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Research Article

Some Properties of Intuitionistic Fuzzy Lie Algebras over a Fuzzy Field

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Abstract The concept of intuitionistic fuzzy Lie algebra over a fuzzy field is introduced. We study the "necessity" and "possibility" operators on intuitionistic fuzzy Lie algebra over a fuzzy field and give some properties of homomorphic images.

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1 Introduction

The fuzzy algebraic structure plays a prominent role in mathematics. It has got wide applications in many other branches of science. After the introduction of fuzzy sets by Zadeh [6], several scholars studied fuzzy substructures of many algebraic structures. Nanda [5] introduced fuzzy fields and fuzzy algebra over a fuzzy field. In [1], we introduced fuzzy Lie algebra over a fuzzy field.

In another direction, Atanassov [2] introduced the notion of intuitionistic fuzzy sets as generalization of fuzzy sets. Certainly, fuzzy subsets are intuitionistic fuzzy subsets.

In this paper, we introduce the notion of intuitionistic fuzzy Lie algebras over a fuzzy field and give some results on it.

2 Preliminaries

We denote the closed interval $[0,1]$ by I. By a fuzzy subset of a nonempty set X, we mean a function from X to I. If $\mu : X \to I$ is a fuzzy set, then $\bar{\mu} : X \to I$ is defined by $\bar{\mu}(x) = 1-\mu(x)$. For $\alpha \in I$ the set $\mu_\alpha = \{\lambda \in X \mid \mu(\lambda) \geq \alpha\}$ is called an α-cut of μ.

Let $\min\{t,s\}$ be denoted by $m(t,s)$ and let $\max\{t,s\}$ be denoted by $M(t,s)$.

Definition 1 (see [4]). Let X be a field and let F be a fuzzy subset of X. Then F is called a fuzzy field of X if

(i) for all λ, γ in X, $F(\lambda-\gamma) \geq F(\lambda) \wedge F(\gamma)$,
(ii) for all $\lambda, \gamma \neq 0$ in X, $F(\lambda\gamma^{-1}) \geq F(\lambda) \wedge F(\gamma)$.

Remark 2. It is seen that if F is a fuzzy field of X, then

$$F(0) \geq F(1) \geq F(\lambda) = F(-\lambda) = F\big(\lambda^{-1}\big) \quad \forall \lambda \neq 0 \text{ in } X.$$

Definition 3 (see [3]). A fuzzy set μ of a set L is said to possess *sup property* if for every non-empty subset S of L there exists $x_0 \in S$ such that

$$\mu(x_0) = \text{Sup}\,\{\mu(x) \mid x \in S\}.$$

Definition 4 (see [2]). An intuitionistic fuzzy set (IFS) A in a nonempty set L is defined as an object of the form

$$A = \big\{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in L\big\},$$

where the functions $\mu_A : L \to I$ and $\nu_A : L \to I$ define the degree of membership and the degree of non-membership of the element $x \in L$, respectively, and for every $x \in L$,

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1.$$

For the sake of simplicity, we will use the symbol $A = (\mu_A, \nu_A)$ for the IFS.

Definition 5. Let $A = (\mu_A, \nu_A)$ be an IFS on a nonempty set L. A is said to be possessing *sup property* if and only if μ_A and $\bar{\nu}_A$ possess *sup property*.

Definition 6 (see [2]). Let $A = (\mu_A, \nu_A)$ be an IFS on a nonempty set L. The set $U(\mu_A, \alpha) = \{x \in L \mid \mu_A(x) \geq \alpha\}$ is called *membership α-level cut* of A and the set $V(\nu_A, \alpha) = \{x \in L \mid \nu_A(x) \leq \alpha\}$ is called *nonmembership α-level cut* of A, where $\alpha \in I$.

Definition 7 (see [7]). Let $A = (\mu_A, \nu_A)$ be an IFS on a nonempty set L and $\alpha \in (0, 1]$. Then the set $W_\alpha(A) = \{x \in L \mid \mu_A(x) \geq \alpha \text{ and } \nu_A(x) \leq 1 - \alpha\}$ is called a weak α-cut of A.

3 Fuzzy Lie algebra over a fuzzy field

Definition 8 (see [1]). Let μ be a fuzzy subset of a Lie algebra L over a field X. Then μ is called a *fuzzy Lie algebra of L over a fuzzy field F of X* if for all $x, y \in L$, $\lambda \in X$,

(i) $\mu(x - y) \geq m(\mu(x), \mu(y))$,
(ii) $\mu(\lambda x) \geq m(F(\lambda), \mu(x))$,
(iii) $\mu([x, y]) \geq m(\mu(x), \mu(y))$.

Definition 9. Let L_1 and L_2 be Lie algebras over a field X and let $f : L_1 \to L_2$ be a homomorphism. If μ is a fuzzy Lie algebra of L_1 over a fuzzy field F of X, then fuzzy set $f(\mu)$ of L_2 is defined by

$$f(\mu)(a) = \begin{cases} \text{Sup}\,\{\mu(x) \mid x \in f^{-1}(a)\}, \\ 0, \quad \text{if } f^{-1}(a) = \phi, \end{cases}$$

for all $a \in L_2$.

Theorem 10. *Let L_1 and L_2 be Lie algebras over a field X, $f : L_1 \to L_2$ an epimorphism and μ a fuzzy Lie algebra of L_1 over a fuzzy field F of X which satisfies* sup property. *Then $f(\mu)$ is a fuzzy Lie algebra of L_2 over the fuzzy field F.*

Proof. Let $a, b \in L_2$. Then $f^{-1}(a)$, $f^{-1}(b)$ are nonempty subsets of L_1. So, by *sup property*, there exist $x \in f^{-1}(a)$, $y \in f^{-1}(b)$ such that

$$\mu(x) = \text{Sup}\,\{\mu(z) \mid z \in f^{-1}(a)\}, \quad \mu(y) = \text{Sup}\,\{\mu(z) \mid z \in f^{-1}(b)\};$$

$f(x) = a$, $f(y) = b$ imply that $f(x - y) = a - b$ and so $x - y \in f^{-1}(a - b)$. Then,

$$\begin{aligned} f(\mu)(a - b) &= \text{Sup}\,\{\mu(z) \mid z \in f^{-1}(a - b)\} \geq \mu(x - y) \geq m\big(\mu(x), \mu(y)\big) \\ &= m\big(\text{Sup}\,\{\mu(z) \mid z \in f^{-1}(a)\}, \text{Sup}\,\{\mu(z) \mid z \in f^{-1}(b)\}\big) \\ &= m\big(f(\mu)(a), f(\mu)(b)\big). \end{aligned}$$

Similarly $f(\mu)([a, b]) \geq m(f(\mu)(a), f(\mu)(b))$.
Let $\lambda \in X$, $b \in L_2$. Then $\lambda b = f(\lambda y)$ imply that $\lambda y \in f^{-1}(\lambda b)$,

$$\begin{aligned} f(\mu)(\lambda b) &= \text{Sup}\,\{\mu(z) \mid z \in f^{-1}(\lambda b)\} \geq \mu(\lambda y) \geq m\big(F(\lambda), \mu(y)\big) \\ &= m\big(F(\lambda), \text{Sup}\,\{\mu(z) \mid z \in f^{-1}(b)\}\big) = m\big(F(\lambda), f(\mu)(b)\big). \end{aligned}$$

Therefore, $f(\mu)$ is a fuzzy Lie algebra of L_2 over the fuzzy field F. □

4 Intuitionistic fuzzy Lie algebra over a fuzzy field

In what follows, let L denote a Lie algebra over a field X.

Definition 11. Let $A = (\mu_A, \nu_A)$ be an IFS on a Lie algebra L. Then A is called an *intuitionistic fuzzy* (IF)*Lie algebra of L over a fuzzy field F* if

(i) $\mu_A(x - y) \geq m(\mu_A(x), \mu_A(y))$ and $\nu_A(x - y) \leq M(\nu_A(x), \nu_A(y))$,
(ii) $\mu_A(\lambda x) \geq m(F(\lambda), \mu_A(x))$ and $\nu_A(\lambda x) \leq M(1 - F(\lambda), \nu_A(x))$,
(iii) $\mu_A([x, y]) \geq m(\mu_A(x), \mu_A(y))$ and $\nu_A([x, y]) \leq M(\nu_A(x), \nu_A(y))$ hold for all $x, y \in L$, and $\lambda \in X$.

Theorem 12. *Let $A = (\mu_A, \nu_A)$ be an IF Lie algebra of L over a fuzzy field F. Then*

(1) $\mu_A(0) \geq \mu_A(x)$ and $\nu_A(0) \leq \nu_A(x)$ for all $x \in L$,
(2) $\mu_A(x) = \mu_A(-x)$ and $\nu_A(x) = \nu_A(-x)$ for all $x \in L$,
(3) $\mu_A(x+y) \geq m(\mu_A(x), \mu_A(y))$ and $\nu_A(x+y) \leq M(\nu_A(x), \nu_A(y))$ for all $x, y \in L$.

Proof. (1) $\mu_A(0) = \mu_A([x,x]) \geq m(\mu_A(x), \mu_A(x)) = \mu_A(x)$. Therefore, $\mu_A(0) \geq \mu_A(x)$. Similarly $\nu_A(0) = \nu_A([x,x]) \leq M(\nu_A(x), \nu_A(x)) = \nu_A(x)$. So $\nu_A(0) \leq \nu_A(x)$. (2) $\mu_A(-x) = \mu_A(0-x) \leq m(\mu_A(0), \mu_A(x)) = \mu_A(x)$ implies that $\mu_A(-x) \leq \mu_A(x)$ and $\mu_A(x) = \mu_A(-(-x)) \geq \mu_A(-x)$. Hence, $\mu_A(x) = \mu_A(-x)$.

Similarly $\nu_A(x) = \nu_A(-x)$. □

Example 1 Let $\mathbb{R}^3 = \{x = (a,b,c) \mid a,b,c \in \mathbb{R}\}$ and Lie bracket $[x,y] = x \times y$, where "$\times$" is cross product for all $x, y \in \mathbb{R}^3$. Then $\mathbb{R}^3$ is a Lie algebra over the field $\mathbb{R}$.

Let $F : \mathbb{R} \to [0,1]$ be defined as follows:

$$F(\lambda) = \begin{cases} 1 & \text{if } \lambda \in \mathbb{Q}, \\ 0.5 & \text{if } \lambda \in \mathbb{Q}(\sqrt{2}) - \mathbb{Q}, \\ 0 & \text{if } \lambda \in \mathbb{R} - \mathbb{Q}(\sqrt{2}), \end{cases}$$

for all $\lambda \in \mathbb{R}$.

Then F is a fuzzy field of $\mathbb{R}$.

Define intuitionistic fuzzy set $A = (\mu_A, \nu_A)$, where $\mu_A : \mathbb{R}^3 \to [0,1]$, $\nu_A : \mathbb{R}^3 \to [0,1]$, as follows: for all $x = (a,b,c) \in \mathbb{R}^3$,

$$\mu_A(x) = \begin{cases} 0.6 & \text{if } a = b = c = 0, \\ 0.4 & \text{if } a \neq 0,\ b = 0,\ c = 0, \\ 0 & \text{otherwise}, \end{cases}$$

$$\nu_A(x) = \begin{cases} 0.3 & \text{if } a = b = c = 0, \\ 0.5 & \text{if } a \neq 0, b = 0, c = 0, \\ 0.8 & \text{otherwise}. \end{cases}$$

Then $A = (\mu_A, \nu_A)$ is an IF Lie algebra of $\mathbb{R}^3$ over the fuzzy field F.

Theorem 13. *Let $A = (\mu_A, \nu_A)$ be an IFS on a Lie algebra L. Then A is an IF Lie algebra of L over a fuzzy field F if and only if μ_A and $\bar{\nu}_A$ are fuzzy Lie algebras over the fuzzy field F.*

Proof. Suppose $A = (\mu_A, \nu_A)$ is an IF Lie algebra of L over a fuzzy field F. Then, by Definition 8, μ_A is a fuzzy Lie algebra of L over the fuzzy field F.

Since $\nu_A(x-y) \leq M(\nu_A(x), \nu_A(y))$,

$$\bar{\nu}_A(x-y) = 1 - \nu_A(x-y) \geq 1 - M\big(\nu_A(x), \nu_A(y)\big) = m\big(\big(1 - \nu_A(x)\big), \big(1 - \nu_A(y)\big)\big) = m\big(\bar{\nu}_A(x), \bar{\nu}_A(y)\big).$$

So $\bar{\nu}_A(x-y) \geq m(\bar{\nu}_A(x), \bar{\nu}_A(y))$. Similarly $\bar{\nu}_A([x,y]) \geq m(\bar{\nu}_A(x), \bar{\nu}_A(y))$. Now, $\nu_A(\lambda x) \leq M(1 - F(\lambda), \nu_A(x))$ shows that

$$\bar{\nu}_A(\lambda x) = 1 - \nu_A(\lambda x) \geq 1 - M\big(1 - F(\lambda), \nu_A(x)\big) = m\big(F(\lambda), 1 - \nu_A(x)\big) = m\big(F(\lambda), \bar{\nu}_A(x)\big).$$

So $\bar{\nu}_A(\lambda x) \geq m(F(\lambda), \bar{\nu}_A(x))$. Therefore, $\bar{\nu}_A$ is a fuzzy Lie algebra of L over the fuzzy field F.

Conversely, suppose μ_A and $\bar{\nu}_A$ are fuzzy Lie algebras over a fuzzy field F of X. Let $x \in L$ and $\lambda \in X$. Then $\mu_A(\lambda x) \geq m(F(\lambda), \mu_A(x))$ and $1 - \nu_A(\lambda x) = \bar{\nu}_A(\lambda x) \geq m(F(\lambda), \bar{\nu}_A(x)) = m(F(\lambda), 1 - \nu_A(x)) = 1 - M(1 - F(\lambda), \nu_A(x))$. So $\nu_A(\lambda x) \leq M(1 - F(\lambda), \nu_A(x))$.

For every $x, y \in L$, we get $\mu_A([x,y]) \geq m(\mu_A(x), \mu_A(y))$ and

$$1 - \nu_A\big([x,y]\big) = \bar{\nu}_A\big([x,y]\big) \geq m\big(\bar{\nu}_A(x), \bar{\nu}_A(y)\big) = m\big(1 - \nu_A(x), 1 - \nu_A(y)\big) = 1 - M\big(\nu_A(x), \nu_A(y)\big).$$

Therefore, $\nu_A([x,y]) \leq M(\nu_A(x), \nu_A(y))$.

Similarly, $\mu_A(x-y) \geq m(\mu_A(x), \mu_A(y))$ and $\nu_A(x-y) \leq M(\nu_A(x), \nu_A(y))$. Hence, $A = (\mu_A, \nu_A)$ is an IF Lie algebra of L over the fuzzy field F. □

Theorem 14. *An IFS $A = (\mu_A, \nu_A)$ on Lie algebra L is an IF Lie algebra over a fuzzy field F of X if and only if $\Box A = (\mu_A, \bar{\mu}_A)$ and $\Diamond A = (\bar{\nu}_A, \nu_A)$ are IF Lie algebras over the fuzzy field F.*

Proof. Suppose $A = (\mu_A, \nu_A)$ is an IF Lie algebra over a fuzzy field F of X. Then by Theorem 13, μ_A and $\bar{\nu}_A$ are fuzzy Lie algebras over the fuzzy field F. But $\overline{\overline{\mu}}_A = \mu_A$. So $\Box A = (\mu_A, \bar{\mu}_A)$ and $\Diamond A = (\bar{\nu}_A, \nu_A)$ are IF Lie algebras over the fuzzy field F.

Conversely, suppose $\Box A$ and $\Diamond A$ are IF Lie algebras over a fuzzy field F. So μ_A and $\bar{\nu}_A$ are fuzzy Lie algebras over the fuzzy field F. Hence by Theorem 13, $A = (\mu_A, \nu_A)$ is an IF Lie algebra over the fuzzy field F. □

Theorem 15. *Let $A = (\mu_A, \nu_A)$ be an IF Lie algebra of L over a fuzzy field F of X. Then for any $\alpha \in I$ and $\beta \in I$ satisfying $\alpha + \beta \geq 1$, the nonempty sets $U(\mu_A, \alpha)$ and $V(\nu_A, \beta)$ of L are Lie subalgebras over F_α, when F_α contains at least two elements.*

Proof. Let $\lambda, \gamma \in F_\alpha$, where $\alpha \in I$. Then $F(\lambda) \geq \alpha$ and $F(\gamma) \geq \alpha$. So $F(\lambda - \gamma) \geq m(F(\lambda), F(\gamma)) \geq \alpha$. Hence, $\lambda - \gamma \in F_\alpha$. Similarly, $F(\lambda\gamma^{-1}) \geq m(F(\lambda), F(\gamma)) \geq \alpha$ implies that $\lambda\gamma^{-1} \in F_\alpha$ for all $\gamma \neq 0$. Therefore, F_α is a subfield of X.

Let $x, y \in U(\mu_A, \alpha)$. Then $\mu_A(x) \geq \alpha$ and $\mu_A(y) \geq \alpha$. Since A is an IF Lie algebra of L, $\mu_A(x + y) \geq m(\mu_A(x), \mu_A(y)) \geq \alpha$ and $\mu_A([x, y]) \geq m(\mu_A(x), \mu_A(y)) \geq \alpha$. This shows that $x + y \in U(\mu_A, \alpha)$ and $[x, y] \in U(\mu_A, \alpha)$. Let $\lambda \in F_\alpha$ and $x \in U(\mu_A, \alpha)$. Then $F(\lambda) \geq \alpha$ and $\mu_A(x) \geq \alpha$. So $\mu_A(\lambda x) \geq m(F(\lambda), \mu_A(x)) \geq \alpha$ implies that $\lambda x \in U(\mu_A, \alpha)$. Therefore, $U(\mu_A, \alpha)$ is a Lie subalgebra of L over the field F_α.

Let $x \in V(\nu_A, \beta)$, $\lambda \in F_\alpha$. Then $\nu_A(x) \leq \beta$ and $1 - F(\lambda) \leq 1 - \alpha \leq \beta$. This shows that $\nu_A(\lambda x) \leq M(1 - F(\lambda), \nu_A(x)) \leq \beta$ and so $\lambda x \in V(\nu_A, \beta)$. Also, $x + y \in V(\nu_A, \beta)$ and $[x, y] \in V(\nu_A, \beta)$. Therefore, $V(\nu_A, \beta)$ is a Lie subalgebra of L over the field F_α. □

Theorem 16. *Let $A = (\mu_A, \nu_A)$ be an IF Lie algebra of L over a fuzzy field F of X. Then for any $\alpha \in I$, the nonempty set weak α-cut $W_\alpha(A)$ of A is a Lie subalgebra of L over F_α, when F_α contains at least two elements.*

Proof. Take $\beta = 1 - \alpha$ and use the argument as in the proof of Theorem 15. Then it follows that $W_\alpha(A)$ is a Lie subalgebra of L over the field F_α. □

Definition 17. Let L_1 and L_2 be Lie algebras over a field X and let $f : L_1 \to L_2$ be a homomorphism. Let $A = (\mu_A, \nu_A)$ be an IFS of L_1 and let $B = (\mu_B, \nu_B)$ be an IFS of L_2. Then IFS $f(A) = (f(\mu_A), f(\nu_A))$ on L_2 is defined by, for all $a \in L_2$,

$$f(\mu_A)(a) = \begin{cases} \text{Sup}\left\{\mu_A(x) \mid x \in f^{-1}(a)\right\}; \\ 0, \quad \text{if } f^{-1}(a) = \phi. \end{cases}$$

$$f(\nu_A)(a) = \begin{cases} \text{Inf}\left\{\nu_A(x) \mid x \in f^{-1}(a)\right\}; \\ 1, \quad \text{if } f^{-1}(a) = \phi. \end{cases}$$

The IFS $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\nu_B))$ of L_1 is defined by, for $x \in L_1$,

$$f^{-1}(\mu_B)(x) = \mu_B(f(x)), \quad f^{-1}(\nu_B)(x) = \nu_B(f(x)).$$

Theorem 18. *Let L_1 and L_2 be Lie algebras over a field X and let $f : L_1 \to L_2$ be an epimorphism. Then IFS $B = (\mu_B, \nu_B)$ is an IF Lie algebra of L_2 over a fuzzy field F of X if and only if $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\nu_B))$ is an IF Lie algebra of L_1 over the fuzzy field F.*

Proof. Suppose $B = (\mu_B, \nu_B)$ is an IF Lie algebra of L_2 over a fuzzy field F of X. Let $x, y \in L_1$ and $\lambda \in X$. Then,

$$\begin{aligned} f^{-1}(\mu_B)(x - y) &= \mu_B(f(x - y)) = \mu_B(f(x) - f(y)) \geq m(\mu_B(f(x)), \mu_B(f(y))) \\ &= m(f^{-1}(\mu_B)(x), f^{-1}(\mu_B)(y)), \\ f^{-1}(\nu_B)(x - y) &= \nu_B(f(x - y)) = \nu_B(f(x) - f(y)) \leq M(\nu_B(f(x)), \nu_B(f(y))) \\ &= M(f^{-1}(\nu_B)(x), f^{-1}(\nu_B)(y)). \end{aligned}$$

Similarly,

$$\begin{aligned} f^{-1}(\mu_B)([x, y]) &\geq m(f^{-1}(\mu_B)(x), f^{-1}(\mu_B)(y)); \\ f^{-1}(\nu_B)([x, y]) &\leq M(f^{-1}(\nu_B)(x), f^{-1}(\nu_B)(y)). \end{aligned}$$

Also

$$f^{-1}(\mu_B)(\lambda x) = \mu_B(f(\lambda x)) = \mu_B(\lambda f(x)) \geq m(F(\lambda), \mu_B(f(x))) = m(F(\lambda), f^{-1}(\mu_B)(x));$$
$$f^{-1}(\nu_B)(\lambda x) \leq M(1 - F(\lambda), f^{-1}(\nu_B)(x)).$$

Therefore, $f^{-1}(B)$ is an IF Lie algebra of L_1 over the fuzzy field F.

Conversely, suppose $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\nu_B))$ is an IF Lie algebra of L_1 over the fuzzy field F. Let $a, b \in L_2$. Since f is a surjection, there exist $x, y \in L_1$ such that $f(x) = a$, $f(y) = b$. Then,

$$\begin{aligned}\mu_B(a - b) &= \mu_B(f(x - y)) = f^{-1}(\mu_B)(x - y) \geq m(f^{-1}(\mu_B)(x), f^{-1}(\mu_B)(y)) \\ &= m(\mu_B(f(x)), \mu_B(f(y))) = m(\mu_B(a), \mu_B(b)), \\ \nu_B(a - b) &\leq M(\nu_B(a), \nu_B(b)).\end{aligned}$$

Similarly,

$$\mu_B([a, b]) \geq m(\mu_B(a), \mu_B(b)), \quad \nu_B([a, b]) \leq M(\nu_B(a), \nu_B(b)),$$
$$\mu_B(\lambda a) = \mu_B(\lambda f(x)) = \mu_B(f(\lambda x)) = f^{-1}(\mu_B)(\lambda x) \geq m(F(\lambda), f^{-1}(\mu_B)(x)) = m(F(\lambda), \mu_B(a)),$$
$$\nu_B(\lambda a) \leq M(1 - F(\lambda), \nu_B(a)).$$

Therefore, $B = (\mu_B, \nu_B)$ is an IF Lie algebra of L_2 over the fuzzy field F. □

Lemma 19. *Let $f : L_1 \to L_2$ be an epimorphism of Lie algebras and let $A = (\mu_A, \nu_A)$ be an IF Lie algebra of L_1 over a fuzzy field F of X. Then $f(\bar{\nu}_A) = \overline{f(\nu_A)}$.*

Proof. Let $a \in L_2$. Then $f^{-1}(a) \neq \phi$. Since $A = (\mu_A, \nu_A)$ is an IF Lie algebra of L_1 over a fuzzy field F of X, by Theorem 13, $\bar{\nu}_A$ is a fuzzy Lie algebra of L_1 over the fuzzy field F. Then by the definition of image of a fuzzy Lie algebra under homomorphism,

$$\begin{aligned}f(\bar{\nu}_A)(a) &= \text{Sup}\,\{\bar{\nu}_A(z) \mid z \in f^{-1}(a)\} = \text{Sup}\,\{1 - \nu_A(z) \mid z \in f^{-1}(a)\} \\ &= 1 - \inf\{\nu_A(z) \mid z \in f^{-1}(a)\} = 1 - f(\nu_A)(a) = \overline{f(\nu_A)}(a).\end{aligned}$$

Therefore, $f(\bar{\nu}_A) = \overline{f(\nu_A)}$. □

Theorem 20. *Let L_1 and L_2 be Lie algebras over a field X, $f : L_1 \to L_2$ an epimorphism and $A = (\mu_A, \nu_A)$ an IF Lie algebra of L_1 over a fuzzy field F of X which satisfies* sup property. *Then $f(A)$ is an IF Lie algebra of L_2 over the fuzzy field F.*

Proof. Let $A = (\mu_A, \nu_A)$ be an IF Lie algebra of L_1 over a fuzzy field F of X which satisfies *sup property*. Then μ_A and $\bar{\nu}_A$ are fuzzy Lie algebras of L_1 over the fuzzy field F which satisfies *sup property*. So, by Theorem 10, $f(\mu_A)$ and $f(\bar{\nu}_A)$ are fuzzy Lie algebras and, by Lemma 19, $f(\mu_A)$ and $\overline{f(\nu_A)}$ are fuzzy Lie algebras of L_2 over the fuzzy field F. This shows that, by Theorem 13, $(f(\mu_A), f(\nu_A)) = f(A)$ is an IF Lie algebra of L_2 over the fuzzy field F. □

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