THE ANNALS of PROBABILITY

AN OFFICIAL JOURNAL OF THE INSTITUTE OF MATHEMATICAL STATISTICS

Articles

Markov systems and their additive functionalsE. B. DYNKIN	653
On the uniqueness and nonuniqueness of proximity processes	
LAWRENCE GRAY AND DAVID GRIFFEATH	678
Convergence rates and r-quick versions of the strong law for stationary mixing se-	
quences	693
Convergence rates for the isotope discrepancy	707
The law of the iterated logarithm on arbitrary sequences for stationary Gaussian processes and Brownian motion	724
The exact Hausdorff measure of the zero set of certain stationary Gaussian processes P. Laurie Davies	740
Quadratic variation of functionals of Brownian motion .	
ALBERT T. WANG	756
An extension of stochastic integrals in the plane	
EUGENE WONG AND MOSHE ZAKAI	770
An inequality for sums of independent random variables indexed by finite dimensional	
filtering sets and its applications to the convergence of series JEAN-PIERRE GABRIEL	779
Two applications of a Poisson approximation for dependent events	•••
Norman Kaplan	787
Short Communications	
Ergodic theorems for the asymmetric simple exclusion process II	
Thomas M. Liggett	795
A note on planar Brownian motion	802
Interpolation of martingales	804
On one-dimensional diffusions with time parameter set (-∞, ∞)	
J. Theodore Cox	807
A generalization of the Karlin-McGregor theorem on coincidence probabilities and an application to clustering	814
Equilibrium measures for semi-Markov processes	
David R. McDonald	818
- square ranction modularly	823
On complete convergence of distributions and expected values of order statistics K. Alyappan Nair	826

Vol. 5, No. 5 October 1977

THE INSTITUTE OF MATHEMATICAL STATISTICS

(Organized September 12, 1935)

The purpose of the Institute of Mathematical Statistics is to encourage the development, dissemination, and application of mathematical statistics.

OFFICERS

President:

C. R. Rao, Indian Statistical Institute, 7, S.J.S. Sansanwal Marg, New Delhi 110001,

President-Elect:

Elizabeth Scott, Department of Statistics, University of California, Berkeley, California 94720

Past President:

Donald L. Burkholder, Department of Mathematics, University of Illinois, Urbana, Illinois 61801

Executive Secretary:

George J. Resnikoff, Department of Statistics, California State University, Hayward, 25800 Hillary Street, Hayward, California 94542

Treasurer:

Robert M. Elashoff, IMS Business Office, 3401 Investment Blvd., Suite 6, Hayward California 94545

Program Secretary:

William L. Harkness, Pennsylvania State University, University Park, Pennsylvania 16801

Editor: Annals of Statistics
Rupert G. Miller, Jr., Department of Statistics, Stanford University, Stanford, California 94305

Editor: Annals of Probability

Patrick Billingsley, Department of Statistics, University of Chicago, Chicago, Illinois 60637

Managing Editor:

Donald R. Truax, Department of Mathematics, University of Oregon, Eugene, Oregon

Membership. Membership dues including a subscription to one Annals and The Institute of Mathematical Statistics Bulletin are \$24.00 per year for residents of the United States or Canada and \$16.00 per year for residents of other countries. Special rates of \$12.00 per year are available to students. Rates in each category are one-third higher for members who wish both Annals as well as the Bulletin. Inquiries regarding membership in the Institute should be sent to the Treasurer at the business office.

Subscription Rates. Current volumes (six issues per calendar year) of the Annals of Probability and the Annals of Statistics are each \$30.00. Members of the Institute of Mathematical Statistics pay different rates (see above). Single issues are \$7.00. Back numbers of both Annals and the Annals of Mathematical Statistics (Volumes 1 through 43) may be purchased from the Treasurer.

The Annals of Probability, Volume 5, Number 5, October 1977. Published bimonthly in February, April, June, August, October, and December by The Institute of Mathematical Statistics, IMS Business Office, 3401 Investment Blvd., Suite 6, Hayward, California 94545.

Mail to the Annals of Probability should be addressed to either the Editor, Managing Editor or the Treasurer, as described above. It should not be addressed to Waverly Press.

> PRINTED AT THE WAVERLY PRESS, Inc., Baltimore, Maryland 21202 U.S.A.

Second-class postage paid at Hayward, California and at additional mailing offices

Copyright © 1977 by the Institute of Mathematical Statistics

EDITORIAL STAFF

EDITOR

PATRICK BILLINGSLEY

Associate Editors

RICHARD E. BARLOW
RABI BHATTACHARYA
J. R. BLUM
DAVID R. BRILLINGER
JEAN PIERRE CONZE
D. A. DAWSON
R. M. DUDLEY
MEYER DWASS

DAVID L. HANSON
C. C. HEYDE
DONALD L. IGLEHART
NARESH C. JAIN
SØREN JOHANSEN
J. F. C. KINGMAN
M. R. LEADBETTER
PETER NEY

WALTER PHILIPP
WILLIAM E. PRUITT
G. E. H. REUTER
DAVID O. SIEGMUND
CHARLES J. STONE
JOHN B. WALSH
MICHAEL J. WICHURA
DONALD YLVISAKER

Managing Editor

D. R. TRUAX

PAST EDITORS ANNALS OF MATHEMATICAL STATISTICS

H. C. Carver, 1930-1938
S. S. Wilks, 1938-1949
T. W. Anderson, 1950-1952
E. L. Lehmann, 1953-1955

T. E. HARRIS, 1955-1958

WILLIAM KRUSKAL, 1958-1961 J. L. HODGES, JR., 1961-1964 D. L..BURKHOLDER, 1964-1967 Z. W. BIRNBAUM, 1967-1970 INGRAM OLKIN, 1970-1972

Annals of Probability Ronald Pyke, 1972-1975

Annals of Statistics Ingram Olkin, 1972–1973 I. R. Savage, 1974–1976

EDITORIAL POLICY

The main purpose of the *Annals of Probability* and the *Annals of Statistics* is to publish contributions to the theory of probability and statistics and to their applications. The emphasis is on importance and interest, not formal novelty and correctness. Especially appropriate are authoritative expository papers and surveys of areas in vigorous development. All papers are refereed.

IMS INSTITUTIONAL MEMBERS

AEROSPACE CORPORATION El Segundo, California

ARIZONA STATE UNIVERSITY Tempe, Arizona

ARTHUR D. LITTLE, INC. Cambridge, Massachusetts

BELL TELEPHONE LABORATORIES, TECHNICAL LIBRARY
Murray Hill, N. J.

BOWLING GREEN STATE UNIVERSITY, DEPT. OF MATHEMATICS Bowling Green, Ohio

CALIFORNIA STATE UNIVERSITY, FULLER-TON, DEPARTMENT OF MATHEMATICS Fullerton, California

CALIFORNIA STATE UNIVERSITY, HAYWARD, DEPARTMENT OF STATISTICS Hayward, California

CASE WESTERN RESERVE UNIVERSITY, DE-PARTMENT OF MATHEMATICS Cleveland, Ohio

CORNELL UNIVERSITY, DEPARTMENT OF MATHEMATICS
Ithaca, New York

FLORIDA STATE UNIVERSITY, DEPARTMENT OF STATISTICS Tallahassee, Florida

FORD MOTOR COMPANY, ENGINEERING AND RESEARCH LIBRARY Dearborn, Michigan

GENERAL MOTORS CORPORATION, RESEARCH LABORATORIES Warren, Michigan

GEORGE WASHINGTON UNIVERSITY, DE-PARTMENT OF STATISTICS Washington, D.C.

INDIANA UNIVERSITY, MATHEMATICS DEPT. Bloomington, Indiana

INSTITUTE FOR DEFENSE ANALYSES Arlington, Virginia

INTERNATIONAL BUSINESS MACHINES COR-PORATION Armonk, New York

IOWA STATE UNIVERSITY, STATISTICAL LABORATORY
Ames. Iowa

JOHNS HOPKINS UNIVERSITY, DEPARTMENT OF BIOSTATISTICS, DEPARTMENT OF MATHEMATICAL SCIENCES
Baltimore, Maryland

LOUISIANA STATE UNIVERSITY, DEPART-MENT OF EXPERIMENTAL STATISTICS Baton Rouge, Louisiana

MASSACHUSETTS INSTITUTE OF TECHNOL-OGY, MATHEMATICS DEPARTMENT Cambridge, Massachusetts MIAMI UNIVERSITY, DEPARTMENT OF MATHEMATICS
Oxford. Ohio

MICHIGAN STATE UNIVERSITY, DEPARTMENT OF STATISTICS AND PROBABILITY East Lansing, Michigan

NATIONAL SECURITY AGENCY Fort George G. Meade, Maryland

NEW MEXICO STATE UNIVERSITY, DEPART-MENT OF MATHEMATICAL SCIENCES Las Cruces, New Mexico

NORTHERN ILLINOIS UNIVERSITY, DEPART-MENT OF MATHEMATICAL SCIENCES De Kalb, Illinois

NORTHWESTERN UNIVERSITY, DEPARTMENT OF MATHEMATICS Evanston, Illinois

OHIO STATE UNIVERSITY, DIVISION OF STATISTICS Columbus, Ohio

OREGON STATE UNIVERSITY, DEPARTMENT OF STATISTICS Corvallis, Oregon

PENNSYLVANIA STATE UNIVERSITY, DE-PARTMENT OF STATISTICS University Park, Pennsylvania

PRINCETON UNIVERSITY, DEPARTMENT OF STATISTICS Princeton, New Jersey

PURDUE UNIVERSITY LIBRARIES Lafayette, Indiana

QUEEN'S UNIVERSITY, DEPT. OF MATHEMATICS, KINGSTON Ontario, Canada

RICE UNIVERSITY, DEPARTMENT OF MATHEMATICAL SCIENCES Houston, Texas

THE ROCKEFELLER UNIVERSITY New York, New York

SANDIA CORPORATION, SANDIA BASE Albuquerque, New Mexico

SIMON FRASER UNIVERSITY, MATHEMATICS DEPARTMENT Burnaby, Canada

SOUTHERN ILLINOIS UNIVERSITY, MATHE-MATICAL STUDIES Edwardsville, Illinois

SOUTHERN METHODIST UNIVERSITY, DE-PARTMENT OF STATISTICS Dallas, Texas

STANFORD UNIVERSITY, GIRSHICK MEMORIAL LIBRARY Stanford, California

STATE UNIVERSITY OF NEW YORK, BUFFALO, DEPARTMENT OF STATISTICS Amherst, New York TEMPLE UNIVERSITY, MATHEMATICS DE-PARTMENT Philadelphia, Pennsylvania

TEXAS A & M UNIVERSITY, DEPT. OF MATH-EMATICS College Station, Texas

TEXAS TECH UNIVERSITY, DEPARTMENT OF MATHEMATICS Lubbock, Texas 79409

THE TOBACCO INSTITUTE Washington, D.C.

UNION OIL COMPANY OF CALIFORNIA, UNION RESEARCH CENTER Brea, California

UNITED STATES ARMY RESEARCH AND DE-VELOPMENT CENTER Aberdeen Proving Ground, Maryland

UNIVERSITY OF ALBERTA, DEPARTMENT OF MATHEMATICS Edmonton, Alberta, Canada

UNIVERSITY OF ARIZONA, DEPARTMENT OF MATHEMATICS Tucson, Arizona

University of British Columbia, De-PARTMENT OF MATHEMATICS Vancouver, B.C., Canada

UNIVERSITY OF CALGARY, MATHEMATICS DEPARTMENT Calgary 44, Alberta, Canada

UNIVERSITY OF CALIFORNIA, BERKELEY, STATISTICAL LABORATORY Berkeley, California

UNIVERSITY OF CINCINNATI, DEPARTMENT OF MATHEMATICAL SCIENCES Cincinnati, Ohio

UNIVERSITY OF GUELPH, MATHEMATICS AND STATISTICS DEPARTMENT Guelph, Ontario, Canada

University of Illinois at Chicago Circle, Department of Mathematics Chicago, Illinois

UNIVERSITY OF ILLINOIS, MATHEMATICS DEPT.
Urbana, Illinois

University of Iowa, Division of Mathematical Sciences Iowa City, Iowa

UNIVERSITY OF MANITOBA, DEPARTMENT OF STATISTICS Winnipeg, Manitoba, Canada

UNIVERSITY OF MARYLAND, DEPARTMENT OF MATHEMATICS College Park, Maryland

UNIVERSITY OF MICHIGAN, DEPARTMENT OF STATISTICS Ann Arbor, Michigan

UNIVERSITY OF MINNESOTA, SCHOOL OF STATISTICS Minneapolis, Minnesota University of Missouri, Department of Statistics Columbia, Missouri

University of Missouri at Rolla, De-PARTMENT OF MATHEMATICS Rolla, Missouri

UNIVERSITY OF MONTREAL, DEPARTMENT OF MATHEMATICS
Montreal, Quebec, Canada

UNIVERSITY OF NEW MEXICO, DEPARTMENT OF MATHEMATICS AND STATISTICS Albuquerque, New Mexico

UNIVERSITY OF NORTH CAROLINA, DEPART-MENT OF STATISTICS Chapel Hill, North Carolina

UNIVERSITY OF OREGON, MATHEMATICS DE-PARTMENT Eugene, Oregon

UNIVERSITY OF OTTAWA, DEPARTMENT OF MATHEMATICS Ottawa, Ontario, Canada

University of Rochester, Library Rochester, New York

UNIVERSITY OF SOUTH CAROLINA, DEPART-MENT OF MATHEMATICS AND COMPUTER SCIENCE Columbia, South Carolina

University of Texas, Department of Mathematics
Austin, Texas

UNIVERSITY OF UTAH, DEPT. OF MATHEMATICS
Salt Lake City, Utah

UNIVERSITY OF VICTORIA, DEPT. OF MATH-EMATICS Victoria, British Columbia, Canada

UNIVERSITY OF WASHINGTON, DEPARTMENT OF MATHEMATICS Seattle, Washington

UNIVERSITY OF WATERLOO, STATISTICS DE-PARTMENT Waterloo, Ont., Canada

UNIVERSITY OF WISCONSIN, MADISON, DE-PARTMENT OF STATISTICS Madison, Wisconsin

UNIVERSITY OF WISCONSIN, MILWAUKEE, DEPARTMENT OF MATHEMATICS Milwaukee, Wisconsin

VIRGINIA COMMONWEALTH UNIVERSITY, DE-PARTMENT OF MATHEMATICAL SCIENCES Richmond, Virginia

WAYNE STATE UNIVERSITY, DEPARTMENT OF MATHEMATICS Detroit, Michigan

WEST CHESTER STATE COLLEGE West Chester, Pennsylvania

WESTINGHOUSE ELECTRIC CORPORATION, RESEARCH LABORATORIES Pittsburgh, Pennsylvania

THE ANNALS OF PROBABILITY

INSTRUCTIONS FOR AUTHORS

Submission of Papers. Papers to be submitted for publication should be sent to the Editor of the Annals of Probability. (For current address, see the latest issue of the Annals.) The original (or xerox copy) should be submitted with two additional copies on paper that will take ink corrections. The manuscript will not normally be returned to the author; when expressly requested by the author, one copy of the manuscript will be returned.

Preparation of Manuscripts. Manuscripts should be typewritten, entirely double-spaced, including references, with wide margins at sides, top and bottom. Dittoed or mimeographed papers are acceptable only if completely legible; xerox copies are preferable. When technical reports are submitted, all extraneous sheets and covers should be removed.

Submission of Reference Papers. Copies (preferably two) of unpublished or not easily available papers cited in the manuscript should be submitted with the manuscript.

Title and Abbreviated Title. The title should be descriptive and as concise as is feasible, i.e., it should indicate the topic of the paper as clearly as possible, but every word in it should be pertinent. An abbreviated title to be used as a running head is also required, and should be given below the main title. This should normally not exceed 35 characters. For example, a title might be "A Limit Theorem for Conditioned Recurrent Random Walk Attracted to a Stable Law," with the running head "Limit Theorem for Recurrent Random Walk" or possibly "Recurrent Random Walk Attracted to a Stable Law," depending on the emphasis to be conveyed.

Summary. Each manuscript is required to contain a summary which will be printed immediately after the title, clearly separated from the rest of the paper. Its main purpose is to inform the reader quickly of the nature and results of the paper; it may also be used as an aid in retrieving information. The length of a summary will clearly depend on the length and difficulty of the paper, but in general it should not exceed 150 words. It should be typed on a separate page, under the heading "Summary," followed by the title of the paper. Formulas should be used as sparingly as possible. The summary should not make reference to results or formulas in the body of the paper—it should be self-contained.

Footnotes. Footnotes should be reduced to a minimum and, where possible, should be replaced by remarks in the text or in the references; formulas in footnotes should be avoided. Footnotes in the text should be identified by superscript numbers and typed together, double-spaced on a separate page.

Key Words. Included as the first footnote on page 1 should be the headings:

American Mathematical Society 1970 subject classifications. Primary—; Secondary—. Key words and phrases.

The classification numbers representing the primary and secondary subjects of the article may be found with instructions for its use, as an Appendix to Mathematical Reviews Index to Volume 39, June 1970. (See, also, The Notices of the American Mathematical Society, June 1970; Bulletin of the Institute of Mathematical Statistics, September 1974; or a current index issue of Mathematical Reviews.) The key words and phrases should describe the subject matter of the article; generally they should be taken from the body of the paper.

Identification of Symbols. Manuscripts for publication should be clearly prepared to insure that all symbols are properly identified. Distinguish between "oh" and "zero"; "ell" and "one"; "kappa" and "kay," etc. Indicate also when special type is required (Greek, German, script, boldface, etc.); other letters will be set in italics. Acronyms should be introduced sparingly.

Figures and Tables. Figures, charts, and diagrams should be prepared in a form suitable for photographic reproduction and should be professionally drawn twice the size they are to be printed. (These need not be submitted until the paper has been accepted for publication.) Tables should be typed on separate pages with accompanying footnotes immediately below the table.

Formulas. Fractions in the text are preferably written with the solidus or negative exponent;

thus, (a + b)/(c + d) is preferred to $\frac{a + b}{c + d}$, and

 $(2\pi)^{-1}$ or $1/(2\pi)$ to $\frac{1}{2\pi}$. Also, $a^{b(c)}$ and $a_{b(c)}$ are pre-

ferred to a^{b_c} and a_{b_c} , respectively. Complicated exponentials should be represented with the symbol exp. A fractional exponent is preferable to a radical sign.

References. References should be typed double-spaced and should follow the style:

[5] Doob, J. L. (1944). The elementary Gaussian processes. Ann. Math. Statist. 15 229-282.

In textual material, the format "... Doob (1944)..." is normally preferred to "... Doob [5]..." Multiple references can be distinguished as "... Doob (1944a)..." Abbreviations for journals should be taken from a current index issue of Mathematical Reviews.

Proofs. Author will ordinarily receive galley proofs. Corrected galley proofs should be sent to the Managing Editor of the Annals of Probability (For current address, see the latest issue of the Annals.)