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Nitrogen Dispersion

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Abstract

A mathematical model of free surface is presented for the study of the nitrogen injection of the gas cap for maintain of pressure, we used one model dispersion a flow in well zone. Nitrogen injected into the cap has a density greater than the original gas which has rushed to the gas-oil contact *goc*. Since the *goc* on the border there is a lot of nitrogen gas, it will spread and disperse into the oil. In the goc around a well about to be invaded by gas coning phenomena occur; therefore the model to interpret the output of the well must include the phenomenon of free surface. This paper proposes a method to measure the degree of contamination of the oil zone. This compositional data based on a production well before being invaded in producing interval by the gas cap. The most contaminated area of oil is close to the *goc*, as nitrogen is expected to exit the current production from the well just before being overcome by gas.

Keywords: Nitrogen injection; Fluids; Displacement; Symmetry

Introduction

Immiscible nitrogen injection has been used in enhanced oil recovery (EOR) processes. This gas can be employed to increment or maintain the reservoir pressure, to facilitate gravitational drainage, and as a pumping gas in a double displacement processes. In EOR processes [1], the nitrogen has disadvantages and advantages. Some advantages in using this gas are:

1. It is a clean technology, nitrogen does not produce greenhouse effect and a leak does not represent a mayor environmental risk,
2. There are no problems to obtain any amount from the atmosphere,
3. It is an inert gas, so there are not pipe corrosion problems and there are not chemical reactions with the reservoir fluids,
4. Because of the density differences between N_2 gas and oil, it is ideal to improve gravitational drainage and to avoid channelization if it is injected in a vertical displacement.

Some disadvantages that must be considered in using N_2 are:

1. There is a commercial limit in the amount of nitrogen in the hydrocarbons, so there must be a strict control in order to reduce dissolved nitrogen in oil,
2. When nitrogen is in contact with oil occurs non-equilibrium phenomena and light hydrocarbons compounds becomes gaseous, this causes an increase in oil viscosity and shrinking,
3. In the surface it is necessary to implement processes to separate the hydrocarbon gas and nitrogen.

To minimize the nitrogen penetration in the oil, it is usually injected in the gas cap, here, because of the density differences with natural gas the nitrogen goes down and reaches very soon the oil gas contact [1].

Physical Models

The domain of study is a cylinder whose z axis coincides with the location of the well and its radius is such that the effects of coning (or deformation of the goc) are negligible in the far radial border. The *goc* is a free surface whose position changes over time depending on the extraction of oil and *goc* near the production well will deform due to pressure gradients caused by oil extraction. The flow of oil carried over to the neighborhood of nitrogen gas-oil interface into the well, causing it to produce oil with nitrogen [1,2].

Using the symmetry of the system, we can simplify the domain of study to the model shown in Figure 1. The reference system (Figure 1) is located so that the z axis is aligned with the producing well and the system's center is located at the bottom of the reservoir. On the other hand, the radial variable, perpendicular to gravity, is r. The radius is R_w producing well and the study area R. The height of the oil zone - or the position of *goc* - is $h(t, r)$, where t is the independent variable time. The producing area of the well is located between the values of z_1 and z_2. It is assumed that the pressure in the *goc* is equal to the gas cap, moreover, that the concentration of nitrogen in the cap is high enough for the oil to see that the interface is the concentration of nitrogen saturation. Finally, it is assumed that the pressure is P_w at z_2

For the model of the pipe, was a system in one phase and low Reynolds numbers, so that the flow never becomes turbulent. The physical system to solve this problem is shown in Figure 2. At $z=z_1$, there is an impermeable barrier so that the velocities are zero in this position and $z=z_2$, the pressure is known $P=P_w$, the pressure depends on the column of oil. In the interval from z_1 to z_2 oil enters the pipe from the reservoir in this boundary, pressure and fluid velocity is equal to the reservoir. The fluid inside the pipe is assumed Newtonian and obeys the Navier-Stokes equations. Consequently field's velocity, pressure and concentration of the reservoir are joint to their counterparts in the tubing, so it is necessary to solve simultaneous systems [1,3].

Theoretical Models

In this section are shown the theoretical bases of two dispersion models of nitrogen in a homogeneous media fully saturated with oil.

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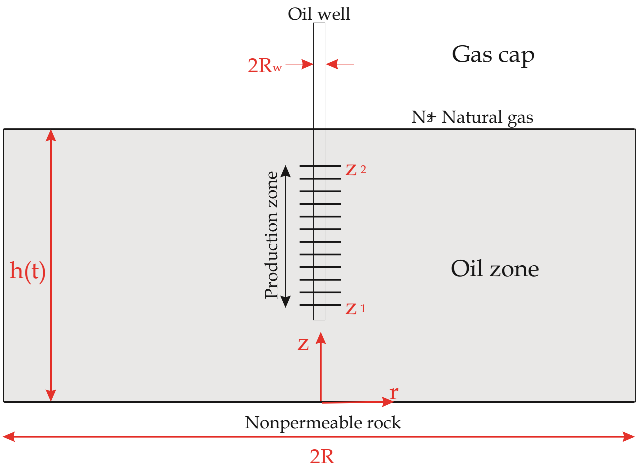

Figure 1: A conceptual reservoir model physics.

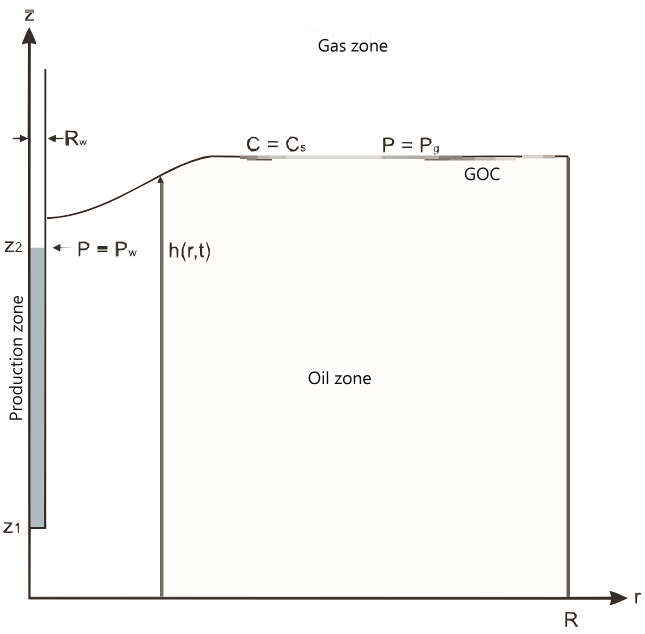

Figure 2: Simplified physical model symmetry.

The first one describes the nitrogen dispersion far away from the wells; the second one describes the nitrogen dispersion in the neighborhood of a producer well. Both cases are formulated in a homogeneous porous media and for simplicity it is considered that the oil saturation in the oil zone, S_{oi}, is constant and that in the gas zone the oil concentration, S_{or}, is also constant.

Far away from the wells

Consider a reservoir zone of porosity ϕ, far away from the producer wells in such way that the convective influence of the producer wells can be neglected. The vertical coordinate z has the origin in the water-oil contact (*woc*). For simplicity, in this work is supposed that the *woc* position is constant. Due the oil production the gas-oil contact (*goc*) is traveling down at velocity $\dot{h}goc$ and the oil column of height $hgoc(t) = h_o - \dot{h}_{goc}t$ is decreasing from the initial height, h_o for each time, t. Inside the oil column the fluid Darcy velocity is in general function of time and space, u(z,t)=ugoc f(z) where f(z) is a monotonic function such that $f(0)=0$ and $f(z=h_{goc(t)}=1)$ (Figure 1) the explicit form of this function depends on the oil extraction design in the field.

To reduce the oil contamination with nitrogen, the injection process is carry out in the gas cap. Because of the differences between the nitrogen and natural gas densities occurs a natural convection process in such way that the nitrogen will segregate to the lower part of the gas cap, this last phenomenon generates a rich nitrogen zone just above the goc line. In the goc line there is enough nitrogen in order to saturate the oil with this gas in the neighborhood, from this zone the nitrogen is transported into the oil by convection, dispersion and molecular diffusion [2]. If it is supposed that the nitrogen does not affect the liquid properties except its composition (passive dispersion and diffusion), such phenomena are governed by

$$\frac{\partial C}{\partial t} - u\frac{\partial C}{\partial z} = \left(\frac{D}{\tau} + \alpha \|u\|\right)\frac{\partial^2 C}{\partial z^2} \qquad (1)$$

Where, u is the Darcy velocity in the porous media, C is the mass fraction of nitrogen, D is the free molecular diffusion coefficient, τ is the porous medium tortuosity and αis the dispersivity coefficient. Initially, there is not nitrogen in the oil, then $C(t=0)=0$. As was mentioned before the nitrogen concentration in the goc line is the saturation one, then, $C(z=h_{goc})=C_{sat}$. For simplicity it is assumed that the nitrogen flux in the woc line is zero, or $\frac{\partial C}{\partial z}|_{z=0}=0$. Also, let os suppose that the Darcy velocity in the porous media obeys a law like (then the $f(z)$ function is:

$$\left(\frac{}{goc}\right) \text{ and } u_{goc} = \phi_e \dot{h}_{goc}) \qquad (2)$$

Where ϕ_e is the porosity that is filled with the gas phase, in the gas cap, this value depends on the residual oil saturation value in the gas cap.

Using the next transformation set:

$$Y = \frac{C}{C_{sat}}$$
$$\eta = \frac{z}{h_0 - \dot{h}_{goc}t} \qquad (3)$$
$$\sigma = \frac{t}{\frac{h_0}{h_{goc}}}$$

Replacing the equation 2 and 3 in 1,and making the corresponding transformations, we obtain the next nondimensional set of equation is obtained:

$$(1-\sigma)2\frac{\partial Y}{\partial \sigma} + (1-\sigma)(\eta - \phi_e\eta^n)\frac{\partial Y}{\partial \eta} = \frac{1}{Pe_d} + \frac{\phi_e\eta^n}{Pe_\alpha}\frac{\partial^2 Y}{\partial \eta^2} \qquad (4)$$

With the initial and boundary conditions

$$Y(\sigma = 0) = 0$$
$$Y(\eta = 1) = 1 \qquad (5)$$
$$\frac{\partial Y}{\partial \eta}|_{\eta=0} = 0$$

Where $Pe_d = \frac{\tau h_{goc} h_0}{D}$ and $Pe_\alpha = \frac{h_0}{\alpha}$ are diffusive and dispersive Peclet number.

The last mathematical model is solved by finite differences and numerical methods in a 10,000 points spatial mesh and for a $\Delta\sigma=0.001$ The solution in terms of real variables is shown in Figure 3, we can

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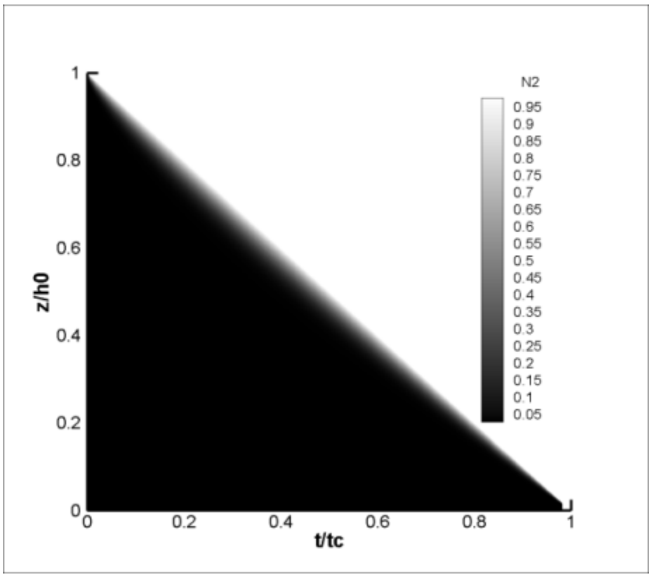

Figure 3: Area contaminated by nitrogen in non-dimensional coordinates.

see that although he was earlier that pollution in terms of percentage increases always in terms of scale this area first increases to a maximum and then decreases monotonically. This model explains why no-show nitrogen in the remote areas of the gas oil contact, but only in the areas of gas-oil contact. Which explains why nitrogen is only present in wells about to be invaded by the gas cap.

In the neighborhood of a well

The neighborhood of a well when its producer zone is nearly to be reached by the goc. Under this situation the goc in no longer only a function on time, here, if the well is vertical the convective forces produce a symmetrical deformation around the well, then it is case $h_{goc}=h_{goc}(r,t)$ here r is the radial coordinate in a cylindrical system of reference (r,z) which origin in located in the base of the oil zone and below the well (Figure 2). As was mentioned before, the nitrogen amount over the h_{goc} surface is big enough is such way that the concentration of this gas in the oil in this face is the saturation one, then $C(z=h_{goc}(r,t))=C_{sat}$. Again, it is supposed that the nitrogen transport in the *woc* line is zero,

$$\frac{\partial C}{\partial z}\Big|_{z=0}=0 \tag{6}$$

The oil into the porous media obeys the Darcy law, for simplicity it is assumed that only the oil phase below the goc is moving, the last is a good approximation if there is in not congenital water in the reservoir or this one stays motionless (Figure 4). For this system the Darcy law can be written as:

$$u_z=-\frac{k}{\mu}\left(\frac{\partial P}{\partial z}+\rho g\right) \tag{7}$$

Where, P is the pressure, (u_r,u_z) is the Darcy velocity vector, k is the relative permeability, μ is the oil viscosity, ρ is the oil density and g is the gravity acceleration. Let is suppose that: 1) the well is far away of others wells, 2) due the injection of nitrogen in the reservoir, the pressure in the goc interface remains constant, 3) the *woc* interface does not move in time 4) The reservoir fluid is assumed incompressible, and then the mass conservation (or continuity equation) has the form in cylindrical coordinates

$$\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_r}{\partial z}=0 \tag{8}$$

The material derived condition has the form

$$\frac{\partial h}{\partial t}=\frac{k}{\mu\phi_e}\left[\frac{\partial P}{\partial r}\frac{\partial h}{\partial r}-\frac{\partial P}{\partial z}-\rho g\right]=0 \tag{9}$$

Where h is the height of the oil zone in the reservoir, and is ϕ_e the effective porosity leaving the oil in the gas zone, when recovery is total effective porosity is equal to the total ϕ.

$$\phi\frac{\partial C}{\partial t}+u.\nabla C=\phi\nabla.(D.\nabla C) \tag{10}$$

Where, C is the concentration of Nitrogen into oil and D is dispersive tensor is given by

$$D=\begin{bmatrix}\frac{D}{\tau}+\frac{\alpha l u_r^2+\alpha_t u_z^2}{\phi|u|} & \frac{(\alpha_l-\alpha_t)u_r u_z}{\phi|u|}\\ \frac{(\alpha_l-\alpha_t)u_z u_r}{\phi|u|} & \frac{D}{\tau}+\frac{\alpha_l u_z^2+\alpha_t u_r^2}{\phi|u|}\end{bmatrix} \tag{11}$$

Here, D is the molecular diffusion coefficient; τ is the tortuosity, and finally, α_l and α_t are the longitudinal and transverse dispersivity, respectively

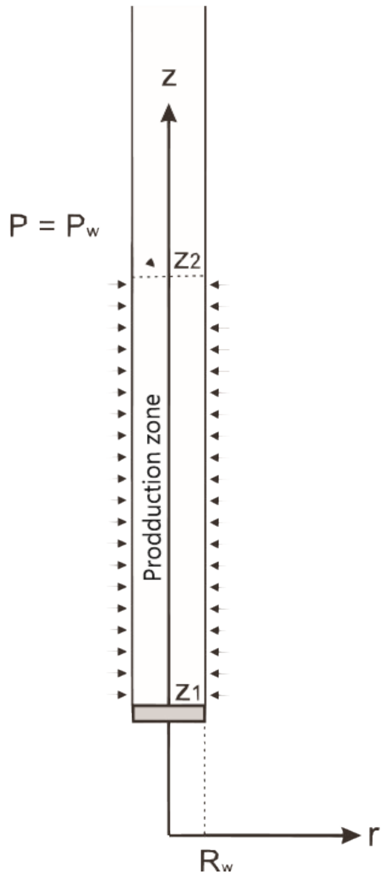

Figure 4: Model for tubing.

The last equations, boundary conditions

$$
\begin{aligned}
&\frac{\partial P}{\partial z}|_{z=0} + \rho g = 0 \\
&\frac{\partial C}{\partial z}|_{z=0} \quad = 0 \\
&\frac{\partial P}{\partial r}|_{r=R} \quad = 0 \\
&\frac{\partial C}{\partial z}|_{r=R} \quad = 0 \\
&P(z=h) \quad = P_g \\
&C(z=h) \quad = C_s
\end{aligned} \tag{12}
$$

At the center of the domain there are two areas, the non-productive (out of the area of fire) we have:

$$
\frac{\partial P}{\partial r}|_{r=Rw} \quad \frac{\partial C}{\partial r}|_{r=R} = 0 = \begin{cases} 0 < z < z_1 \\ z_1 < z < h \end{cases} \tag{13}
$$

Within the production area of the well pressure ($z_1 < z < z_2$ is Nitrogen Dispersion

$$
\begin{aligned}
&\frac{\partial P}{\partial r}|_{r=R_w^+} = \frac{R_w^3}{16k}\frac{d^2 p}{dz^2}|_{r=R_w^-} \\
&P(r=R_w^+) = P(r=R_w^-)
\end{aligned} \tag{14}
$$

And nitrogen concentration is given by:

$$
\left[\left(\frac{D}{\tau\phi} + \frac{\alpha l u_r^2 + \alpha_t u_z^2}{\phi |u|}\right)\frac{\partial C}{\partial r} + \frac{(\alpha_l - \alpha_t) u_r u_z}{\phi |u|}\frac{\partial C}{\partial z}\right]_{r=R_w^+} = \frac{D}{\phi}\frac{\partial C}{\partial r}|_{r=R_w^-} \quad C(r=R_w^+) = C(r=R_w^-) \tag{15}
$$

We utilized the follow transformation

$$
\begin{aligned}
&\eta = \frac{z}{\lambda h_0} \\
&\xi = \ln\left(\frac{r}{R_w}\right) \\
&p = \frac{P = P_g}{\rho g h0} \\
&\lambda = \frac{h}{h_0} \\
&Y = \frac{C}{C_s}
\end{aligned} \tag{16}
$$

This set of transformations has a first objective re-scale the radial variable in order to observe the effects on the scale of the well, and a second, mapping the domain in which resides the porous medium to a rectangle that does not change over time [3].

The resulting system of equations for the pressure and goc has the form:

$$
\begin{aligned}
&e^{-2\xi}\frac{\partial^2 P}{\partial \xi^2} + \left[\frac{\eta^2 e^{-2\xi}}{\lambda^2}\left(\frac{\partial \lambda}{\partial \xi}\right)^2 + \frac{\Gamma^2}{\lambda^2}\right]\frac{\partial^2 P}{\partial \eta^2} - \frac{2\eta e^{-2\xi}}{\lambda}\frac{\partial \lambda}{\partial \xi}\frac{\partial^2 P}{\partial \eta \partial \xi} \\
&-\frac{\eta e^{-2\xi}}{\lambda}\left[\frac{\partial^2 \lambda}{\partial \xi^2} - \frac{2}{\lambda}\left(\frac{\partial \lambda}{\partial \xi}\right)^2\right]\frac{\partial P}{\partial \eta} = 0
\end{aligned} \tag{17}
$$

Where the equations of Darcy's law and continuity were combined:

$$
\Gamma = \frac{R_w}{h_0}\cdot \tag{18}
$$

Aspect ratio of the reservoir compared to the radius of the pipe. The equation is obtained for the concentration of nitrogen has the form:

Where

$$
\begin{aligned}
&d_1 = \frac{1}{Pe_d} + \frac{u_r^2 + \frac{\alpha_t}{\alpha_l} u_z^2}{Pe_\alpha |u|} \\
&d_2 = \frac{2\left(1 - \frac{\alpha_t}{\alpha_l}\right) u_z u_r}{Pe_\alpha |u|} \\
&d_3 = d1 e^{-\xi} + e^{-\xi}\frac{\partial d1}{\partial \xi} - e^{-\xi}\frac{\eta}{\lambda}\frac{\partial \lambda}{\partial \xi}\frac{\partial d_1}{\partial \eta} + \frac{\Gamma}{2\lambda}\frac{\partial d_2}{\partial \eta} \\
&d_4 = \frac{d2}{2}e^{-\xi} + \frac{1}{2}e^{-\xi}\frac{\partial d_2}{\partial \xi} - \frac{1}{2}e^{-\xi}\frac{\eta}{\lambda}\frac{\partial \lambda}{\partial \xi}\frac{\partial d_1}{\partial \eta} + \frac{\Gamma}{\lambda}\frac{\partial d_1}{\partial \eta}
\end{aligned} \tag{19}
$$

Model for pipe production

$$
\begin{aligned}
&\rho\left(u_r\frac{\partial u_r}{\partial r} + u_z\frac{\partial u_r}{\partial z}\right) = -\frac{\partial P}{\partial r} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r u_r)}{\partial r}\right) + \frac{\partial^2 u_r}{\partial z^2}\right] \\
&\rho\left(u_r\frac{\partial u_z}{\partial r} + u_z\frac{\partial u_z}{\partial z}\right) = -\frac{\partial P}{\partial z} + \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r u_z)}{\partial r}\right) + \frac{\partial^2 u_z}{\partial z^2}\right] - \rho g \\
&\frac{1}{r}\frac{\partial (r u_r)}{\partial r} + \frac{\partial u_z}{\partial z} = 0
\end{aligned} \tag{20}
$$

Where *rho* and μ are the density and dynamics viscosity of the oil; u_r and u_z are the radial and axial velocities and g is the gravity acceleration. The boundary conditions are given by: The velocities in $z=z_1$ are

$$
u_r(z=0) = u_z(z=0) = 0 \tag{21}
$$

For symmetry, the variations of the velocity en z and r are

$$
\frac{\partial u_z}{\partial r}|_{r=0} = u_r(r=0) = 0 \tag{22}
$$

On the wall of the tube (r=Rw) is divided in two zones, for oil production (z1 >z2) are:

$$
u_r(r=R_w) = u_z(r=R_w) = 0 \tag{23}
$$

In the oil production zone, the radial velocity in the tube and reservoir are like

$$
u_r(r=R_w^+) = u_r(r=R_w^-) \tag{24}
$$

In this same region the axial velocity is also equal, yet here you can use an approach often used when the characteristic axial velocity in the pipe is much greater than the axial velocity in the porous medium, then the boundary condition is as

$$
u_z(r=R_w) \simeq 0 \tag{25}
$$

The pressure in $z=z_2$ is known, then, the boundary condition is (25)

$$
P(z=z2) = P_w \tag{26}
$$

We use the thin tube approximation ($u_z >> u_r$) and use the transformation set

$$U_\chi = \frac{uR\mu R_w}{k\rho g h_0}$$
$$U_\xi = \frac{u_z \mu R_w^2}{2zk\rho g h_0}$$
$$\chi = \overline{R_w}$$
$$\zeta = \frac{z}{z_2 - z_1} \qquad (27)$$
$$p = \frac{P + \rho g z}{\rho g h_0}$$

$$\mathrm{Re}_w \zeta \left(\mathring{a}_\chi \frac{\partial u\zeta}{\partial \chi} + 2\zeta u_\zeta \frac{\partial u\zeta}{\partial \zeta} + 2u_\zeta^2 \right) = -\frac{\partial p}{\partial \zeta} + \left[\frac{\zeta}{\chi} \frac{\partial}{\partial \chi} \left(\chi \frac{\partial u_\zeta}{\partial \chi} \right) + 2\ {}_W^2 \frac{\partial u_\zeta}{\partial \zeta} + \varepsilon_W^2 \zeta \frac{\partial^2 u\zeta}{\partial \zeta^2} \right]$$
$$\varepsilon_W^2 Re_W \left(u_\chi \frac{\partial u_\chi}{\partial \chi} + 2\zeta u\zeta \frac{\partial u_\chi}{\partial \zeta} \right) = -2\frac{\partial p}{\partial \chi} + \varepsilon_W^2 \left[\frac{\partial}{\partial \chi} \left(\frac{1}{\chi} \frac{\partial (\chi u_\chi)}{\partial \chi} \right) + \varepsilon_W^2 \frac{\partial^2 u_\chi}{\partial \zeta^2} \right] \qquad (28)$$
$$\frac{1}{\chi} \frac{\partial (\chi u_\chi)}{\partial \chi} + 2\zeta \frac{\partial u_\zeta}{\partial \zeta} + 2u\zeta = 0$$

Where k is the absolute permeability of porous medium and h0 is the initial height of the reservoir. To simplify the parameter space defined the following dimensionless parameters

$$\varepsilon = \frac{R_w}{z_2 - z_1}$$
$$\mathrm{R}_{ew} = \frac{R_w \rho U_{CR}}{\mu} \qquad (29)$$

The first represents the ratio of the pipe, and the second the Reynolds number based on tube radius (Figure 5).

Asymptotic Solution

For much of case studies must be the pipe is slender, which is much longer than wide. This results in a dimensionless parameter that gives geometric information of the pipe is very small, i.e.

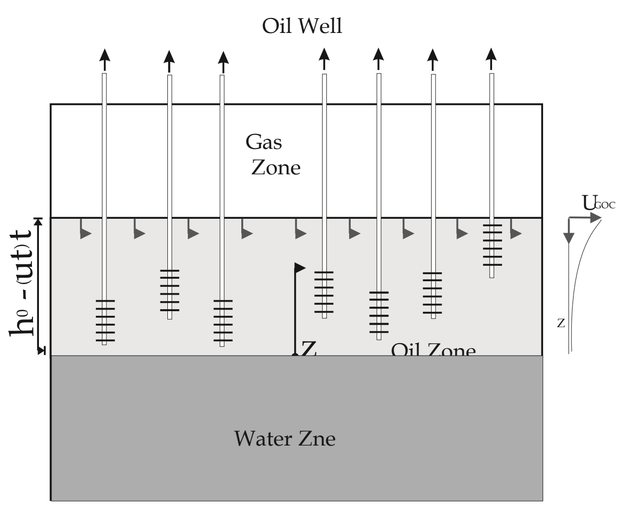

Figure 5: Reservoir model.

$$\varepsilon_w << 1 \qquad (30)$$

Given this characteristic of the parameter space is possible to use asymptotic methods of disturbance, so a solution is proposed such as:

$$u_\chi = \sum_{n=0}^{\infty} u_\chi, n\varepsilon_w^n$$
$$u_\zeta = \sum_{n=0}^{\infty} u_\zeta, n\varepsilon_w^n$$
$$p = \sum_{n=0}^{\infty} P_n \varepsilon_w^n \qquad (31)$$

Substituting this proposal in the original equations and using the fact that it ε is

linearly independent, the solution to order ε_w^0 is:

$$u_\chi,_0 = (\chi^3 - 2\chi) u_\chi (\zeta, \chi = 1^+)$$
$$u_\zeta,_0 = \frac{2(1-\chi^2)}{\zeta} \int_0^\zeta u_\chi (\zeta, \chi = 1^+) d\zeta \qquad (32)$$

Where $u_\chi(\zeta,\chi=1^+)$is the radial velocity with which the fluid reaches the pipe from the reservoir. On the other hand, the solution to order ε_w^1 is

$$u_{\chi,1=}0$$
$$u_{\zeta,1=}0 \qquad (33)$$

Conclusion

We show the problem of dispersion of a pollutant of nitrogen that comes from the gas cap of a reservoir in the vicinity of a well. It was shown that the effects are local, the increase in the dispersion of pollutants is important in a region only a few meters around the well, and only occurs in wells about to be invaded by the gas cap. This model explains why no-show nitrogen in the remote areas of the gas oil contact, but only in the areas of gas-oil contact. Which explains why nitrogen is only present in wells about to be invaded by the gas cap.

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