the different states for the training of this class of engineers. The authors go into these in detail. A composite cross-sectional view of present practise would show about the following.

At least one or two years of practical training under the guidance of a regularly appointed surveyor. This practical experience may precede or follow the course of instruction in the technical schools. An average of about four semesters in a technical school offering the special courses required. An opportunity to advance in the profession. Advancement based solely on merit.

The courses offered necessarily include trigonometry, algebraic analysis, analytic and descriptive geometry, the calculus, mechanics, sometimes the method of least squares, astronomy, geodesy, drafting, map-making, seminar.

A general survey is made of the methods of handling these various subjects. And one is not surprised to find that throughout there are emphasized all those which aim to develop the order, accuracy, simplicity, and efficiency so desirable in any engineer.

Among suggestions intended to secure a greater efficiency the authors include such as the founding of higher schools, a three-year course of study, and a longer apprenticeship.

ERNEST W. PONZER.

Algebraic Invariants. By LEONARD EUGENE DICKSON. (No. 14, Mathematical Monographs, edited by Mansfield Merriman and Robert S. Woodward.) New York, John Wiley and Sons, 1914. x+100 pp.

IN this brief introduction to the classical theory of invariants Professor Dickson puts the reader under a further debt of gratitude for the excellent and entertaining way in which he is led to a first acquaintance with the important subject of invariants. It is difficult to conceive how one could be more comfortably drawn into a knowledge of invariants and covariants than by the gradual and lucid processes of the early part of this book. In the first ten pages there is a progressive approach to the full notion of invariant, carried forward from things well known to the beginner by means of processes and ideas which are of intrinsic interest and value in themselves. After a similar gradual development of the notion of covariant the formal definitions of invariants and covariants are given on pages 14 and 15.

The book falls into three parts of nearly equal length. Part I (pages 1–29) treats of linear transformations both from the standpoint of a change of points of reference and from the standpoint of projective geometry. Its purpose is to lead the reader by easy stages into a proper conception of the nature of the subject. How this purpose is achieved may be seen in part by the way in which the notions of invariant and covariant are introduced and illustrated by means of the simplest examples. Certain covariants such as Jacobians and Hessians are discussed and their algebraic and geometrical interpretations are given. In particular, the Hessian is employed in the solution of the cubic equation and in the discussion of the points of inflection of the plane cubic curve.

In connection with the interesting illustrative examples and applications of Part I the author finds opportunity to derive or illustrate several general elementary theorems so that the reader is making substantial progress in the theory as he learns its meaning through examples.

Part II (pages 30–62) is devoted to a systematic development, in non-symbolic notation, of the algebraic properties of invariants and covariants, chiefly of binary forms. In the preceding part the reader has been prepared so as to be able to follow this treatment with comfort. There is here a compact but lucid treatment of the following topics: homogeneity, weight, annihilators, alternants, seminvariant leaders of covariants, number of linearly independent seminvariants, Hermite's law of reciprocity, fundamental systems of covariants, canonical form of the binary quartic, properties of invariants and covariants as functions of the roots, and differential operators producing covariants. Moreover, irrational invariants are illustrated (in § 35) by a carefully selected set of exercises.

In part III (pages 63–97) is given an introduction to the symbolic notation of Aronhold and Clebsch. This notation is first explained at length for a simple case and the reader is led gradually into its use through a carefully constructed proof of the fundamental theorem on the types of symbolic factors of a term of a covariant of binary forms. That a fundamental system of covariants is finite is proved (pages 70–76) by a method due to Hilbert. The theory of transvectants is developed so far as needed in the treatment of apolarity and its application to rational curves and in the inductive determination of fundamental systems of covariants. Finally, there is a discussion of the concomitants of ternary forms in symbolic notation.

R. D. CARMICHAEL.

Leçons sur la Théorie des Fonctions. Par EMILE BOREL. Deuxième édition. Paris, Gauthier-Villars, 1914. xi+259 pp.

FOR purposes of review the second edition of this valuable book by Borel may be divided into three parts: the body of the text exclusive of the extensive notes at the end (Chapters I to VI, pages 1–101); notes contained in the first edition (pages 102–134); notes added in the second edition (pages 135–256). The first two parts are reprinted without modification except for a single change in a matter of terminology in the theory of point sets. Since these parts have been before the mathematical public for many years (having been originally published in 1898), they call for no further review now. The third part consists of three notes numbered IV, V, VI; it makes up about half of the present volume.

Note IV (pages 135–181) is devoted to polemics concerning the transfinite and the demonstration of Zermelo. It contains a reprint of seven articles, principally by Borel, published from time to time during the years 1899 to 1914. These discussions are perhaps of more interest to philosophers than to mathematicians. A perusal of this note in comparison with earlier statements by Borel shows that his thought on some of the matters in consideration has undergone a marked evolution.

Note V (pages 182–216) is devoted to denumerable probabilities and their arithmetic applications.

Finally, Note VI (pages 217–256) is given to a development of the theory of measure and of integration from the point of view adopted by Borel in his definition of measurable sets. It contains the most important matter added in this new edition of the work. The subject is approached in a very elementary and simple way and the treatment is carried through to results of considerable generality both in the theory of measure and in the theory of integration. The treatment will serve conveniently as an introduction to the fundamental researches of Lebesgue in the theory of integration.

R. D. CARMICHAEL.