dr. Walsh, and Professors Emch, Morenus, and Hollcroft were read by title.

1. Professors G. D. Birkhoff and O. D. Kellogg: *Invariant points in function space.*

In this paper, the authors announce some of their results on the problem communicated to the Society on Dec. 30, 1920 (this BULLETIN, vol. 27 (1921), p. 307). The existence is established of invariant points in a region of n-space which is convex toward an interior point, under a continuous, one-valued transformation which carries points of the region into points of the region; also of the inverses of points on the hypersphere in n-space (n odd) with respect to a parametric transformation containing the identity. A number of analogous theorems are thus inferred for function space, first by a method of interpolation, and second, by a transition through a Hilbert space. Some theorems on linear transformations are derived directly. Applications are given to existence theorems for differential equations and integral equations, non-linear and linear. Incidentally, a simple proof is given of the important theorem that the general algebraic m-spread in n-space is non-singular, of which the authors have not found a demonstration in the literature.

2. Professor O. D. Kellogg: *A property of certain functions whose Sturmian developments do not terminate.*

In this note, the author reports the generalization to developments in terms of solutions of differential equations of the Sturm type of a result previously announced for Fourier series (see this BULLETIN, vol. 18 (1911), p. 234). A theorem on the rate of growth of the maximum of the absolute value of the nth derivative of an analytic function, on a circle interior to a region of analyticity, as a function of n, is also given.

3. Professor G. D. Birkhoff and Mr. R. E. Langer: *The boundary problems and developments associated with a system of ordinary linear differential equations of the first order.*

This paper develops the theory of a system of n ordinary linear differential equations of the first order containing a parameter and subject to certain boundary conditions; the notation of matrices is used. The major portion of the treatment is devoted to the vector equation $\gamma'(x)\cdot = \{A(x)\lambda + B(x)\}\gamma(x)\cdot$

and to the system comprised of this equation and linear boundary conditions. The elements of $A(x)$ and $B(x)$ are assumed to be differentiable, and λ is a complex parameter. The asymptotic form of the solutions of the equation, and the distribution of the characteristic values of the system under regular and particular irregular conditions are obtained if the roots $\gamma_j(x)$ $(j = 1, 2, \cdots, n)$ of the equation $|a_{ij}(x) - \delta_{ij}\gamma(x)| = 0$ are non-vanishing and distinct, and satisfy the condition $\arg\{\gamma_j(x) - \gamma_i(x)\} = c_{ij}$ (c_{ij} a constant). Finally the formal development of a vector of arbitrary functions, $F(x)\cdot$, into a series of characteristic functions is derived and is proved to converge when the points $\gamma_j(x)$ $(j = 1, 2, \cdots, n)$ are either constrained to lie on a line through the origin of the complex plane (in particular are all real), or form the vertices of a polygon which, as x varies, at most expands or contracts about the origin.*

This paper will appear in the PROCEEDINGS OF THE AMERICAN ACADEMY.

4. Mr. R. E. Langer: *Developments associated with a boundary problem not linear in the parameter.*

It appears that the ordinary differential equations which have heretofore been used as sources of characteristic functions in expansion problems are each equivalent to a system of linear, first order equations of the type

$$g_i(x)u_i'(x) = L_i(u) + \lambda M_i(u) \quad (i = 1, 2, \cdots, n), \tag{1}$$

where $g_i \not\equiv 0$, while L_i and M_i are linear expressions in $u_1, \cdots, u_n$, $M_i \equiv 0$ for $i \neq n$, $M_n \not\equiv 0$.† The author discusses in this paper the expansion problem which results from choosing as a source of characteristic functions an equation which is equivalent to a system of type (1) in which $g_i \equiv 0$ for $i \neq n$, $g_n \not\equiv 0$, $M_i \not\equiv 0$ $(i = 1, 2, \cdots, n)$. The equation in question with boundary conditions is $P(\lambda)u'(x) = Q(\lambda, x)u(x)$, $u(a) = hu(b)$, $P(\lambda)$ and $Q(\lambda, x)$ being polynomials in λ which satisfy certain restrictions. It is shown that the characteristic values of the system cluster about certain points, and that n arbitrary functions may be formally developed with one determination of coefficients into series. These developments are shown to reduce under certain conditions to Four-

* TRANSACTIONS OF THIS SOCIETY, vol. 9 (1908), pp. 373–395.

† See a joint paper by Professor Birkhoff and the author, reported at this meeting, where a system in which $g_i \not\equiv 0$, $M_i \not\equiv 0$ $(i = 1, 2, \cdots, n)$, is treated.

ier's expansions and are proved in every case (with usual restrictions on $f_i(x)$) to converge like Fourier's series.

5. Professor L. P. Eisenhart: *Ricci's principal directions for a Riemann space and the Einstein theory.*

In 1904 Ricci developed the idea of principal directions in a Riemann space of n dimensions. In doing so he introduced the contracted curvature tensor, which is fundamental in the Einstein theory, and thus gave a geometrical interpretation to it. The principal directions are those for which the mean curvature takes maximum and minimum values, the mean curvature at a point for a direction h being the sum of the Riemann curvatures for the $n - 1$ directions determined by h and each of $n - 1$ directions orthogonal to h. The present paper points out that a space in which the principal directions are completely indeterminate may be thought of as possessing a homogeneous character. Applying these considerations to spaces of four dimensions, it is shown that the three types of space chosen by Einstein in 1914, 1917 and 1919 as spaces free from matter are of this homogeneous character, and include all types of such spaces.

6. Dr. Jesse Douglas: *Normal congruences and quadruply infinite families of curves.*

The present paper deals with a general geometric problem suggested by a theorem of Thomson-Tait and a converse theorem of E. Kasner: to classify quadruply infinite families of curves in space, $y'' = F(x, y, z, y', z')$, $z'' = G(x, y, z, y', z')$, with respect to normal congruences contained within them. The analysis is based on the equations of variation of the differential equations of the family, and leads to an infinite system of partial differential equations of the Monge-Ampère type. The infinite system of equations is discussed as to its complete integrability, resulting in the following classification of families of ∞^4 curves: (1) ∞^∞ normal congruences, where the infinitude is that of all the surfaces of space. This is the case of the theorems of Thomson-Tait and Kasner, where the family is of the "natural" type. (2) ∞^∞ normal congruences, where the infinite exponent represents the generality of two arbitrary functions of one argument. (3) ∞^∞ normal congruences, where the infinite exponent corresponds to one arbitrary function of one argument. (4) ∞^3 normal congruences. (5) ∞^2 normal congruences. (6) No normal congruences, or a finite number, or an infinite number (which must be ∞^1 or ∞^∞), but all confined to a triply infinite sub-family. This is the general case.

7. Dr. T. H. Gronwall: *Qualitative properties of the ballistic trajectory. Second paper.*

In the last section of the author's first paper with this title (ANNALS OF MATHEMATICS (2), vol. 21 (1920), pp. 44–65), it is shown that under certain very broad assumptions on the G- and H-functions, the velocity of the projectile has only a finite number of maxima and minima. The present paper gives further information regarding the existence and location of these extremes.

8. Dr. T. H. Gronwall: *The reflection of X-rays in a finite number of equidistant parallel planes.*

In the PHYSICAL REVIEW (May, 1921), Dr. Lamson has solved the corresponding, and simpler, problem for an infinite number of planes. The present paper reduces the problem for a finite number of planes to a non-linear difference equation of the second order, the solution of which is obtained algebraically.

9. Professor J. L. Coolidge: *The basis of mathematical probability.*

Certain historical and philosophic aspects of the mathematical foundations of the theory of probability were presented in this paper, which was read at the request of the programme committee.

10. Professor E. V. Huntington: *On the Alabama paradox in the problem of apportionment.*

If the problem of apportionment is regarded as a problem in minimizing some form of *total error*, there are 80 or more summation formulas to be considered. (See, for example, the writer's paper presented at the Wellesley meeting, September, 1921; this BULLETIN, vol. 28 (1922), p. 15.) Two of these formulas are known to lead to the method of major fractions, and two to the method of equal proportions. It is here shown that all the other known summation formulas lead to the *Alabama paradox*, that is, an increase in the total size of the House may involve a decrease in the representation of one of the states.

(On the general problem of apportionment, see papers in the QUARTERLY PUBLICATION OF THE AMERICAN STATISTICAL ASSOCIATION, September 1921, and December, 1921.)

11. Professor E. V. Huntington: *On the d'Hondt method of apportionment, and its counterpart.*

If the problem of apportionment is regarded as a problem in minimizing the inequality between every state and every other state, there are five different comparison formulas to be considered, leading to five distinct methods of apportionment. The "amount by which the population of the over-populated state must be reduced to bring it down to parity with the other state," taken in the sense of absolute differences, is shown to lead to the d'Hondt method. The "amount by which the population of the under populated state must be increased to bring it up to parity with the other state," taken in the sense of absolute differences, is shown to lead to the contra-d'Hondt method, a new method which favors the small states extremely, just as the d'Hondt method favors the large states extremely. But each of these tests, if taken in the sense of relative differences, leads to the method of equal proportions.

12. Dr. G. A. Pfeiffer: *Theorems on sequences of sets of points.*

The first theorem of this paper is the following: If the sum of the sequence of sets of points, $\{A_n\}$, is compact in a metric space, then there exists a subsequence of $\{A_n\}$ whose lower closed limit* set is identical with its upper closed limit set and is between the upper and lower closed limit sets of the given sequence. An obvious modification of the definitions and proof of this theorem gives an analogous theorem for sets of point sets. Essential use is made of the above theorem in proving the following theorems: (1) If the sum of a sequence of connected sets of diameter $\leqq \delta$ is compact in a metric space, then the lower closed limit set of the sequence is contained in a closed connected set (a continuum) which has a diameter $\leqq \delta$ and which is contained in the upper closed limit set of the sequence. (2) If the sum of a sequence of continua $\{A_n\}$ is compact in a metric space and if A_n and A_{n+1} have a point in common and $\Sigma\delta_n$, where δ_n is the diameter of A_n, is convergent, then the closed limit set of $\{A_n\}$ contains one and only one point.

* Lower (upper) closed limit set ≡ untere (obere) abgeschlossene Limes. See Hausdorff, *Grundzüge der Mengenlehre*, p. 236.

13. Professor C. A. Fischer: *The Fredholm theory of Stieltjes integral equations.*

A large part of the Fredholm theory has been developed for the equation $f(x) = \varphi(x) - \int_a^b \varphi(y) d_y K(x, y)$, by F. Riesz, where the integral defines a completely continuous transformation, without making use of the Fredholm determinant. In a previous paper Professor Fischer has found necessary and sufficient conditions which $K(x, y)$ must satisfy, in order that the transformation be completely continuous. In the present paper, modified definitions of Stieltjes integrals are given, and a set of conditions for $K(x, y)$ is found, under which the Fredholm determinant is defined for the above equation, and the classical Fredholm theory is applied directly. This theory neither includes, nor is included by, the Riesz theory.

14. Dr. J. L. Walsh: *A closed set of normal orthogonal functions.*

This paper presents a simple example of a closed normal orthogonal set of functions on the interval (0, 1). Each function takes on no value other than $+1$ or -1, except at a finite number of points; the nth function has precisely $n + 1$ zeros (that is, sign-changes) in the interior of the interval; and a large class of arbitrary functions can be expanded in terms of these functions. The set is somewhat analogous to one considered by Haar, but has properties more closely allied to those of the more familiar sets of orthogonal functions.

15. Professor Arnold Emch: *Kinematics in a complex plane and some geometric applications.*

This paper appears in this number of this BULLETIN.

16. Professor J. F. Ritt: *On functions with integrals of elementary character.*

Let w be an integral of any algebraic function of z. It is a result of Liouville's that if w can be expressed in terms of elementary functions, then

$$(1) \qquad w = \alpha_0 + c_1 \log \alpha_1 + c_2 \log \alpha_2 + \cdots + c_n \log \alpha_n,$$

where each α is an algebraic function and each c a constant. The present paper gives an extension of Liouville's result. It is shown that if w is a solution of an elementary transcendental equation, then w is of the form (1). It follows that no elliptic function is a solution of an elementary transcendental equation. The same fact can be proved with respect to the integral logarithm and to many other transcendental functions.

17. Professor Eugenie M. Morenus: *Geometric properties of the system of all the curves of constant pressure in a plane field of force.*

The problem here considered is: In a plane field of force, to find smooth curves along which a constrained motion is possible such that the pressure of a moving particle against the curve shall remain constant. For one required constant pressure c there are ∞^3 such curves in the plane. The triply infinite system S_c has four geometrical properties which are shown to be sufficient to identify a set of ∞^3 curves as curves of constant pressure in a field of force. The quadruply infinite set of curves obtained by employing all possible values of c has eight geometrical properties which are sufficient to characterize completely a set of ∞^4 curves as curves of constant pressure. Eight additional properties of the quadruply infinite set are demonstrated.

18. Professor W. C. Graustein: *Spherical representation of conjugate systems and asymptotic lines.*

It is well known that a system of curves on the Gauss sphere represents a conjugate system of curves on each of infinitely many surfaces. It is here shown that, if the ratio of the radii of normal curvature in the conjugate directions is prescribed subject to a certain condition, one of the required surfaces is determined to within its homothetics. The general result is peculiarly adaptable to the important special cases of conjugate systems which have equal point or equal plane invariants or are isothermal-conjugate. It also lends itself readily to the treatment of lines of curvature. By similar methods applied to asymptotic lines a generalization of the theorem of Dini, giving the condition under which a system of curves on the sphere is the spherical representation of the asymptotic lines on a surface, is obtained. The paper will appear in the ANNALS OF MATHEMATICS.

19. Mr. Charles Manneback: *The distribution of current in a long cylindrical conductor.*

The problem of determining the distribution of density of an alternating current in a cylindrical conductor under the influence of a remote return parallel wire leads to the well known partial differential equations of Laplace and Poisson, with boundary conditions. The solution of the problem has not received hitherto a satisfactory answer. In the present paper the equivalent integral equation

$$i(r,\theta) = \frac{\alpha^2 I}{2\pi} \log \frac{\delta}{D} + \frac{\alpha^2}{2\pi} \iint_s i\,(\rho,\phi) \log \frac{d}{\rho}\, dS\,(\rho,\phi)$$

is set up and the following solution, which is a uniformly convergent series,*

$$i(r,\theta) = -\sum_{n=1}^{\infty} \frac{I}{\pi a^2} \left(\frac{a}{\delta}\right)^n j\alpha a \frac{J_n(j\alpha r)}{J_{n-1}(j\alpha a)} \cos n\theta,$$

is derived. Several related problems are also treated. The method appears capable of quite wide application.

20. Mr. A. Press: *Operational solution of equations of nth degree.*

An equation of the form $f(y) = a$ can be solved operationally if a mathematical operator f^{-1} can be found such that operating on both sides of the equation gives $f^{-1}\cdot f(y) = f^{-1}(a) = y$; f^{-1} is the inverse functional form of f. The operator multiplication (X) of ordinary algebra provides the basis of the required functional form with the understanding that $X^m \cdot y = f(y) = y^m$. The f in an equation of the nth degree then takes the form $f = a_m X^m + a_{m-1} X^{m-1} + \cdots = \Sigma a_m X^m$. The corresponding inverse of f is assumed of the form $f^{-1} = \Sigma a_p X^p$ which may be an ascending or descending series of implied multiplications or roots. By developing the corresponding algebra of the multiplication X as an operator either the inversion of series is accomplished or the form of resolution first obtained by McClintock. The operational developments in ascending form give the real roots and in descending form the complex roots. The algebra is particularly valuable in determining the complex roots of Heaviside's determinantal equations.

21. Professor T. R. Hollcroft: *Maximal cuspidal curves.*

A maximal cuspidal curve is an algebraic plane curve which has the greatest number of cusps possible for a given order n and genus $p \leqq \frac{1}{6}(n-2)(n-3)$. For $p > \frac{1}{6}(n-2)(n-3)$ all the double points may be cusps so there is no limit to the number of cusps except the genus. First, formulas are found

* r, θ and ρ, ϕ = polar coordinates of points of the conductor's cross-section; I = total return current; i = density of induced current in the conductor; a = radius of the conductor; δ = distance between centers of the parallel wires; D = distance between the return wire and point r, θ; d = distance between points r, θ and ρ, ϕ; dS = element of the conductor's cross-section area; α = a complex quantity of argument $\pi/4$.

giving the singularities of maximal cuspidal curves from genus 0 to p_0, the genus of the curve of lowest genus of the minimum class m_0, and the number of curves of class m_0 (which are always maximal cuspidal curves) and their singularities. Next, beginning with the self-dual maximal cuspidal curves, formulas are derived giving the number of maximal cuspidal curves for each class from n down to m_0. From these the maximum and minimum genus of each class is found, thence an inequality giving the class of any maximal cuspidal curve of given order and genus $p > p_0$, and from this an inequality giving the maximum number of cusps for any algebraic curve of given order and genus $p > p_0$. Using this inequality together with the one found for $p \leqq p_0$, we have the following: The maximum number of cusps for any algebraic plane curve of order n and genus $p \leqq \frac{1}{6}(n-2)(n-3)$ is given by the greatest integer k satisfying the smaller of these two inequalities when both are real or the first when the second is imaginary: (1) $k \leqq \frac{3}{2}(n-2)+3p$; (2) $k \leqq 2(n+p) - \frac{1}{2}(11+\sqrt{24p-8n+25})$.

22. Professor I. J. Schwatt: *Method for the separation into partial fractions of powers of trigonometric functions.*

The separation into partial fractions of $\tan^p x$, $\operatorname{ctn}^p x$, $\sec^p x$ and $\csc^p x$ depends on the evaluation of expressions like

$$\left(1 \pm \frac{2x}{(2n+1)\pi}\right)^p \tan^p x, \quad x = \mp \frac{2n+1}{2}\pi;$$

$$\left(1 \pm \frac{x}{n\pi}\right)^p \operatorname{ctn}^p x, \quad x = \mp n\pi; \text{ etc.}$$

The methods devised and the results obtained in this paper are believed to be new.

23. Professor I. J. Schwatt: *The expansion of the continued product* $\Pi\,(x+k)$.

The methods developed in the preceding paper have enabled the author to obtain the general term of the expansion of the continued product $\Pi\,(x+k)$.

R. G. Richardson,
Secretary