149. A Necessary Unitary Field Theory as a Non-Holonomic Parabolic Lie Geometry Realized in the Three-Dimensional Cartesian Space. II

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The purpose of the present paper consists in the following five points: to deduce (i), (ii), (ii′), (iii), (iii′) mentioned below from Part I (these Proc., **29** (1953).

Since the three-dimensional non-holonomic Laguerre (parabolic Lie) geometry is in law a four-dimensional teleparallelism geometry keeping the Riemann (Weyl) metric, it is remarkable that the following conjecture of Prof. Einstein of 1928, which seems to be now scarcely considered, must acquire a renaissance: "Es ist denkbar, dass diese Theorie die ursprüngliche Fassung der allgemeinen Relativitätstheorie verdrängen wird".

(i) A Unitary Field Theory of a Single Particle

6. *A Necessary Unitary Field Theory of a Single Particle Charged with Rest-mass m_0 and Constant Electricity $-e$.* In Art. 4, we have solved a two particles problem stated in Art. 2 and the resulting generalizations of the Maxwell's equations were (4.24), (4.25), (4.26) and (4.27). Thereby the continuity condition (4.23) was assumed. Now in the case of a single particle P, we have

$$(6.1)\qquad \overline{m}_0=\overline{e}=\overline{\varepsilon}^i=\overline{\varepsilon}^4=\overline{\sigma}^i=\overline{\sigma}^4=\overline{\mathfrak{X}}^i=\overline{X}^i=\overline{a}^i=\overline{\alpha}^i=0.$$

Hence (4.24), (4.25), (4.26) and (4.27) become the *necessary-unitary-field-theoretical generalization of the Maxwell's equations*:

$$(6.2)\quad \frac{\partial}{\omega^i}(\mathfrak{X}^i+eX^i)=\varepsilon^4+\sigma^4,\ \frac{\partial}{\omega^4}(a^i+e\alpha^i)+\frac{\partial}{\omega^j}(\mathfrak{X}^k+eX^k)-\frac{\partial}{\omega^k}(\mathfrak{X}^j+eX^j)=0,$$

$$(6.2')\quad \frac{\partial}{\omega^j}(a^k+e\alpha^k)-\frac{\partial}{\omega^k}(a^j+e\alpha^j)-\frac{\partial}{\omega^4}(\mathfrak{X}^i+eX^i)=\varepsilon^i+\sigma^i,\ \frac{\partial}{\omega^i}(a^i+e\alpha^i)=0.$$

7. *General-Relativistic and Necessary-Unitary-Field-Theoretical Generalization of the Dirac Equation in the Case of a Single Particle.* In the case of a single particle P, the *general-relativistic and necessary-unitary-field-theoretical generalizations*

$$(5.3)\quad \left[\gamma_i\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^i}+e\phi^i+\frac{h}{2\pi i}\overline{E}\frac{\partial}{\omega^i}+\overline{e}\overline{\phi}^i\right)+\gamma_4\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^4}+e\phi^4+\overline{m}_0\overline{E}^2\right)\right.$$
$$\left.+\gamma_5\left(\frac{h}{2\pi i}\overline{E}\frac{\partial}{\omega^5}+\overline{e}\overline{\phi}^5+m_0E^2\right)\right]\psi=0,$$

$$(7.1)\qquad \left[\gamma_l\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^l}+e\phi^l+\frac{h}{2\pi i}\overline{E}\frac{\partial}{\omega^l}+\bar{e}\bar{\phi}^l\right)\right.$$
$$\left.+\gamma_5\left[\frac{h}{2\pi i}E\frac{\partial}{\omega^5}+e\phi^5+\frac{h}{2\pi i}\overline{E}\frac{\partial}{\omega^5}+\bar{e}\bar{\phi}\right)\right]\psi=0$$

of the Dirac equation become

$$(7.2)\qquad \left[\gamma_l\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^l}+e\phi^l\right)+\gamma_5 m_0E^2\right]\psi=0,$$

$$(7.3)\qquad \left[\gamma_l\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^l}+e\phi^l\right)+\gamma_5\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^5}+e\phi^5\right)\right]\psi=0,$$

$$(7.4)\qquad \psi\equiv-\gamma_4\gamma_i(\mathfrak{X}^i+eX^i)+\sum\gamma_j\gamma_k(a^i+e\alpha^i).$$

8. *General-Relativistic and Necessary-Unitary-Field-Theoretical Generalization of the Schrödinger Equation in the Case of a Single Particle.* In the case of a single particle P, the general-relativistic and necessary-unitary-field-theoretical generalizations

$$(8.1)\qquad \left[\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^t}+e\phi^t+\frac{h}{2\pi i}\overline{E}\frac{\partial}{\omega^t}+\bar{e}\bar{\phi}^t\right)^2-\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^4}+e\phi^4+\overline{m}_0\overline{E}^2\right)^2\right.$$
$$\left.+\left(\frac{h}{2\pi i}\overline{E}\frac{\partial}{\omega^5}+\bar{e}\bar{\phi}^5+m_0E^2\right)^2\right]\psi=0,$$

$$(8.2)\qquad \left[\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^t}+e\phi^t+\frac{h}{2\pi i}\overline{E}\frac{\partial}{\omega^t}+\bar{e}\bar{\phi}^t\right)^2-\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^4}+e\phi^4+\frac{h}{2\pi i}\overline{E}\frac{\partial}{\omega^t}\right.\right.$$
$$\left.\left.+\bar{e}\bar{\phi}^4\right)^2+\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^5}+e\phi^5+\frac{h}{2\pi i}\overline{E}\frac{\partial}{\omega^5}+\bar{e}\bar{\phi}^5\right)^2\right]\psi=0$$

become

$$(8.3)\qquad \left[\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^t}+e\phi^t\right)^2+(m_0E^2)^2\right]\psi=0,$$

$$(8.4)\qquad \left[\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^t}+e\phi^t\right)^2-\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^4}+e\phi^4\right)^2+\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^5}+e\phi^5\right)^2\right]\psi=0.$$

(ii) An Exact Gravitational (ii′) An Electromagnetic Wave Theory of Two Particles

9. *Gravitational Wave Theories*

A. S. Eddington [1][1]	Prof. A. Einstein [4]

has given an approximative wave theory of gravity within the

general relativity.	teleparallelism geometry.

10. *Problem Formulation.* Consider two particles O and P charged with constant

1) The ciphers in the square brackets refer throughout this paper to the References at the end of this paper.

rest-masses $\overline{m}_0$ and m_0 | electricity $-\overline{e}$ and $-e$

respectively, which move relative to each other. Then both O and P emit gravitational | P emit electromagnetic

energy radially in such a manner that the action is non-holonomic, the energy levels being spherical. The law of motion is required.

Solution. Introducing our conditions:

$$(10.1)\quad \begin{cases} \phi^i=0,\ \phi^4=0,\ \phi^5\neq 0,\ \overline{\phi}^i=0, \\ \overline{\phi}^4\neq 0,\ \overline{\phi}^5=0,\ E\phi^5\neq 0, \\ \overline{E}\overline{p}^4\neq 0,\ Ep^5\neq 0,\ \overline{E}\overline{p}^5=0, \end{cases} \qquad \begin{matrix} \phi^i\neq 0,\ -\phi^5=0,\ -\phi^4\neq 0,\ \overline{\phi}^i\neq 0, \\ \overline{\phi}^4=0,\ \overline{\phi}^5\neq 0,\ Ep^4\neq 0, \\ \overline{E}\overline{p}^4=0,\ Ep^5=0,\ \overline{E}\overline{p}^5\neq 0, \end{matrix}$$

$$(10.2)\quad (Ep^i+\overline{E}\overline{p}^i) \qquad (Ep^i+e\phi^i+\overline{E}\overline{p}^i+\overline{e}\overline{\phi}^i)$$
$$=(mE+\overline{m}\overline{E})\frac{d\xi^i}{dt},\ \left(m=m_0\frac{dr}{dS},\ \overline{m}=\overline{m}_0\frac{dS}{dr}\right),$$

$$(10.3)\quad (\overline{E}\overline{p}^4+\overline{e}\overline{\phi}^4)=(mE+\overline{m}\overline{E})\frac{d\xi^4}{dt}, \qquad (Ep^4+e\phi^4)=(mE+\overline{m}\overline{E})\frac{d\xi^4}{dt},$$

$$(10.4)\quad (Ep^5+e\phi^5)=(mE+\overline{m}\overline{E})\frac{d\xi^5}{dt}. \qquad (\overline{E}\overline{p}^5+\overline{e}\overline{\phi}^5)=(mE+m\overline{E})\frac{d\xi^5}{dt}.$$

Introducing these values into $(mE+\overline{m}\overline{E})$-times of

$$(10.5)\quad -i\gamma_5\omega^5/dt=\gamma_t\omega^t/dt,$$

$$(10.6)\quad \gamma_i(Ep^i+\overline{E}\overline{p}^i)+\gamma_4(\overline{E}\overline{p}^4+\overline{e}\overline{\phi}^4) =-i\gamma_5(Ep^5+e\phi^5), \qquad \gamma_i(Ep^i+e\phi^i+\overline{E}\overline{p}^i+\overline{e}\overline{\phi}^i)+\gamma_4(Ep^4 +e\phi^4)=-i\gamma_5(\overline{E}\overline{p}^5+\overline{e}\overline{\phi}^5),$$

which becomes

$$(10.7)\quad EP+e\Psi+\overline{E}\overline{P}+\overline{e}\overline{\Psi}=0$$

for

$$(10.8)\quad \begin{cases} \gamma_ip^i+i\gamma_5p^5=P, \gamma_i\overline{p}^i+\gamma_4\overline{p}^4=\overline{P}, \\ i\gamma_5\phi^5=\Psi,\ \gamma_4\overline{e}\overline{p}^4=\overline{\Psi}. \end{cases} \Bigg| \quad \begin{cases} \gamma_ip^i+\gamma_4p^4=P,\ \gamma_i\overline{p}^i+i\gamma_5\overline{p}^5=\overline{P}, \\ \gamma_i\phi^i+\gamma_4\phi^4=\Psi,\ \gamma_i\overline{\phi}^i+i\gamma_5\overline{\phi}^5=\overline{\Psi}. \end{cases}$$

Applying the operator (4.17) to (10.7), we obtain

$$(10.9)\quad 2\gamma_5\frac{\partial}{\omega^5}(EP+e\overline{\Psi}+\overline{E}\overline{P}+\overline{e}\overline{\Psi})=\frac{\partial}{\omega^l}(Ep^l+e\phi^l+\overline{E}\overline{p}^l+\overline{e}\overline{\phi}^l)$$
$$-\gamma_4\gamma_i(\mathfrak{X}^i+eX^i+\overline{\mathfrak{X}}^i+\overline{e}\overline{X}^i)+\sum\gamma_j\gamma_k(a^i+ea^i+\overline{a}^i+\overline{e}\overline{a}^i)$$
$$+2i\frac{\partial}{\omega^5}(Ep^5+ep^5+\overline{E}\overline{p}^5+\overline{e}\overline{\phi}^5)=0.$$

In the present case, we have

$$(10.10)\quad \mathfrak{X}^i=\frac{\partial(Ep^i)}{\omega^4}, \qquad \mathfrak{X}^i=\frac{\partial(Ep^4)}{\omega^i}+\frac{\partial(Ep^i)}{\omega^4}=0,$$

$$(10.11)\quad \overline{\mathfrak{X}}^i=i\frac{\partial(\overline{E}\overline{p}^i)}{\omega^5}, \qquad \overline{\mathfrak{X}}^i=\frac{\partial(\overline{E}\overline{p}^5)}{\omega^i}+i\frac{\partial(\overline{E}\overline{p}^i)}{\omega^5}=0,$$

$$(10.12)\quad a^i=\frac{\partial(Ep^k)}{\omega^j}-\frac{\partial(Ep^j)}{\omega^k}, \qquad\Big|\qquad a^i=\frac{\partial(Ep^k)}{\omega^j}-\frac{\partial(Ep^j)}{\omega^k},$$

$$(10.13)\quad \bar{a}^i=\frac{\partial(\overline{E}\bar{p}^k)}{\omega^j}-\frac{\partial(\overline{E}\bar{p}^j)}{\omega^k}, \qquad\Big|\qquad \bar{a}^i=\frac{\partial(\overline{E}\bar{p}^k)}{\omega^j}-\frac{\partial(\overline{E}\bar{p}^j)}{\omega^k}=0,$$

$$(10.14)\quad X^i=0,\ \ \overline{X}^i=0, \qquad\Big|\qquad X^i=\frac{\partial\phi^4}{\omega^i}+\frac{\partial\phi^i}{\omega^4},\ \ \overline{X}^i=i\frac{\partial\bar{\phi}^i}{\omega^5},$$

$$(10.15)\quad \mathscr{X}^i=\frac{\partial(Ep^i)}{\omega^4},\ \ \overline{\mathscr{X}}^i=i\frac{\partial(\overline{E}\bar{p}^i)}{\omega^5}, \qquad\Big|\qquad \mathscr{X}^i=0,\ \ \overline{\mathscr{X}}^i=0,$$

$$(10.16)\quad \alpha^i=0,\ \ \bar{\alpha}^i=0 \qquad\Big|\qquad \alpha^i=\frac{\partial\phi^k}{\omega^j}-\frac{\partial\phi^j}{\omega^k},\ \ \bar{\alpha}^i=\frac{\partial\bar{\phi}^k}{\omega^j}-\frac{\partial\bar{\phi}^j}{\omega^k}$$

and (10.9):

$$(10.17)\qquad 2\gamma_5\frac{\partial}{\omega^5}(EP+e\overline{\Psi}+\overline{E}\,\overline{P}+\bar{e}\overline{\Psi})$$

$$=\frac{\partial}{\omega^l}(Ep^l+\overline{E}\bar{p}^l+\bar{e}\bar{\phi}^l)-\gamma_4\gamma_i(\mathscr{X}^i+\overline{\mathscr{X}}^i)+\sum\gamma_j\gamma_k(a^i+\bar{a}^i)+2i\frac{\partial}{\omega^5}(Ep^5+ep^5)=0.$$

$$\Big|\quad =\frac{\partial}{\omega^l}(Ep^l+e\phi^l+\overline{E}\bar{p}^l+\bar{e}\bar{\phi}^l)-\gamma_4\gamma_i(eX^i+\bar{e}\overline{X}^i)+\sum\gamma_j\gamma_k(e\alpha^i+\bar{e}\bar{\alpha}^i)+2i\frac{\partial}{\omega^5}(\overline{E}\bar{p}^5+\bar{e}\bar{\phi}^5)=0.$$

Introducing the continuity condition

$$(10.18)\quad \frac{\partial}{\omega^i}(Ep^i+\overline{E}\bar{p}^i)+\frac{\partial}{\omega^4}(\overline{E}\bar{p}^4+\bar{e}\bar{\phi}^4)+2i\frac{\partial}{\omega^5}(Ep^5+e\phi^5)=0$$

$$\Big|\quad \frac{\partial}{\omega^l}(Ep^l+ep^l)+\frac{\partial}{\omega^i}(\overline{E}\bar{p}^i+\bar{e}\bar{\phi}^i)+2i\frac{\partial}{\omega^5}(\overline{E}p^5+\bar{e}\bar{\phi}^5)=0$$

and then applying the operator (4.17) to (10.18) once more, we obtain the *generalized Maxwell's equations*

$$(10.19)\quad \frac{\partial}{\omega^i}(\mathscr{X}^i+\overline{\mathscr{X}}^i)=\varepsilon^4+\bar{\varepsilon}^4, \qquad\Big|\qquad \frac{\partial}{\omega^i}(eX^i+\bar{e}\overline{X}^i)=\sigma^4+\bar{\sigma}^4,$$

$$(10.20)\quad \frac{\partial}{\omega^i}(a^i+\bar{a}^i)+\frac{\partial}{\omega^j}(\mathscr{X}^k+\overline{\mathscr{X}}^k)-\frac{\partial}{\omega^k}(\mathscr{X}^j+\overline{\mathscr{X}}^j)=0,$$

$$\Big|\quad \frac{\partial}{\omega^i}(e\alpha^i+\bar{e}\bar{\alpha}^i)+\frac{\partial}{\omega^j}(eX^k+\bar{e}\overline{X}^k)-\frac{\partial}{\omega^k}(eX^j+\bar{e}\overline{X}^j)=0,$$

$$(10.21)\quad \frac{\partial}{\omega^j}(a^k+\bar{a}^k)-\frac{\partial}{\omega^4}(a^j+\bar{a}^j)-\frac{\partial}{\omega^k}(\mathscr{X}^i+\overline{\mathscr{X}}^i)=\varepsilon^i+\bar{\varepsilon}^i,$$

$$\Big|\quad \frac{\partial}{\omega^i}(e\alpha^k+\bar{e}\bar{\alpha}^k)-\frac{\partial}{\omega^k}(e\alpha^j+\bar{e}\bar{\alpha}^j)-\frac{\partial}{\omega^4}(eX^i+\bar{e}\overline{X}^i)=\sigma^i+\bar{\sigma}^i,$$

$$(10.22)\quad \frac{\partial}{\omega^i}(a^i+\bar{a}^i)=0 \qquad\Big|\qquad \frac{\partial}{\omega^i}(e\alpha^i+\bar{e}\bar{\alpha}^i)=0$$

for the two particles O and P.

11. *General-Relativistic Analogue to the Dirac Equation for the Case of Gravitation* | *Case of Electromagnetism* *for Two Particles.* The (5.1) becomes

(11.1)
$$\psi = -\gamma_4\gamma_i(\mathcal{X}^i+\bar{\mathcal{X}}^i)+\sum\gamma_j\gamma_k(a^i+\bar{a}^i) \quad\Big|\quad \psi = -\gamma_4\gamma_i(eX^i+\bar{e}\bar{X}^i)+\sum\gamma_j\gamma_k(ea^i+\bar{e}\bar{a}^i)$$

and (5.3) and (7.1) to

(11.2)
$$\left[\gamma_i(E+\bar{E})\frac{h}{2\pi i}\frac{\partial}{\omega^i}+\gamma_4\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^4}+\bar{m}_0\bar{E}^2\right)+\gamma_5\left(\frac{h}{2\pi i}\bar{E}\frac{\partial}{\omega^5}+m_0E^2\right)\right]\psi=0,$$

$$\left[\gamma_i\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^i}+e\phi^i+\frac{h}{2\pi i}\bar{E}\frac{\partial}{\omega^i}+\bar{e}\bar{\phi}^i\right)+\gamma_4\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^4}+e\phi^4\right)+\gamma_5\left(\frac{h}{2\pi i}\bar{E}\frac{\partial}{\omega^5}+\bar{e}\bar{\phi}^5\right)\right]\psi=0,$$

(11.3)
$$\gamma_i\left[\frac{h}{2\pi i}(E+\bar{E})\frac{\partial}{\omega^i}+\gamma_4\left\{\frac{h}{2\pi i}(E+\bar{E})\frac{\partial}{\omega^4}+\bar{e}\bar{\phi}^4\right\}+\gamma_5\left\{\frac{h}{2\pi i}(E+\bar{E})\frac{\partial}{\omega^5}+e\phi^4\right\}\right]\psi=0.$$

$$\left[\gamma_i\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^i}+e\phi^i+\frac{h}{2\pi i}\bar{E}\frac{\partial}{\omega^i}+\bar{e}\bar{\phi}^i\right)+\gamma_4\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^4}+e\phi^4+\frac{h}{2\pi i}\bar{E}\frac{\partial}{\omega^4}\right)+\gamma_5\left\{(E+\bar{E})\frac{h}{2\pi i}\frac{\partial}{\omega^5}+\bar{e}\bar{\phi}^5\right\}\right]\psi=0.$$

12. *General Relativistic Analogue to the Schrödinger Equation for the Gravitation* | *for the Electromagnetism* *for Two Particles.* (8.1) and (8.2) become our *general-relativistic analogues to the Schrödinger equation*:

(12.1)
$$\left[-\frac{h^2}{4\pi^2}(E+\bar{E})^2\left(\frac{\partial}{\omega^i}\right)^2-\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^4}+\bar{m}_0\bar{E}^2\right)^2+\left(\frac{h}{2\pi i}\bar{E}\frac{\partial}{\omega^5}+m_0E^2\right)^2\right]\psi=0,$$

$$\left[\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^i}+e\phi^i+\frac{h}{2\pi i}\bar{E}\frac{\partial}{\omega^i}+\bar{e}\bar{\phi}^i\right)^2-\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^4}+e\phi^4\right)^2+\left(\frac{h}{2\pi i}\bar{E}\frac{\partial}{\omega^5}+\bar{e}\bar{\phi}^5\right)^2\right]\psi=0,$$

(12.2)
$$\left[-\frac{h^2}{4\pi^2}(E+\bar{E})^2\left(\frac{\partial}{\omega^i}\right)^2-\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^4}+\frac{h}{2\pi i}\bar{E}\frac{\partial}{\omega^4}+\bar{e}\bar{\phi}^4\right)^2+\left(\frac{h}{2\pi i}(E+\bar{E})\frac{\partial}{\omega^5}+e\phi^5\right)^2\right]\psi=0.$$

$$\left[\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^i}+e\phi^i+\frac{h}{2\pi i}\bar{E}\frac{\partial}{\omega^i}+\bar{e}\bar{\phi}^i\right)^2-\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^4}+e\phi^4+\frac{h}{2\pi i}\bar{E}\frac{\partial}{\omega^4}\right)^2+\left(\frac{h}{2\pi i}(E+\bar{E})\frac{\partial}{\omega^5}+\bar{e}\bar{\phi}^5\right)^2\right]\psi=0.$$

(iii) An Exact Gravitational | (iii′) An Electromagnetic Wave Theory of a Single Particle

13. *Problem Formulation.* Consider a single particle P charged with constant rest-mass m_0, | with constant electricity $-e$,

which makes a motion emitting

gravitational | electromagnetic

energy radially in such a manner, that the action is non-holonomic, the energy level being spherical. The law of motion is required.

Solution. Introducing the condition that the particle O has no

rest-mass ($\overline{m}_0=0$) | electric charge ($e=0$)

into (10.19), (10.20), (10.21) and (10.22), we obtain

(13)

$$\frac{\partial\mathfrak{X}^i}{\omega^i}=\varepsilon^4,\quad \frac{\partial\alpha^i}{\omega^i}+\frac{\partial\mathfrak{X}^k}{\omega^j}-\frac{\partial\mathfrak{X}^j}{\omega^k}=0,\quad \frac{\partial\alpha^k}{\omega^j}-\frac{\partial\alpha^j}{\omega^k}-\frac{\partial\mathfrak{X}^i}{\omega^4}=0,\quad \frac{\partial\alpha^i}{\omega^i}=0.$$

$$\frac{\partial X^i}{\omega^i}=\frac{\sigma^4}{e},\quad \frac{\partial\alpha^i}{\omega^i}+\frac{\partial X^k}{\omega^j}-\frac{\partial X^j}{\omega^k}=0,\quad \frac{\partial\alpha^k}{\omega^j}-\frac{\partial\alpha^j}{\omega^k}-\frac{\partial X^i}{\omega^4}=\frac{\sigma^i}{e},\quad \frac{\partial\alpha^i}{\omega^i}=0.$$

The lefthand side is | The righthand side is

gravitational analogues | *general-relativistic generalizations*

of the Maxwell's equations.

14. *General-Relativistic Analogues to the Dirac Equations for*

the Case of Gravitation | *the Case of Electromagnetism*

for a Single Particle. For the case of a single particle P, (11.2) and (11.3) becomes respectively to

(14.1)

$$\left(\left[\gamma_l E\frac{h}{2\pi i}\frac{\partial}{\omega^l}+\gamma_5 m_0 E^2\right]\psi=0,\qquad \left[\gamma_l\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^l}+e\phi^l\right)\right]\psi=0,$$

(14.2)

$$\left[\gamma_l\frac{h}{2\pi i}E\frac{\partial}{\omega^l}+\gamma_4\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^4}+\bar{e}\phi^4\right)+\gamma_5\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^5}+e\phi^5\right)\right]\psi=0.$$

$$\left[\gamma_l\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^l}+e\phi^l\right)+\gamma_5\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^5}\right)\right]\psi=0.$$

15. *General-Relativistic Analogue to the Schrödinger Equations for the Case of*

Gravitation | *Electromagnetism*

for a Single Particle. For the case of a single particle P, (12.1) and (12.2) become respectively to

(15.1)

$$\left(-\frac{h^2}{4\pi^2}E^2\left(\frac{\partial}{\omega^i}\right)^2-\frac{h^2}{4\pi^2}E^2\left(\frac{\partial}{\omega^l}\right)^2+(m_0E^2)^2\right]\psi=0,$$

$$\left[\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^l}+e\phi^l\right)^2-\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^4}+e\phi^4\right)^2\right]\psi=0,$$

(15.2)

$$\left[\frac{h^2}{4\pi^2}E^2\left(\frac{\partial}{\omega^l}\right)^2+\frac{h^2}{4\pi^2}E^2\left(\frac{\partial}{\omega^4}\right)^2+\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^5}+e\phi^5\right)^2\right]\psi=0.$$

$$\left[\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^l}+e\phi^l\right)^2-\left(\frac{h}{2\pi i}E\frac{\partial}{\omega^4}+e\phi^4\right)^2-\frac{h^2}{4\pi^2}E^2\left(\frac{\partial}{\omega^5}\right)^2\right]\psi=0.$$

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