61. Images of l_2-Manifolds under Approximate Fibrations

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Let $\mathcal{U}$ be an open cover of a space Y. Two maps $f, g: X \to Y$ are said to be *$\mathcal{U}$-near* (or f is said to be *$\mathcal{U}$-near* to g) if for each $x \in X$, there is some $U \in \mathcal{U}$ which contains both $f(x)$ and $g(x)$. A map $f: X \to Y$ be *proper* if $f^{-1}(C)$ is compact for each compact set C in Y.

A proper surjection $p: E \to B$ is said to be an *approximate fibration* [2], provided that given a space X, maps $f: X \times \{0\} \to E$ and $F: X \times I \to B$ such that $pf = F|X \times \{0\}$, and an open cover $\mathcal{U}$ of B, there exists a map $\tilde{F}: X \times I \to E$ such that $\tilde{F}|X \times \{0\} = f$ and $p\tilde{F}$ is *$\mathcal{U}$-near* to F. From [3], *CE-maps of ANR's are approximate fibrations.*

J. Mogilski [5] proved that *among ANR's the CE-images of l_2-manifolds are l_2-manifolds* (also see Theorem 4.1 in [6]). In this note, we prove the following generalization:

Theorem. *Let M be an l_2-manifold and B a separable complete-metrizable ANR. If there is an approximate fibration $p: M \to B$ from M onto B, then B is an l_2-manifold.*

To prove the above theorem, we use the Toruńczyk characterization of l_2-manifolds (Corollary 3.3 in [6]) which states that a separable complete-metrizable ANR X is an l_2-manifold if and only if X satisfies the following two condition:

(*1) For any two maps $f, g: Q \to X$ of the Hilbert cube Q and an open cover $\mathcal{U}$ of X, there are two maps $f', g': Q \to X$ such that $f'(Q) \cap g'(Q) = \emptyset$ and f' and g' are $\mathcal{U}$-near to f and g, respectively.

(*2) For any map $f: \sum_{n \in N} P_n \to X$ of a topological sum of compact polyhedra and an open cover $\mathcal{U}$ of X, there is a map $f': \sum_{n \in N} P_n \to X$ such that $\{f'(P_n) | n \in N\}$ is locally finite and f' is $\mathcal{U}$-near to f.

Using this characterization, we will prove that each point $x_0 \in B$ has an l_2-manifold neighbourhood. Since B is locally contractible, x_0 has an open neighbourhood U which is contractible in B. Then U is a separable complete-metrizable ANR. Since $p: M \to B$ is an approximate fibration, it is easy to see that U has the following property:

($*$) For any open cover $\mathcal{U}$ of U and any map $f: Y \to U$ of any compact space Y, there exists a map $\tilde{f}: Y \to p^{-1}(U)$ such that $p\tilde{f}$ is $\mathcal{U}$-near to f.

Now we will see that U satifies the conditions (*1) and (*2), that is, U is an l_2-manifold.

Condition (*1). Let $f, g: Q \to U$ be maps and $\mathcal{U}$ an open cover of U. From (*), there are maps $\tilde{f}, \tilde{g}: Q \to p^{-1}(U)$ such that $p\tilde{f}$ and $p\tilde{g}$ are $\mathcal{U}$-near to f and g, resp. Since $p^{-1}(p\tilde{f}(Q))$ is compact, it is strongly negligible in M (see [1]), so there is a homeomorphism $h: M \to M \setminus p^{-1}(p\tilde{f}(Q))$ which is $p^{-1}(\mathcal{U})$-near to id. Thus we have maps $f' = p\tilde{f}$, $g' = ph\tilde{g}: Q \to U$ such that $f'(Q) \cap g'(Q) = \emptyset$ and f' and g' are $st(\mathcal{U})$-near to f and g, resp.

Condition (*2). Let $f: \sum_{n \in N} P_n \to U$ be a map of a topological sum of compact polyhedra and $\mathcal{U}$ an open cover of U. From (*), there exists a map $f: \sum_{n \in N} P_n \to p^{-1}(U)$ such that pf is $\mathcal{U}$-near to f. By the Henderson Approximation Theorem ([4], p. 49, a)), there exists a closed embedding $g: \sum_{n \in N} p_n \to p^{-1}(U)$ which is $p^{-1}(\mathcal{U})$-near to f. Then pg is $st(\mathcal{U})$-near to f. Since pg is proper, it is easy to see that $\{pg(P_n) \mid n \in N\}$ is locally finite.

References

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