entheorie der Liniengeometrie," *Sitzungsberichte der k. Akademie der Wissenschaften*, Wien, 1889). The author states that he developed the subject without knowledge of Waelsch's work so that there naturally is divergence of treatment.

In many places the author has not made his meaning clear, and this coupled with frequent typographical errors makes the reading rather difficult. But after one has gone through with it the point of view gained is well worth the trouble.

Two kinds of symbols are defined, viz., ordinary and complex. Ordinary symbols are those which obey the commutative law of multiplication, complex symbols are those which do not. The ordinary coefficients are of the first kind, and the symbols used to represent the line coordinates are of the second kind. Thus for line coordinates we have

$$p_i p_k = p_{ik} = -p_{ki} = -p_k p_i.$$

The quantities p_i and p_k have no meaning except as symbols. After these definitions the general properties of complex symbols are discussed and applied to the linear and quadratic complex in three dimensions and to finding the invariants, covariants, and contravariants of systems of lines.

Then follows the discussion of linear systems of lines in higher dimensions. Here the author has contributed much new material. After reading the discussion of the linear complex in s_4 one appreciates how much more direct and simple is the treatment of the same subject by Castelnuovo. But nevertheless after the symbolism is built up it enables one to see much that might otherwise escape him.

Throughout the book the author has made good use of the idea of defining a line, curve, or in higher dimensions, a plane, etc., by the system of lines which cut it. This has a decided advantage for certain problems, since a single equation then represents the line, curve, etc.

The book closes with an excellent chapter outlining a general symbolic analytic geometry and setting forth some of its advantages.

C. L. E. MOORE.

Analytische Geometrie des Punktpaares, des Kegelschnittes und der Fläche zweiter Ordnung. Zweiter Teilband. Von Dr. OTTO STAUDE. Leipzig and Berlin, Teubner, 1910. iv + 452 pp., with 47 figures.

THIS volume continues the work reviewed in the October (1910) BULLETIN, the page and paragraph numbering being continuous, and the entire work containing one thousand pages. The valuable bibliography and notes relating to the whole work cover sixty pages at the end of the present volume. Since the character of the treatment of the various topics is the same as that in the earlier volumes, little comment seems necessary.

After classifying surfaces of the second class in various ways, the author devotes nearly one hundred pages to a detailed study of plane sections of surfaces of the second order. Circular and equilateral hyperbolic sections receive particular attention. The second part, containing three chapters, is devoted to confocal systems of quadrics, and to general focal properties, both of the surfaces and of their plane sections. In the first chapter, by regarding x, y, z, as homogeneous coordinates of the lines of a sheaf on a point O, instead of ordinary point coordinates in space, the properties of confocal systems of cones are very neatly established. The so-called elliptic coordinates of the lines of a sheaf, and of the points of space are explained and used to some extent in subsequent proofs. The axis complexes of confocal systems are discussed and a large number of theorems relating to confocal systems proven. The second chapter is devoted to the Amiot, the MacCullagh, and the Jacobi focal properties. To any one not familiar with the theorems, this chapter will prove interesting reading. The third chapter on broken focal distances contains a large number of theorems and formulas relating to metrical relations existing between the points of the focal ellipse and the points of the corresponding focal hyperbola, and between the points of two conjugate focal parabolas. From these focal properties, the "thread" construction of the ellipsoid is deduced, and the corresponding property and the construction of the ellipse on the plane are shown analytically to be a special case of this.

The third part consists of two long chapters on surfaces of the second order and of the second class in tetraedral coordinates. In the first chapter surfaces are again classified, conditions for degeneracy stated, plane sections discussed, etc., as previously done in ordinary coordinates. The complex of tangents to a surface and the six generator complexes are studied. The theorem is established that the generators fall into two systems, each system being composed of the lines common to three of the six linear generator complexes. These six complexes are thus divided into two groups of three each. The last chapter is devoted largely to polar properties. The orthogonal linear substitutions corresponding to the real polar tetraedra; the properties of tetraedra determined by four generators, two from each system; and the Pascal and Brianchon theorems in space are other topics in this chapter.

This work with its extensive bibliography, and hundreds upon hundreds of references to the literature on the subject, must represent a prodigious amount of labor and pains on the part of the author. As an encyclopaedic reference book it will undoubtedly be useful, although the index does not seem very satisfactory. For example, twenty or more pages are devoted to a discussion and comparison of the Amiot and MacCullagh focal properties, yet neither of these names appears in the index.

D. D. LEIB.

Festschrift zur Feier des 100 Geburtstages Eduard Kummers mit Briefen an seine Mutter und an Leopold Kronecker. Herausgegeben vom Vorstande der Berliner Mathematischen Gesellschaft. Leipzig, Teubner, 1910. 103 pp.

IT was very appropriate that the memorial address on Kummer should be delivered by K. Hensel, whose remarkable investigations on algebraic numbers entitle him to speak with authority on Kummer's chief triumph, the creation of ideal numbers. We find here an elementary and entertaining account of Kummer's early interest in Fermat's equation $x^\lambda+y^\lambda+z^\lambda=0$, where λ is a prime, which led him to investigate the numbers $a + b\alpha + \cdots + k\alpha^{\lambda-1}$, where α is a complex λ-th root of unity, and $a, b, \cdots, k$ are integers. It appears on excellent authority that Kummer at an early period supposed that he had a complete proof of the impossibility of Fermat's equation and laid before Dirichlet a manuscript purporting to give such a proof. The latter pointed out to Kummer that, although he had proved that any number $f(\alpha)$ was the product of indecomposable factors, he had assumed that such a factorization was unique, whereas this was not true in general. After years of study, Kummer concluded that this chaotic state of affairs following from the non-uniqueness of factorization was due to the fact that the domain of the numbers $f(\alpha)$ was too small to permit the presence in it of the true prime numbers, and was led to his epoch-making creation of ideal numbers. Without attempting to explain the elaborate machinery of Kummer's