is the exponent of the highest power of p dividing the order of G. A fortunate choice of notation gives the results and proofs a more compact and luminous form than is usual in so general an investigation in group theory. Repeated use is made of the theory of commutator subgroups. The paper gives a wide generalization, by different methods of proof, of the results in the author's article in the BULLETIN for May, 1904.

24. The second paper by Professor Dickson forms a preliminary chapter in his investigation, as research assistant to the Carnegie Institution of Washington, on the resolvents for the p-section of the periods of hyperelliptic functions of $2m$ periods. The group for this p-section contains subgroups isomorphic with the binary group Γ of determinant unity. Having determined all the subgroups of Γ, we derive at once the subgroups of the quotient group of linear fractional transformations. As the latter are required for the elliptic modular theory, the present procedure is an instance of mathematical economy.

M. W. HASKELL,
H. S. WHITE.

THE OCTOBER MEETING OF THE SAN FRANCISCO SECTION.

THE sixth regular meeting of the San Francisco Section of the AMERICAN MATHEMATICAL SOCIETY was held on Saturday, October 1, 1904, at the University of California. The following fifteen members were present:

Dr. E. M. Blake, Professor H. F. Blichfeldt, Professor G. C. Edwards, Professor R. L. Green, Professor M. W. Haskell, Professor L. M. Hoskins, Professor D. N. Lehmer, Professor A. O. Leuschner, Dr. J. H. McDonald, Professor G. A. Miller, Professor H. C. Moreno, Professor C. A. Noble, Dr. T. M. Putnam, Professor Irving Stringham, Professor A. W. Whitney.

Major P. A. MacMahon presided during the morning session and Professor Stringham during the afternoon session. During the morning session the following officers were elected for the ensuing year: Professor M. W. Haskell, chairman; Professor G. A. Miller, secretary; Professors M. W. Haskell, D. N. Lehmer, G. A. Miller, programme committee.

The following papers were read at this meeting:

(1) Professor E. J. WILCZYNSKI: "General projective theory of space curves."

(2) Professor L. M. HOSKINS: "Stresses in an elastic sphere due to a superficial layer of heavy matter of uniform thickness bounded by a circle."

(3) Dr. T. M. PUTNAM: "Concerning the factors of $(p^2-1)^2$ that are of the form $px+1$, and allied problems."

(4) Major P. A. MACMAHON: "Groups of linear differential operators."

(5) Professor H. F. BLICHFELDT: "On primitive continuous groups."

(6) Professor D. N. LEHMER: "Figures invariant in space of three dimensions under the most general projective transformation."

(7) Professor G. A. MILLER: "Determination of all the groups of order 2^m which contain an odd number of cyclic subgroups of composite order."

The programme provided also for a "conference on recent investigations in the foundations of geometry." This conference was opened by Professor Stringham, who was followed by Dr. J. H. McDonald. A number of the high school teachers of mathematics took part in the discussion, which had reference mainly to the influence of the recent investigations on the teaching of elementary geometry.

Major MacMahon was introduced by Professor Stringham. In the absence of the author, Professor Wilczynski's paper was read by Professor Haskell. Abstracts of the papers follow below: The abstracts are numbered to correspond to the titles in the list above.

1. Professor Wilczynski considers a linear differential equation

$$(1) \qquad y'''' + 4p_1y''' + 6p_2y'' + 4p_3y' + p_4y = 0,$$

together with its invariants and covariants under the transformation

$$(2) \qquad y = \lambda(x)y, \quad \xi = \xi(x),$$

λ and ξ being arbitrary functions. The covariants are functions of the semicovariants y and

$$(3)\qquad \begin{gathered} z = y' + p_1 y, \quad \rho = y'' + 2p_1 y' + p_2 y, \\ \sigma = y''' + 3p_1 y'' + 3p_2 y' + p_3 y. \end{gathered}$$

If $y_1, \cdots, y_4$ form a fundamental system of (1), these functions y_k may be taken as homogeneous coördinates of a point P_y on a curve C_y, the integral curve of (1). By means of (3), three other curves C_z, C_ρ, C_σ are obtained, which are closely connected with C_y. These curves are denoted as the derivatives of C_y with respect to x of the first, second and third kind respectively. If the variable x be changed, all of these curves except C_y itself are transformed into others. The paper is concerned with these curves and with the ruled surfaces generated by the edges of the tetrahedron $P_yP_zP_\rho P_\sigma$, each of which, corresponding to all of the transformations of the independent variable, depends upon an arbitrary function.

The author succeeds in defining these curves geometrically by introducing the notions osculating plane, conic, cubic and linear complex of the curve C_y. Among the surfaces generated by P_yP_σ there exists a one-parameter family of developables which corresponds to the reduction of (1) to the Laguerre-Forsyth canonical form. If upon the generator of each of these developables the point be marked where it meets the cuspidal edge, the locus of these points is a cubic curve called the torsal cubic, which has contact of the fourth order with C_y at P_y. This torsal cubic also enters largely into the theory. It coincides with the osculating cubic if the curve belongs to a inear complex.

The covariants may be defined geometrically by introducing the notion due to Halphen of the principal tangent plane of the curve C_y and its osculating cubic. By means of the osculating linear complex this defines a covariant curve C_z on the developable of C_y, and as a consequence also covariant curves C_ρ and C_σ. If, however, the curve belongs to a linear complex all of these curves coincide, so that a special treatment of this case becomes necessary. This is left for a future paper. The author notices, however, a very remarkable theorem for this case. If the curve belongs to a linear complex we may, in an infinity of ways, choose the fundamental tetrahedron so that four of its edges give rise to developables whose cuspidal edges are described by the four vertices. The other two edges of the tetrahedron will then give rise to ruled surfaces upon each of which the vertices of the tetrahedron trace a pair of asymptotic curves. The latter coincide with the two branches of the complex curve for the derived surface of the second kind. The paper will be offered to the *Transactions* for publication.

2. The general problem of determining the elastic displacements and stresses in a homogeneous spherical shell due to the action of known bodily forces and surface forces has been known as Lamé's problem. A general solution was first given by Lamé, another afterward by Thomson. The application of the general solution to special cases in which the surface and bodily forces are arbitrarily assumed is likely to be laborious. The case of a sphere acted upon by bodily forces due to the attraction of a thin surface layer of matter and by surface forces due to the weight of the layer was discussed by Darwin with reference to the problem of the stresses produced in the earth by the weight of continents. His computations were restricted to surface distributions which could be represented by even zonal harmonics of order not exceeding 12. The present paper by Professor Hoskins refers to the case in which the surface layer is of uniform thickness and bounded by any circle. This distribution is represented by an infinite series of zonal harmonics, each term of which introduces a term into the general solution. The summation of the resulting series is effected and general formulas are found for the stresses at all points in the axis of symmetry of the layer. Numerical results are given for several special cases.

3. Dr. Putnam considers a special case of the problem, to find the factors of a number which belong to a non-homogeneous linear form in any number of variables, the conjugate factor belonging to the same form. Besides the six factors which are always present in $(p^2 - 1)^2$, he shows that $px + 1$ (p being a prime) is a factor if $x = \frac{1}{2}(1 + \sqrt{4p + s})$ and $4p + s$ is a square. The latter condition requires that p be of the form $10n \pm 1$. Different statements of the problem were considered and certain other sufficient conditions for additional factors were developed.

4. Major MacMahon's paper was devoted to a study of linear differential operators closely related to his earlier investigations along this line. He considered the operators depending upon $\phi(u) \equiv \phi$ and $\psi(u) \equiv \psi$, where ϕ and ψ are analytic functions capable of expansion in ascending powers of x, and $u = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$. If

$$\phi = \phi(u) = \phi_0 + \phi_1 x + \phi_2 x^2 + \cdots,$$
$$\psi = \psi(u) = \psi_0 + \psi_1 x + \psi_2 x^2 + \cdots,$$

the derived operations may be written

$$(\mu, \nu, n, \phi) = \mu\phi_0\partial_{a_n} + (\mu + \nu)\phi_1\partial_{a_{n+1}} + (\mu + 2\nu)\phi_2\partial_{a_{n+2}} + \cdots,$$
$$(\mu', \nu', n', \psi) = \mu'\psi_0\partial_{a_{n'}} + (\mu' + \nu')\psi_1\partial_{a_{n'+1}} + (\mu' + 2\nu')\psi_2\partial_{a_{n'+2}} + \cdots.$$

It is found that the alternant of two such "functionally derived operators" is expressible as a linear function of operators derived from the three functions

$$\phi\psi', \qquad \phi'\psi, \qquad \int_0^u \phi'\psi'\,du,$$

where the accent denotes differentiation in regard to u.

The condition that the operator derived from

$$\int_0^u \phi'\psi'\,du$$

may be absent is that $\mu/\nu - n = \mu'/\nu' - n' = c$, a constant, and in that case the alternant is an operator derived from the single function $(c + n)\phi\psi' - (c + n')\phi'\psi$.

Designating c as the characteristic of the operator, the author considered the subgroups of operators of constant characteristic. The paper concludes with the following functional group, appertaining to operators which involve a finite number of variables: Let $x^{p+1} + b_1x^p + b_2x^{p-1} + \cdots + b_{p+1} = 0$ be a fixed relation, so that

$$u_p^m = (a_0 + a_1x + a_2x^2 + \cdots + a_px^p)^m$$
$$= A_{m,0} + A_{m,1}x + \cdots + A_{m,p}x^p,$$

and take an operator

$$\Omega_m = A_{m,0}\partial_{a_0} + A_{m,1}\partial_{a_1} + \cdots + A_{m,p}\partial_{a_p}.$$

It is easy to prove that the alternant of Ω_m and Ω_n is expressible in the manner

$$\Omega_m\Omega_n - \Omega_n\Omega_m = (\Omega_m.\Omega_n) = (n-m)\Omega_{n-m+1}.$$

These operators therefore form a transitive group of infinite order involving $p+1$ variables $a_0, a_1, \cdots, a_p$ and $p+1$ parameters $b_1, b_2, \cdots, b_{p+1}$. This group admits in all probability of further generalization.

5. At a previous meeting of the Society, Professor Blichfeldt stated the theorem that the number of parameters of a primitive continuous group in n variables is limited, being less than a number λ which depends only upon n. In the present paper it is shown that if the structure ("Zusammensetzung") of a primitive continuous group G be given, the construction of the infinitesimal transformations of G is reduced to solving a given set of algebraic equations and (possibly) to changing a given set of infinitesimal transformations with algebraic coefficients, generating an intransitive group G' in $n' > n$ variables, into a set generating the required group G, the invariant functions of G' acting as arbitrary constants of G. If the presence of arbitrary constants be allowed in the coefficients of the infinitesimal transformations of a group-type, it follows that the number of types of primitive groups in n variables is limited.

6. The results in Professor Lehmer's paper are well known (Muth, Elementartheiler, 1899, page 217). The methods used are believed to be new and the connection of the invariant figures with the character of the roots of the characteristic quartic is clearly shown.

7. In a recent number of the *Proceedings of the London Mathematical Society* Professor Miller proved the theorem, "The number of the cyclic subgroups of order $p^a (a > 1, p > 2)$ in any group G is of the form kp whenever the Sylow subgroups of order p^m in G are non-cyclic." In the present paper

he supplements this theorem by considering the case when $p = 2$. The main results may be stated as follows: If the Sylow subgroups of order $2^m (m > 1)$ contained in any group G are either cyclic or contain a cyclic subgroup of order 2^{m-1} which includes only two invariant operators under one of these Sylow subgroups, then the number of operators of order 2 in G is of the form $1 + 4k$. When this condition is not satisfied the number of these operators is always of the form $3 + 4k$. When $m = 1$, G contains an invariant subgroup which is composed of all its operators of odd order, and the number of the subgroups of order 2 may have either of the two forms $1 + 4k$, $3 + 4k$. This is the only case where the form of the number of the subgroups of order 2 is not determined by the form of this number in a Sylow subgroup.

G. A. MILLER,
Secretary of the Section.

THE FOUNDATIONS OF MATHEMATICS.*

The Principles of Mathematics. By BERTRAND RUSSELL. Volume I. Cambridge. The University Press, 1903. xxix + 534 pp.

Essai sur les Fondements de la Géométrie. Par BERTRAND RUSSELL. Traduction par A. CADENAT, revue et annotée par l'auteur et par L. COUTURAT. Paris, Gauthier-Villars, 1901. x + 274 pp.

1. *The Problem.* — Pure mathematics has always been conceived in the minds of its votaries and by the world at large to be a science which makes up for whatever it lacks in human interest, and in the stimulus of close contact with the infinite variety of nature, by the sureness, the absolute accuracy, of its methods and results. Yet what has been accepted as sure and accurate in one generation has frequently required fundamental revision in the next. Euclid and his pupils could doubtless have complained of the lack of rigor and logical precision in his predecessors just as forcibly as some modern pupils of Weierstrass berate their scientific ancestors and companions.

* We may also refer our readers to the review by L. Couturat, *Bulletin des Sciences Mathématiques*, vol. 28, pp. 129–147 (1904). So large is the work of Russell that Couturat's review and our own supplement rather than overlap one another.