THE OCTOBER MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

A REGULAR meeting of the AMERICAN MATHEMATICAL SOCIETY was held in New York City on Saturday, October 25, 1902. About forty-five persons attended the morning and afternoon sessions, including the following thirty-five members of the Society :

Professor Maxime Bôcher, Professor Joseph Bowden, Dr. J. E. Clarke, Professor F. N. Cole, Dr. W. S. Dennett, Professor A. M. Ely, Dr. William Findlay, Professor T. S. Fiske, Miss Ida Griffiths, Dr. E. R. Hedrick, Dr. G. W. Hill, Dr. A. A. Himowich, Dr. E. V. Huntington, Dr. S. A. Joffe, Dr. Edward Kasner, Dr. G. H. Ling, Mr. H. B. Mitchell, Dr. I. E. Rabinovitch, Professor J. K. Rees, Mr. C. H. Rockwell, Dr. P. L. Saurel, Miss I. M. Schottenfels, Professor C. A. Scott, Professor D. E. Smith, Dr. Virgil Snyder, Dr. H. F. Stecker, Miss Mary Underhill, Professor E. B. Van Vleck, Professor L. A. Wait, Mr. H. E. Webb, Professor J. B. Webb, Professor A. G. Webster, Miss E. C. Williams, Dr. R. G. Wood, Professor R. S. Woodward.

Vice-President Professor Maxime Bôcher presided at the morning session, Ex-President Professor R. S. Woodward at the afternoon session. The Council announced the election of the following persons to membership in the Society : Professor Sir R. S. Ball, Cambridge University, England ; Dr. Otto Dunkel, Wesleyan University, Middletown, Conn. ; Mr. W. H. Osborne, Purdue University, Lafayette, Ind.; Professor H. L. Rietz, Butler College, Indianapolis, Ind. ; Professor G. H. Scott, Yankton College, Yankton, So. Dak. ; Professor J. N. Van der Vries, University of Kansas, Lawrence, Kan. ; Professor B. F. Yanney, Mount Union College, Alliance, Ohio ; Mr. W. H. Young, Cambridge University, England. Seven applications for membership in the Society were received.

A list of nominations to be placed on the official ballot for the annual election of officers and other members of the Council was reported. A committee consisting of the Secretary and Professors W. F. Osgood, Oskar Bolza, James Pierpont, and

G. A. Miller was appointed to make arrangements for the next summer meeting and colloquium of the Society.

The following papers were presented at this meeting:

(1) DR. E. R. HEDRICK: "On the foundations of mechanics" (preliminary communication).

(2) DR. E. V. HUNTINGTON: "Definitions of a commutative group by independent postulates."

(3) PROFESSOR PETER FIELD: "On the infinite branches of plane curves which have no point singularities."

(4) DR. EDWARD KASNER: "The apolarity of double binary forms."

(5) PROFESSOR MAXIME BÔCHER: "An application of the Riemann-Darboux generalization of Green's theorem."

(6) PROFESSOR MAXIME BÔCHER: "Note on Laplace's equation."

(7) DR. VIRGIL SNYDER: "On the quintic scrolls having three double conics."

(8) MISS I. M. SCHOTTENFELS: "Note on the types of groups of order p^n every element of which, except identity, is of order p" (preliminary communication).

(9) DR. L. P. EISENHART: "Surfaces referred to their lines of length zero."

(10) PROFESSOR L. E. DICKSON: "Three sets of generational relations defining the abstract simple group of order 504."

(11) PROFESSOR L. E. DICKSON: "Generational relations defining the abstract simple group of order 660."

(12) DR. G. H. LING: "The approximate representation of functions by means of functions defined by quadratic equations."

(13) DR. C. N. HASKINS: "On the invariants of differential forms of degree higher than two."

Dr. Haskins's paper was presented to the Society through Professor Bôcher. In the absence of the authors, Professor Field's paper was read by Dr. Snyder, that of Dr. Haskins by Professor Bôcher, and the papers of Dr. Eisenhart and Professor Dickson were read by title.

The papers of Dr. Snyder, Dr. Eisenhart and Professor Dickson will appear in the BULLETIN. Abstracts of the other papers follow below. The abstracts are numbered to correspond to the titles in the list above.

1. In Dr. Hedrick's preliminary report, an attempt is made to establish a logical system of axioms, sufficient for the deriva-

tion of the elementary theorems in the mechanics of a system of particles. The ideas of space and time are made fundamental, space being taken as that configuration of points, lines and planes defined by Hilbert's set of axioms in his Grundlagen der Geometrie, while time is defined as a parameter, in terms of which the equations of any curve in space may so be expressed that these equations represent the motion of the particle in the ordinary sense. Axioms are then laid down which fix (or define) the ideas "before," "after," etc.

Mass is now defined in terms of time and space, on the basis of Newton's law of universal gravitation, previous axioms having required a certain form of motion in the case when two particles are isolated in space. Force is then defined as usual, and the remaining axioms necessary to the development of the elementary theorems are laid down. This preliminary report is intended to be a forerunner of a more complete discussion, and the purpose of presenting it at this time is to invite suggestion and criticism.

2. The definitions of an abelian (commutative) group given in Dr. Huntington's paper are simple modifications of the author's definitions of a general group published in the last volume of the BULLETIN.* According to the first definition, a set of elements, with a rule of combination $\circ$, will be an abelian group when the following three postulates are satisfied: 1° $a \circ b = b \circ a$, whenever a, b and $b \circ a$ belong to the set. 2° $(a \circ b) \circ c = a \circ (b \circ c)$, whenever a, b, $a \circ b$, $b \circ c$ and $a \circ (b \circ c)$ belong to the set. 3° For every two elements a and b there is an element x in the set, such that $a \circ x = b$. The second definition involves four postulates. The independence of the postulates of each set is established both for finite and for infinite groups. The paper will appear in the *Transactions*.

3. Professor Field's paper considers the form of sextic curves which have no point singularities and which have two essential infinite branches. It is shown that such curves may be unipartite or multipartite. The form and equation of a multipartite curve of degree $3n$ with no point singularities and with n infinite branches is thus obtained, and also the form and equation of a non-singular unipartite curve of degree $12n$ with

* Cf., on this subject an article by Professor E. H. Moore in the *Transactions* for October, 1902.

$4n$ essential infinite branches. The question as to the maximum number of essential infinite branches which a curve without point singularities may possess is then considered, it being shown that such a curve may have $n-4$ essential infinite branches and that this is the maximum number.

4. In a paper published in the first volume of the *Transactions* Dr. Kasner established a connection between the theories of double binary and quaternary forms. In the present paper the author applies this connection to certain questions as to apolarity. A double binary form $f_{nm}=f(x_1, x_2; y_1, y_2)$ of the nth degree in $x_1 : x_2$ and the mth degree in $y_1 : y_2$, may be interpreted as the general algebraic curve of species (n, m) upon a quadric surface Q. When the two degrees are equal the curve is a complete intersection curve, and there corresponds to the form f_{nn} a unique surface of nth order F_n and also a unique surface of nth class ϕ_n. If two forms f_{nn} and $f'_{n'n'}$, where $n \geqq n'$, are apolar, then every surface of the n'th order through the curve $f'_{n'n'}=0$ is apolar to the surface ϕ_n which corresponds to f_{nn}; conversely, if ϕ_n is apolar to any surface of n'th order through $f'_{n'n'}=0$, then the double binary forms are apolar. This theorem, applied to the case where one of the forms is bilinear, reduces the problem of constructing all the curves on a quadric surface which are apolar to a conic to the corresponding problem of plane geometry. The discussion of apolar relations with respect to forms f_{nm} where $m \neq n$ occupies the final part of the paper. The method is based on the introduction of certain systems of polar forms and the related systems of surfaces.

5. The form of Green's theorem here used will be found in the Encyclopädie, II, *A*7*c*, pages 513, 514. Using the notation and terminology there explained, it is here proved that if throughout a certain region S of the (x, y)-plane the partial differential equation $L(u)=0$ belongs to the elliptic type, and if u is a solution of this equation analytic throughout S, and if a function v exists continuous and having continuous first and second partial derivatives throughout S and satisfying the inequality $AM(v) \leqq 0$, then u cannot vanish at all points of a closed curve in S, unless it vanishes identically.

6. It is well known that if $f_0(y)$ and $f_1(y)$ are two functions which can be developed in power series about the point $y=0$,

there exists one and only one solution $u(x, y)$ of Laplace's equation in two dimensions which, when $x = 0$, reduces to $f_0(y)$ while its partial derivative with regard to x reduces to $f_1(y)$, and that this function $u(x, y)$ can itself be developed in a power series about the point $x = 0$, $y = 0$. The chief theorem proved in the present paper is that if the series f_0 and f_1 both converge when $|y| < r$, the double power series for $u(x, y)$ converges within the square whose vertices are the points $(\pm r, 0)$, $(0, \pm r)$.

8. Miss Schottenfels's note on types of groups of order p^n, all of whose elements (I excluded) are of order p, contains a proof of the existence of a group of indices (modulo p) where p is a prime, and the generational definition of the abelian group G_{p^n} every element of which is of prime order p.

12. Dr. Ling's paper contains a treatment of a problem of a type suggested by Hermite, viz., that of the approximate representation of a function by means of algebraic functions. Padé, after showing the relation of the representation of functions by means of power series to the general problem, treated the case of functions defined by linear equations. In the present paper is considered the case of functions defined by quadratic equations of the type $y^2 + Qy + P = 0$, where Q and P are polynomials. After a definition of *reduced function* has been given, the paper treats of the determination of such functions in the general case and of the order of approximation to a specified function which is obtained in each case. The results obtained are generalizations of the two theorems proved by Padé.

13. Dr. Haskins's paper is an extension of the work in the author's paper on the invariants of quadratic differential forms to forms of higher degree. The methods used are the same, but the difficulties of the problem are much less. The result is as follows: The number of invariants of order μ for the general homogeneous differential form of degree m in n variables is

$$(n, \mu)\left\{(n, m) - \frac{n(n+\mu)}{\mu+1}\right\}, \quad (n, m) = \frac{(n+m-1)!}{(n-1)!\,m!}$$

The case of simultaneous invariants of several forms is also considered.

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