angles. This approximation is then made closer by using the values of f at points where AB cuts the curve $f(x, y) = \text{const.}$ If the second approximation is not close enough, the process is repeated.

23. Herr Wagenmann correlates successive steps in the theory of evolution with series $-\infty, \cdots -2, -1, 0, 1, 2, \cdots, \infty$ along three coördinate axes developing successively the ideas of motion, mass, the nebular hypothesis and evolution of living organisms and of civilization. He finds that his method leads to a monistic philosophy — in fact to a pan-monism.

A. B. FRIZELL.

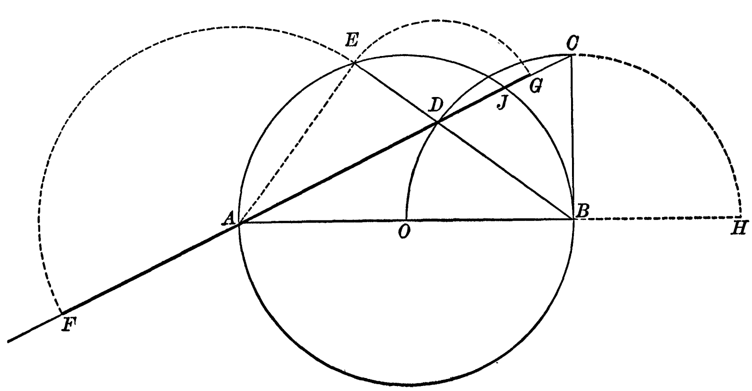
GÖTTINGEN,
November, 1906.

A NEW APPROXIMATE CONSTRUCTION FOR π.

BY MR. GEORGE PEIRCE.

GIVEN a circle with radius r and center at O; to find an approximate construction for πr.

Draw the diameter AOB and the tangent BC at right angles to it. Describe the arc ODC with radius r and center at B.



Draw the line AC cutting the arcs ODC and AB at D and J; also draw the line BDE through B and D cutting the given circle at E. Then $AD + 3DE = \pi r$ approximately.

Proof:

$$AC = \sqrt{(AB^2 + BC^2)} = r\sqrt{5}$$

$$AD = \frac{AO \cdot AH}{AC} = \frac{r \cdot 3r}{r\sqrt{5}} = \tfrac{3}{5}\sqrt{5}r,$$

$$JC = \frac{BC^2}{AC} = \frac{r^2}{r\sqrt{5}} = \tfrac{1}{5}\sqrt{5}r$$

$$DJ = AC - AD - JC = \tfrac{1}{5}\sqrt{5}r,$$

$$DE = \frac{AD \cdot DJ}{BD} = \frac{\frac{3}{5}\sqrt{5}r \cdot \frac{1}{5}\sqrt{5}r}{r} = \tfrac{3}{5}r,$$

$$AD + 3DE = \tfrac{3}{5}\sqrt{5}r + 3(\tfrac{3}{5}r) = 3.141641r.$$

By making use of the fact that in the triangle ABE

$$AE = \sqrt{(AB^2 - BE^2)} = \sqrt{(2r)^2 - (\tfrac{8}{5}r)^2} = \tfrac{6}{5}r = 2DE,$$

we can obtain a single line of the same length as $AD + 3DE$. We can therefore draw the arc EG with radius DE and center at D and the arc EF with radius AE and center at A. Then $AD + 3DE = AD + AE + DE = AD + FA + DJ = FG$.

There are many other approximate constructions for πr. A summary of those that have been worked out according to the method of geometrography is given below. A, B, C and D are to be found in the BULLETIN for January, 1902, page 137; E is in Cantor's Geschichte der Mathematik, volume 3, page 23; F is the construction given above.

			WITHOUT SQUARE.				WITH SQUARE.			
	Author.	Δ	S.	E.	Lines.	Circles.	S.	E.	Lines.	Circle
A	G. Peirce	+ .0012	22	14	4	4	17	11	4	2
B	Kühn	+ .0047	14	9	2	3	14	9	2	3
C	Lemoine	+ .0030	21	13	2	6	20	13	2	5
D	Pleskot	— .00016	24	16	3	5	24	16	3	5
E	Kochansky	— .000060	33	20	6	7	23	13	6	4
F	G. Peirce	+ .000048	24	15	4	5	19	12	4	3

Δ is the difference between the mechanically exact construction and π. S stands for simplicity and E for exactitude. For the technical meanings of these two words see the article in the BULLETIN for January, 1902. The lower these numbers are, the better the construction.