to the classes in our American colleges, and we hope that they will be extensively used.

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SHORTER NOTICES.

Analytic Geometry of Space. By VIRGIL SNYDER AND C. H. SISAM. New York, Henry Holt and Company, 1914. xi+289 pp.

THIS is one of the series of mathematical texts prepared under the general editorship of E. J. Townsend. The announced plan, however, of selecting as joint authors a mathematician and an engineer or physicist has not been adhered to in this case. As would be expected accordingly, the book will make its first and strongest appeal to the student of pure mathematics. If there is a single " practical problem " in the entire volume the reviewer has failed to discover it.

The authors are well fitted for their task since each is a specialist in the geometry of space and both are teachers of wide experience. Moreover their book possesses remarkable homogeneity of style and spirit, due possibly to the fact that the junior author was a pupil of the senior. At any rate, if there was any sharp division of labor the internal evidence is difficult to detect.

The book in size is an unpretentious volume of some 250 pages exclusive of the last chapter, and the modest preface states that it is intended as an introductory course. But even a casual examination will disclose an astonishing number and variety of topics, while a detailed reading emphatically confirms the first impression. Thus besides the usual equations of lines and planes and the metric formulas for angles and distances are introduced polar, cylindrical, and spherical coordinates, linear systems of planes, the notion of duality, homogeneous coordinates, and the plane at infinity, all in the first 37 pages! Surely this is information and ideas in a form sufficiently concentrated to stagger the average American undergraduate. It is only the very large number of excellent exercises (about 150 in the two chapters) which saves him from the otherwise inevitable confusion.

Indeed in our opinion the chief fault of the work as a *text book* is an excessive diversity of topics, many of which are treated very sketchily. As an extreme instance, the Kummer surface is introduced and dismissed with a half page. Whereas the worst blemish as a *book* is the frequent monotony of the expository style. This reaches a climax in Chapter VI in which the short, choppy, declarative sentences grow exceedingly tedious, the more because of the " vain repetitions."

While it is true, as indicated in the preface, that a knowledge of the usual elementary courses is all that is logically presupposed, it must be admitted that the first approach to such subjects as homogeneous coordinates, duality, linear systems, and the absolute is much simpler in the plane. And a student would be far better prepared for the present course after a thoroughgoing course in advanced plane analytic geometry. This statement applies to the entire book, but nowhere have we found the slightest hint of the natural method of attacking most topics in space, viz., that of generalization from the plane.

The book is divided broadly into two parts of approximately 100 and 150 pages respectively. The treatment in the first part is chiefly metric and " can be regarded as a first course not demanding more than 30 or 40 lessons." The subject matter of the first two chapters has already been indicated. The exposition is for the most part clear and concise but there are some obscurities of language. On page 1 it is stated that any point in space has three real coordinates. If one is to speak of real numbers, why not real points? " Orthogonal projection " is defined on page 3, while the theorems of page 4 use " projection " as the equivalent. The proof, page 12, that the three points are not collinear might have been made a little more explicit for the beginner who has not yet regarded the formulas of § 6 as parametric equations of the line. On page 21 " real value " is again associated with " point " without qualification. Article 23 is the first echo of the classical C. Smith, which has given analytic geometry of space such an awesome reputation in our colleges. Such puzzles are doubtless stimulating to the occasional American student, but the statement should be free from ambiguity. The wording of the paragraph is awkward and the figure is inappropriately lettered. PP' ought rather to be P_1P_2 and should be defined. In any case d refers to the other common secant.

The introduction of plane coordinates and elements at infinity is admirable, being simple, direct, and lucid. Moreover we commend the early discussion of these subjects (pages 31–35), since the notions are not intrinsically difficult and opportunity is afforded for the needed practice in their use. Not only is euclidean geometry greatly enriched, but the foundations are laid for the easy and natural transition to projective geometry.

Chapter IV is prefaced by a satisfactory treatment of geometric imaginaries. It is a matter of regret that imaginary elements are not recognized in our current texts on an equal footing with real,—that, e. g., such expressions as "point sphere" (§ 48), "the ellipse reduces to a point" (§ 56) should survive in the present text.

Chapter V is devoted to the sphere. We are glad to find a discussion of the absolute in this connection, for the subject belongs properly to metric geometry, though curiously enough it is usually included with projective.

In Chapter VI forms of quadric surfaces are studied from the standard equation by means of plane sections. This is a useful chapter, serving as an introduction to the following. Chapter VII contains an excellent treatment of the general equation of the second degree following the usual lines. Some of the important results are employed in § 75 to formulate a method of attack in discussing any particular quadric. This article would be improved pedagogically if it were amplified into a summary by including the criteria of §§ 66 and 73. The reduction of the general equation to the canonical forms and hence the complete classification of quadrics is effected with small labor and without actually applying the formulas for the transformation of coordinates.

The authors mention the definitions of page 76 among the features of Part I. In our opinion they are a bad feature unless the associated type of surface is indicated. What student would ever guess from the definition that "line of vertices" is the common line of two intersecting planes, or even that the quadric degenerates? Worse still is "plane of centers" and "plane of vertices" with the even greater specialization involved.

The second part of the book (Chapters IX–XIII) is devoted almost exclusively to the projective geometry of space. Many advanced students will find this part an attractive introduction to the modern methods in the subject, while the variety of topics and the completeness of some of the results make it a valuable reference book as well.

Chapter IX is introductory and has to do with the tetrahedral coordinate system, duality, and the transformation of coordinates. The projective properties of quadric surfaces are developed in Chapter X and simplified forms of the general equation obtained for various reference systems.

A special feature of the entire book is the extensive space given to linear systems. Systems of planes and spheres are considered in Part I. The whole of Chapter XI, the longest in the book (40 pages), is occupied with linear systems of quadrics. An exhaustive classification of pencils of quadrics by the theory of elementary divisors is obtained. And this study is paralleled in Chapter XII by a complete classification of the types of projective transformations of space. Bundles (systems of three) are also examined at some length. Webs (systems of four) are considered briefly and the Weddle and Kummer surfaces defined by means of them.

The polar theory of surfaces is outlined in Chapter XIII. Space curves are introduced as intersections of surfaces and their properties deduced largely from this point of view. Fifteen pages are given to cubic and quartic curves, including a classification, metric in the case of the cubic since all space cubics are rational. The characteristic symbols for curves are likely to be confused (in § 186, e. g.) with those for pencils of quadrics, since they differ in some cases only by a comma. Parentheses might be used in one case instead of brackets.

The book closes with a chapter (23 pages) on metric differential geometry, after the modern fashion of appending a few pages on a subject itself requiring a treatise. Such supplements are necessarily fragmentary but it is surprising to see how much can be condensed into these introductory sketches.

A review would be incomplete without a statement regarding the exercises. It has been our fortune to see few books with such a wealth of good exercises. There is hardly a topic in all the incredible variety which is not amply illustrated, and they are suited to all classes and abilities of students. It is safe to say that any one who solves any considerable proportion of them must possess a pretty comprehensive grasp of the methods of analytic geometry as well as a very respectable store of its subject matter.

The following typographical errors have been noted:

P. 8, l. 4	for NP	read NP_2
P. 14, l. 23	" ON	" OM
P. 21, first eq. (18)	" b_2	" b_1
P. 28, l. 5	" D^2	" D_2
P. 63, last line	" $a^2\left(1-\frac{2}{c^2}\right)$	" $a^2\left(1-\frac{k^2}{c^2}\right)$
P. 64, eq. at bottom	" y	" z
P. 71, l. 4	" section	" sections
P. 72, § 64, eq. 2	" y^2/a^2	" y^2/b^2
P. 77, eq. (9)	interchange f and g	
P. 84, l. 17, second term	for κ	read κ^2
P. 85, last eq.	" f, g, h	" h, f, g
P. 92, l. 4	" (5)	" (6)
P. 97, l. 17	" y^3	" y^2
P. 101, mid. page	" $a\sqrt{\ }$	" $\pm a\sqrt{\ }$
P. 102, l. 5	" $c\sqrt{\ }$	" $\pm c\sqrt{\ }$
P. 103, l. 9	" hyperbolic	" parabolic
P. 115, l. 15	" $x)$	" (x)
P. 132, l. 2	" $A(xz)$	" $A(xy)$
P. 169, l. 2 (bottom)	" substantiated	" substituted
P. 177, l. 8	" poin	" point
P. 200, l. 17	" conic	" conics
P. 243, l. 9	" cubic	" quartic

R. M. WINGER.

Elementary Mathematical Analysis, a text-book for first year college students. By CHARLES S. SLICHTER, Professor of Applied Mathematics, University of Wisconsin. New York, McGraw-Hill Book Company, 1914. Price $2.50. xiv + 490 pp.

IN the older texts on pure mathematics the intellectual interest of the student in the subject was assumed, and the practical interest in applications was not given recognition. In many modern discussions of the place of mathematics in instruction the possibility of an intellectual interest in the subject, the possibility that a real need of reasoning, intelligent beings is satisfied by pure mathematics is denied and only that which ministers quite directly to the physical being is recognized. The present text, while it gives some attention to the intellectual side, places the real stress upon the applications to practical affairs, apparently justifying the study of mathematics because of its service to other sciences and to business.

Trigonometry, analytical geometry, and calculus have undoubtedly been made the fundamental mathematical studies