

Tangents and normals receive discussion in a separate chapter, and the analytic geometry of space is treated in the two concluding chapters as a direct generalization of the developments for the plane.

The "Elements," while designed to meet the needs of the same class of students as the shorter book, constitutes a somewhat longer course, and is sufficiently rich in material to allow of considerable latitude of choice. A valuable chapter is added on equations in parametric form, also one on invariants of the quadric with respect to motions of the plane, one on euclidean transformations, one on inversion, one on poles and polars, including polar reciprocation, and the analytic geometry of space receives a much fuller treatment. The chapter on "Line and quadric" treats of tangent lines and planes, diametral and polar planes, and circumscribed and asymptotic cones.

The general appearance of the book is exceedingly attractive. A variety of styles of type has been skillfully used to make clear the different characters of various parts of the text, as theorem, proof, rule, and example. The figures are excellent, clear and distinct, and mention should be made of the half-tones of models of quadric surfaces toward the end of the books, which will do a great deal in helping the student to a correct idea of these surfaces.

No estimate of a book can be quite satisfactory without a basis of class-room experience; but these two books certainly merit a trial, especially in institutions where the needs of the students are similar to those at Yale University.

O. D. KELLOGG.

PRINCETON, N. J.,  
July 25, 1905.

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## NOTES.

THE German mathematical society held its annual meeting, in affiliation with the association of German scientists and physicians, at Meran, Tyrol, September 24–30. Twenty-seven titles were announced on the preliminary programme, including reports "On the introduction of the calculus in the gymnasia," by Professor E. CZUBER; "On partial differential equations of physics," by Professor W. WIEN; "New investigations of Riemann's  $\zeta$ -functions," by Dr. P. EPSTEIN; "The status of

the arithmetic theory of algebraic numbers," by Professor J. WELLSTEIN. A full report of the meeting will appear in a later number of the BULLETIN.

THE annual meeting of the French association for the advancement of science was held at Cherbourg, August 3 to 8, under the presidency of Professor A. GIARD. Professor M. D'OCAGNE was president of sections I and II, mathematics, mechanics and geodesy. Besides various papers on astronomical subjects, the following mathematical papers were read: "Difficulties in rational mechanics," by Professor P. APPELL; "The forms of functional singularities defined by a vanishing discriminant," by Professor CLARK; "On kinematic methods of determining the radius of curvature," by Professor D'OCAGNE; "Construction of magic squares for the first three degrees," by M. TARRY; "On the number of primes between 1 and  $n$ ," by Professor LEBON; "Arithmetic spaces," by M. ARNOUX; "On the theory of polygonal numbers," by M. A. CABREIRA. After the meeting various excursions were made to points of interest in the vicinity of Cherbourg.

THE annual list of American doctorates published in *Science* presents for the academic year 1904-1905 324 names, of which 213 are credited to the sciences. The following 21 successful candidates offered mathematics as major subject (the titles of the theses are appended): O. P. AKERS, Cornell, "On the congruence of axes in a bundle of linear complexes"; R. B. ALLEN, Clark, "On hypercomplex number systems belonging to an arbitrary domain of rationality"; C. E. BROWN, Clark, "A study of the simple arithmetic processes"; Miss E. M. CODDINGTON, Columbia, "The historical development of pseudo-spherical surfaces"; W. B. FORD, Harvard, "On the problem of analytic extension as applied to functions defined by power series"; J. S. FRENCH, Clark, "On the theory of the pertingents to a plane curve"; J. N. GATES, Clark, "Cubic and quartic surfaces in four-fold space"; O. E. GLENN, Pennsylvania, "The determination of the abstract groups of order  $p^2qr$ ,  $p$ ,  $q$ ,  $r$  being distinct primes"; U. S. HANNA, Pennsylvania, "The bitangentials of the plane quintic and plane sextic"; A. M. HILTEBEITEL, Princeton, "The problem of two fixed centers and certain of its generalizations"; H. E. JORDAN, Chicago, "Group characters of various types of linear groups"; H. G. KEPPEL, Clark, "The cubic three-spread ruled with

planes in four-fold space"; R. B. McCLENON, Yale, "On simple integration with variable limits"; Miss A. M. MCKELDON, Pennsylvania, "Groups of order  $2^m$  that contain cyclic subgroups of orders  $2^{m-1}$ ,  $2^{m-2}$ ,  $2^{m-3}$ "; T. E. MCKINNEY, Chicago, "Concerning a certain type of continued fractions depending upon a variable parameter"; W. A. MANNING, Stanford, "Studies on the class of primitive substitution groups"; R. L. MOORE, Chicago, "Sets of metrical hypotheses of geometry"; J. C. MOREHEAD, Yale, "Numbers of the form  $2^a - 1$  and Fermat's numbers"; H. B. PHILLIPS, Johns Hopkins, "Some invariants and covariants of ternary collineations"; A. W. SMITH, Chicago, "The symbolic treatment of differential geometry"; R. P. STEPHENS, Johns Hopkins, "I. On a curve of the fifth class, II. On a system of parastroids." The number of American doctorates in mathematics for each year from 1898 to 1905 is 11, 13, 11, 18, 8, 7, 14, 21.

IN the new edition of the *Encyclopedia Americana* about to be issued by the *Scientific American*, special attention has been given to mathematics. Over forty articles have been written for the encyclopedia by American mathematicians on their several specialties. Professor C. J. KEYSER is in editorial charge of this department of the encyclopedia.

THE publishing house of B. G. Teubner in Leipzig has announced a collection of mathematical models which will henceforth be sold by the firm. The collection at present consists of 59 models, selected from the exhibit prepared by Professor H. WIENER for the third congress of mathematicians at Heidelberg in 1904. It is to be increased and systematized and is intended for service as illustration in the class room rather than for original investigation.

IN its issue of September 15, 1905, the *Enseignement mathématique* begins the publication of the results of its inquiry regarding the method of work of mathematicians, noticed in the BULLETIN, volume 11, page 95. The first installment of eight pages is mainly concerned with answers to question 1a, "At what age and under what circumstances did you acquire a taste for mathematics?"

THE various foreign universities named below offer courses in mathematics during the winter semester of 1905-1906 as follows:

UNIVERSITY OF BERLIN. — By Professor H. A. SCHWARZ: Analytic geometry, four hours; Theory of analytic functions, I., four hours; Applications of elliptic functions, 2 hours; Colloquium and seminar, two hours. — By Professor G. FROBENIUS: Theory of numbers, four hours; Seminar, two hours; — By Professor F. SCHOTTKY: Theory of abelian functions, four hours; Theory of plane potential, two hours; Seminar, two hours. — By Professor J. KNOBLAUCH: Approximate calculation of definite integrals, one hour; Theory of elliptic functions, four hours; Analytic mechanics, four hours. — By Professor G. HETTNER: Infinite series, products and continued fractions, two hours. — By Professor L. LEHMANN-FILHÉS: Integral calculus, four hours; with exercises, one hour. — By Dr. I. SCHUR: Differential calculus, four hours; with exercises, one hour; Irrational numbers, one hour. — By Dr. E. LANDAU: Algebra, four hours.

UNIVERSITY OF BONN. — By Professor E. STUDY: Differential geometry, four hours; Introduction to the calculus of variations, two hours; Seminar, two hours. — By Professor G. KOWALEWSKI: Elements of analytic geometry of the plane and of space, four hours; with exercises, one hour; General theory of functions, three hours; with exercises, one hour; Seminar, one hour. — By Professor F. LONDON: Differential and integral calculus, II., four hours; with exercises, one hour; Synthetic geometry, two hours; Seminar, one hour.

UNIVERSITY OF Breslau. — By Professor J. ROSANES: Elements of determinants, two hours; Modern methods in analytic geometry, three hours; Seminar, one hour. — By Professor R. STURM: Theory of geometric correspondences, II., four hours; Line geometry, two hours; Seminar, two hours. — By Professor A. KNESER: Differential and integral calculus, five hours; Theory of functions, three hours; Seminar, one hour. — By Professor G. LANDSBERG: Algebraic exercises, two hours; Theory of equations, four hours; Introduction to the theory of algebraic functions, two hours.

UNIVERSITY OF FREIBURG. — By Professor J. LUROTH: Plane analytic geometry and differential calculus, five hours; Calculus of variations, three hours; Seminar, one hour. — By Professor L. STICKELBERGER: Theory of differential equations, four hours; Elements of the theory of numbers, two hours. — By Professor P. KÖNIGSBERGER: Elements of par-

tial differential equations, three hours. — By Professor A. LOEWY: Algebraic analysis, four hours; Introduction to the mathematics of insurance, two hours; Seminar, one hour. — By Dr. K. SEITH: Projective geometry, two hours.

UNIVERSITY OF GIESSEN. — By Professor M. PASCH: Introduction to algebraic equations, four hours; Seminar, one hour. — By Professor E. NETTO: Differential and integral calculus, four hours; Analytic geometry of space, four hours; Seminar, two hours. — By Professor H. GRASSMANN: Geometric theory of functions with application to depiction, four hours; Graphical statics, three hours.

UNIVERSITY OF GREIFSWALD. — By Professor W. THOMÉ: Theory of potential, four hours; Calculus of variations, two hours; Seminar, two hours. — By Professor F. ENGEL: Analytic mechanics, I, four hours; Algebra, four hours; Differential equations and groups of transformations, two hours, Seminar, two hours. — By Professor K. TH. VAHLEN: Differential calculus, four hours; Theory of probabilities, two hours, with exercises, one hour.

UNIVERSITY OF HEIDELBERG: Professor L. KOENIGSBERGER: Analytic mechanics, four hours; Elliptic functions, two hours; Selected chapters of integral calculus (calculus of variations), two hours; Seminar, two hours. — By Professor E. VALENTINER: Planetary orbits, two hours. — By Professor M. CANTOR: Differential and integral calculus, four hours, with exercises, one hour; Political arithmetic, two hours. — By Professor K. KOEHLER: Analytic geometry of space, three hours. — By Professor K. BOEHM: Introduction to higher mathematics, three hours; Reading of memoirs, one hour.

UNIVERSITY OF KIEL. — By Professor L. POCHHAMMER: Theory of surfaces, three hours; Differential equations of one variable, three hours; Seminar, one hour. — By Professor L. HEFFTER (not yet announced): By Dr. E. WEINNOLDT: Graphical statics, two hours.

UNIVERSITY OF LEIPZIG. — By Professor C. NEUMANN: Differential and integral calculus, four hours; Seminar, one hour. — By Professor A. MAYER: Analytic mechanics, four hours, with exercises, one hour. — By Professor O. HÖLDER: Elliptic functions, four hours; Galois's theory of algebraic equa-

tions, two hours ; Seminar, two hours. — By Professor K. ROHN : Analytic geometry of space, five hours, with exercises, one hour ; Descriptive geometry, II, two hours, with exercises, two hours ; Seminar, two hours. — By Professor F. HAUSDORFF : Theory of transformation groups, three hours. — By Professor H. LIEBMANN : Theory of potential, two hours ; Graphical statics, two hours.

UNIVERSITY OF MARBURG. — By Professor K. HENSEL : Algebra, four hours ; Theory of surfaces and twisted curves, four hours ; Seminar, two hours. — By Professor E. NEUMANN : Theory of functions, four hours ; Analytic geometry of space, two hours ; Seminar, two hours. — By Dr. F. v. DALWIGK : Statics, two hours ; Applied mathematics, two hours ; Higher questions of elementary mathematics, one hour. — By Dr. F. JUNG : Integral calculus, five hours.

UNIVERSITY OF ROSTOCK. — By Professor O. STAUDE : Differential and integral calculus, four hours ; Theory of curves and surfaces, four hours ; Seminar, two hours. — By Professor J. WACHSMUTH : Theory of potential, three hours.

UNIVERSITY OF STRASSBURG. — By Professor TH. REYE : Modern methods in the analytic geometry of space, three hours ; Mathematics of elasticity of rigid bodies, two hours ; Seminar, two hours. — By Professor H. WEBER : Differential and integral calculus, four hours ; Encyclopedia of elementary mathematics, three hours ; Seminar, two hours. — By Professor M. SIMON : History of mathematics in antiquity, two hours. — By Professor J. WELLSTEIN : Elliptic integrals, two hours ; Determinants and matrices, three hours ; Seminar, two hours. — By Professor E. TIMERDING : Plane analytic geometry with exercises, five hours ; Graphical statics with exercises, three hours. — By Dr. P. EPSTEIN : Modern investigations in the theory of analytic functions, one hour ; Seminar (with Professor Weber), two hours.

UNIVERSITY OF TÜBINGEN. — By Professor A. v. BRILL : Introduction to higher mathematics, four hours ; Theory of algebraic curves, three hours ; Seminar, two hours. — By Professor H. v. STAHL : Higher analysis, II (integral calculus), four hours ; Partial differential equations, three hours ; Seminar, two hours. — By Professor L. MAURER : Descriptive geometry, II, with exercises, three hours ; Elliptic functions, two hours.

UNIVERSITY OF BASEL. — By Professor H. KINKELIN : Differential and integral calculus, I, three hours ; Definite integrals, two hours ; Partial differential equations, two hours ; Algebraic analysis, two hours ; Seminar, one hour. — By Professor O. SPEISS : Algebraic equations, two hours ; Synthetic geometry, two hours ; Elliptic functions, three hours.

UNIVERSITY OF BERN. — By Professor J. H. GRAF : Bessel functions, three hours ; Definite integrals and gamma functions, three hours ; Differential equations, two hours ; Differential and integral calculus, two hours ; Theory of functions, two hours ; Mathematics of investments and insurance, two hours ; Seminar, two hours. — By Professor K. OTT : Integral calculus, two hours ; Plane analytic geometry, II, two hours. — By Professor G. HUBER : Celestial mechanics, two hours ; Introduction to the theory of algebraic surfaces, three hours.

UNIVERSITY OF GENEVA. — By Professor C. CAILLER : Differential and integral calculus, three hours ; Rational mechanics, three hours ; Seminar, two hours. — By Professor H. FEHR : Algebra, two hours ; Analytic geometry, two hours ; Seminar, one hour. — By Dr. J. LYON : Algebraic theory of quadratic forms, one hour. — By Dr. R. DE SAUSSURE : Geometry of motion, two hours ; Mechanics of fluids, one hour.

UNIVERSITY OF LAUSANNE. — By Professor M. AMSTEIN : Differential and integral calculus, I, six hours ; II, two hours ; with exercises, two hours ; theory of functions, three hours. By Professor H. JOLY : Descriptive geometry, I, five hours ; Analytic geometry, two hours ; Synthetic geometry, two hours ; Plane curves, two hours. — By Professor O. MAYOR : Rational mechanics, five hours ; with exercises, one hour. — By Dr. C. JACCOTTET : Spherical harmonics, two hours.

ACADEMY OF NEUCHÂTEL. — By Professor L. ISELY : Calculus with geometric applications, three hours ; Projective geometry, two hours. — By Professor L. GABEREL : Analytic mechanics, two hours.

UNIVERSITY OF ZÜRICH. — By Professor H. BURKHARDT : Elements of differential and integral calculus, four hours ; Theory of potential, four hours ; Seminar, two hours. — By Professor A. WEILER : Descriptive geometry, I, with exercises four hours ; Synthetic geometry, three hours ; Analytic

geometry, two hours. — By Dr. E. GUBLER: Algebraic analysis with exercises, two hours; Determinants, one hour; Spherical trigonometry, one hour; Geometric pedagogy, one hour.

BEGINNING on Friday, December 1, 1905, Professor V. F. BJERKNES, of the University of Stockholm, will deliver at Columbia University a course of lectures on the subject of "Fields of Force." These lectures are open, without charge, to all teachers and advanced students of physics who may desire to attend. They will be given on Fridays from 4 to 6 P. M. and on Saturdays from 10 to 12 A. M., December 1–23 inclusive. The topics to be treated in the several lectures are enumerated below:

*December 1–2.*—Elementary investigation of the geometric properties of hydrodynamic fields (with experiments).

*December 8–9.*—Geometric and dynamic properties of electromagnetic fields according to Maxwell's theory.

*December 15–16.*—Transformation of the hydrodynamic equations to forms which prove the analogy of hydrodynamic and stationary electromagnetic fields. Further development and discussion of this analogy.

*December 22–23.*—General conclusions. Remarks on methods of research and of instruction in theoretical physics. Supplementary lecture on the hydrodynamic fields of force in the atmosphere and the sea. Discussion of the fundamental problem of meteorology and hydrography.

The lectures of December 1–2 are introductory and require no special preparation. The most important lecture is that of December 15. Vector analysis as developed by Gibbs will be employed in the course.

PROFESSOR J. H. GRAF, of the University of Bern, and Professor R. v. LILIENTHAL, of the University of Münster, have been elected rectors of their respective institutions for the present academic year.

PROFESSOR R. HAUSSNER, of the technical school of Karlsruhe, has been appointed professor of mathematics at the University of Jena.

PROFESSOR F. SCHUR, of the technical school at Karlsruhe, has declined a call to the technical school at Charlottenburg, as successor to the late Professor Guido Hauck.

PROFESSOR O. TUMLIRZ, of the University of Czernowitz, has been appointed professor of mathematical physics at the University of Innsbruck, as successor to PROFESSOR K. F. J. EXNER, who has retired from active service.

PROFESSOR CH. MÉRAY, of the University of Dijon, has retired from active teaching with the title of honorary professor.

DR. J. BENDIXON has been appointed professor of mathematics at the University of Stockholm.

THE following have been appointed masters of mathematical conferences at the various French universities: ESCLANGON at Bordeaux, DULAC at Grenoble, CLAIRIN at Lille, AUTONNE at Lyon, LEBESGUE at Rennes, BOURGET at Toulouse.

MR. J. H. JEANS, M.A., of Trinity College, Cambridge, England, has been appointed to a professorship of applied mathematics at Princeton University.

AT Columbia University Mr. A. R. MAXSON has been appointed tutor in mathematics.

MR. G. C. DALY, Mr. H. W. POWELL, and Mr. L. P. SIGELOFF have been appointed tutors in mathematics in the College of the City of New York.

DR. SAUL EPSTEEN has retired from the editorial staff of the *American Mathematical Monthly*, and has been succeeded by Professor O. E. GLEN, of Drury College.

AT Iowa College, Grinnell, Iowa, Professor S. J. BUCK has retired, having been professor of mathematics at that institution for over forty years. Assistant Professor W. J. RUSK has been promoted to a full professorship of mathematics.

DR. S. KRUSPER, professor emeritus of mathematics and geodesy at the technical school of Budapest, died July 2 at the age of 87 years.