23. *Supplement to "Homomorphisms of a Left Simple Semigroup onto a Group"*

By Tôru SAITÔ

Tokyo Gakugei University

(Comm. by K. SHODA, M.J.A., March 12, 1959)

In this supplementary note of [1], we show that Theorem 3 in [1] holds even when we omit the condition that 'N contains the core of S' in its assumption, and that, indeed, any normal and left unitary subsemigroup of a left simple semigroup S contains the core of S.

Let N be a normal and left unitary subsemigroup of a left simple semigroup S. Then, *without the assumption that N contains the core of S*, from Lemma 6 onwards in [1], we can argue in the same way as in [1], except the parts which we shall discuss in the following.

At first, we show that Lemma 7, asserting that $a \in Na$ for any $a \in S$, holds in this case. In fact, we take an element $x \in N$. By the left simplicity of S, there exist two elements y and n of S such that $yx=a$ and $nx=x$. By Lemma 6, we have $n \in N$. Moreover, since N is normal, there exists an element $n' \in N$ such that $yn=n'y$. Then we have

$$a=yx=ynx=n'yx=n'a \in Na.$$

Also, in the proof of Lemma 10 in [1], the statement "Then there exists an element $u \in S$ such that $ua=a$. The element u belongs to U and so belongs to N" is replaced in this case by the statement "Then by Lemma 7, there exists an element $u \in U$ such that $a=ua$".

After discussing in the same way as in [1] except the parts abovementioned, we obtain Theorem 3 in [1] in a better form:

Theorem 3'. *If N is a normal and left unitary subsemigroup of S, then there exists a homomorphism θ of S onto a group such that the kernel of θ is N.*

As an immediate consequence of the above theorem, we obtain the following

Theorem 4. *If N is a normal and left unitary subsemigroup of S, then N contains the core of S.*

In fact, by Theorem 3', there exists a homomorphism θ of S onto a group G such that the kernel of θ is N. But by Theorem 2 of [1], N, being the kernel of θ, contains the core of S.

Reference

[1] T. Saito: Homomorphisms of a left simple semigroup onto a group, Proc. Japan Acad., **34**, 664-667 (1958).