SHORTER NOTICES.

The Collected Mathematical Papers of James Joseph Sylvester. (Edited by H. F. BAKER.) Volume III. xv + 688 pp. Cambridge, University Press, 1909.

THE first two volumes of Sylvester's works appeared in 1904 and 1908, and were reviewed by the present writer in the BULLETIN, volume 15 (1909), pages 232–239. The third volume contains chiefly the papers written while Sylvester was at the Johns Hopkins University. The majority of these papers relate to the theory of invariants, in particular with the use of generating functions to enumerate the complete system of concomitants of a system of quantics. In two of the earlier papers of this series Sylvester proved the dependence of two of the thirty fundamental forms of two binary quartics given by Gordan, and emphasized the importance of the English enumerative method in questions on the dependence of the invariants. Among the longer papers is that "On an application of the new atomic theory to the graphical representation of the invariants and covariants of binary quantics." On the origin of this paper on chemistry and algebra, Sylvester says: "Casting about, as I lay awake in bed one night, to discover some means of conveying an intelligible conception of the objects of modern algebra to a mixed society, mainly composed of physicists, chemists, and biologists, interspersed only with a few mathematicians, to whom I stood engaged to give some account of my recent researches in this subject of my predilection, and impressed as I had long been with a feeling of affinity if not identity of object between the inquiry into compound radicals and the search for 'Grundformen' or irreducible invariants, I was agreeably surprised to find, of a sudden, distinctly pictured on my mental retina a chemico-graphical image serving to embody and illustrate the relations of these derived algebraical forms to their primitives and to each other which would perfectly accomplish the object I had in view."

Mention should be made of the elaborate memoir on ternary cubic-form equations in regard to their solvability in rational numbers, in particular the cases in which an integer can be expressed as a sum of two cubes. The paper also treats of the arithmetical properties of cyclotomic functions and of polygons which can be inscribed and escribed about a cubic. The volume contains various short papers on the theory of numbers, in particular one on the totality of primes comprised within given limits.

The memoir on subinvariants and perpetuants belongs to this period. At the end of the volume appear some short papers on the theory of partitions; but Sylvester's chief work in this field will appear in the fourth (final) volume of his collected papers.

L. E. DICKSON.

Niedere Zahlentheorie. Zweiter Teil. By DR. P. BACHMANN. B. G. Teubner's Sammlung von Lehrbüchern auf dem Gebiete der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen. Band X, 2. Leipzig, 1910. x + 480 pp.

THE first volume of this work appeared in 1902 and was reviewed by the writer in the BULLETIN, volume 9 (1903), page 555. The present second volume has the subtitle Additive Zahlentheorie, a term suggested by Kronecker for the properties of numbers relating to their additive combinations.

The first chapter deals with arithmetical series of the nth order, polygonal and figurate numbers, the sum of the kth powers of the first n integers, the sum of the kth powers of n numbers in arithmetical progression, and at length with the theory of Bernoullian numbers. The second chapter deals with recurring series, in particular with those bearing the names Farey, Fibonacci, Fermat, Pell, and Dupré. Application is made to factorization of numbers of certain forms, perfect numbers, Fermat and Mersenne numbers. Here results are omitted that have been known for several years. For instance the number $2^{67} - 1$ is left in doubt although Cole has given two factors (BULLETIN, 1903, page 137). Chapters III–V relate to the theory of partitions. Chapter VI is entitled recursion formulæ and deals with various number theoretic functions. Chapters VII–VIII treat of the representation of a given number as a sum of like powers or as a sum of multiples of powers. Every positive integer can be expressed as the sum of four squares (but not always as the sum of three); as the sum of nine cubes (but not always as the sum of eight); as the sum of 37 fourth powers (though doubtless this limit is too high). Further theorems relate to the number of representations of an integer as the sum of four squares. The related investigations by Liouville are given at length. The final