these construct the pole of b, and connect it with C' by means of x' ; these lines are respectively conjugate to b. Thus a (ν, ν) correspondence between the lines of C' is established, having 2ν coincidences made up of π central collineations and μ having a, b and c' conjugate points, hence $2\nu = \mu + \pi$; and by duality $2\mu = \nu + \lambda$. If the number of correlations μ in the pencil having a given pair of conjugate points be denoted by $\mu(\alpha, \beta, \gamma, \delta)_7$, we may therefore write

$$\mu(\alpha, \beta, \gamma, \delta)_7 = (\alpha, \beta, \gamma + 1, \delta)_8.$$

Finally, if two conditions be suppressed, in $(\alpha, \beta\, \gamma, \delta)_8$ are ∞^1 axial and ∞^1 central collineations. The conjugate points will describe curves, the conjugate lines will envelop others. By use of the chapter on multiple correspondence developed in the first volume it is now possible to determine $(\alpha, \beta, \gamma, \delta)_8$ in each case.

Many of the details are omitted; in order to follow the argument in the later cases of the bundle frequent use must be made of Schubert's Enumerative geometry, Hirst's article in the *Mathematische Annalen*, volume 8, and in particular to the comprehensive memoir of the author in volume 12 of the *Mathematische Annalen*. Contrary to the promise in the preface of the first volume, the treatment is synthetic throughout the volume, thus making the development quite one-sided and restricted.

VIRGIL SNYDER.

Arithmétique Graphique. Les Espaces Arithmétiques, leurs Transformations. Par G. ARNOUX. Paris, Gauthier-Villars, 1908. xii + 84 pp.

THIS book treats of arithmetic spaces which have been defined and studied by the author in two previous books written under the same general title, "Arithmétique Graphique."* The second of these was reviewed by the writer in May, 1907.† The reader is referred to this review for the definition of arithmetic spaces and for a short account of Arnoux's use of them to furnish a graphical representation in the theory of numbers.

In this third book, the author is more interested in the properties of the arithmetic spaces themselves and not so much

* Les espaces arithmétiques hypermagiques, Paris, 1894. Introduction a l'étude des fonctions arithmétiques, Paris, 1906.

† BULLETIN, vol. 13, pp. 402–403.

in their applications. He has in mind the creation of what he calls a géométrie analytique arithmétique which shall be to the theory of numbers what ordinary analytic geometry is to algebra and analysis. He does not claim to have made more than a beginning in the working out of his idea, but clearly states that he has tried only to point out the possibility and the usefulness of such an attainment.

In the book under review the author uses coordinates more extensively than in his earlier work. He finds that the general idea of geometric transformation may be conveniently applied to arithmetic spaces, and that the general linear transformation in a two-dimensional arithmetic space changes arithmetic lines into arithmetic lines; that is, it is a collineation. This is true whether the arithmetic space be infinite or modular. In an infinite arithmetic space, the domain for the coordinates of points is the totality of integers, but in an arithmetic space of modulus m the domain consists of integers, modulo m. If m is a prime number, these integers form a field or finite algebra, and consequently the analytic geometry of the space is more like ordinary analytic geometry. In particular, the transformation called inversion is possible. Arnoux calls attention to this fact.

The reviewer takes this opportunity to mention the fact that Arnoux's k-dimensional arithmetic space of prime modulus p is what Veblen and Bussey have called a finite euclidean geometry and have denoted by the symbol $EG[k, p]$.* Arnoux has not given the theorem that the linear transformation in a k-dimensional modular arithmetic space ($k > 2$) of prime modulus is a collineation, nor has he made any study of groups of transformations in these finite euclidean geometries.

The book closes with a chapter of applications to number theory, magic squares, and Euler's problem of the 36 officers.

W. H. BUSSEY.

Fragen der Elementargeometrie, gesammelt und zusammengestellt von FEDERIGO ENRIQUES. Deutsche Ausgabe von DR. HERMANN FLEISCHER. II Teil. pp. xii + 348. Leipzig, B. G. Teubner, 1907.

ONE of the few unique books of the last few years on elementary mathematics was the collection of related monographs

* "Finite projective geometries," *Transactions Amer. Math. Society*, vol. 7 (1906), pp. 241–259. In particular, see §7.