7. *Correction to the Paper "On the Behaviour of Analytic Functions"*

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On Dec. 9, 1962, C. Constantinescu wrote to me that Theorem 4, b)[1] is false. The purpose of the present paper is to show the root of my mistake and to prove the theorem under a little changed conditions.

Let R be a Riemann surface of positive boundary over a basic surface $\underline{R}$. Suppose a metric defined on $\overline{\underline{R}}=\underline{R}+\underline{B}$, where $\underline{B}$ is the ideal boundary of $\underline{R}$. Put $\underline{B}_n=E\left[w\in\overline{\underline{R}}: \text{dist}\,(w,\underline{B})<\dfrac{1}{n}\right]$. Let $C(r,p)$ be a circle$=E[w\in\overline{\underline{R}}: \text{dist}\,(w,p)<r]$, $p\in\overline{\underline{R}}$. Suppose that $\overline{\underline{R}}$ is a Riemann surface with positive boundary. Put $\Omega_{1-\varepsilon}=E[w\in\underline{R}: w(\partial C(r_2,p),w)>1-\varepsilon]$. If $\lim\limits_{\varepsilon\to 0} w(\Omega_{1-\varepsilon}\cap C(r_1,p)\cap\underline{B},w)$[2]$=0$ for $r_1<r_2$, we call the topology is H.S. (harmonically separative), where $w(\partial C(r_2,p)\cap\underline{B},w)$ is H.M. (harmonic measure) of $\partial C(r_2,p)\cap\underline{B}$. Let $\{\underline{R}_n\}$ $(n=0,1,2,\cdots)$ be an exhaustion of $\underline{R}$ with compact relative boundary $\partial\underline{R}_n$. Suppose C.P. (capacitary potential) of $C(r_1,p)\cap\underline{B}$ $\omega(C(r_1,p)\cap\underline{B},w)>0$, where $\omega(C(r_1,p)\cap\underline{B},z)=\lim\limits_n\omega_n(z)$ and $\omega_n(z)$ is a harmonic function in $\underline{R}-\underline{R}_0-(C(r_1,p)\cap\underline{B}_n)$ such that $\omega_n(z)=0$ on $\partial\underline{R}_0$, $\omega_n(z)=1$ on $C(r_1,p)\cap\underline{B}_n$ and $\omega_n(z)$ has M.D.I. (minimal Dirichlet integral). If there exists a increasing sequence of domains $\{V_n\}$ such that $\omega(C(r_1,p)\cap CV_n\cap\underline{B},w)$[3] $\downarrow 0(\omega(C(r_1,p)\cap V_n,w)>0)$ as $n\to\infty$ and that there exists at least one continuous function $U_n(w)$ in $C(r_2,p)-(C(r_1,p)\cap V_n)$ such that $U_n(w)=1$ on $(V_n\cap C(r_1,p))$, $U_n(w)=0$ on $\partial C(r_2,p)$ and $D(U_n(w))<L_n<\infty$ for every n, we call such a topology is D.S. (Dirichlet separative). We proved that K and N-Martin's topologies, Green's and Stoilow's topologies are H.S.[4] and N-Martin's, Green's, and Stoilow's topologies

1) Z. Kuramochi: On the behaviour of analytic functions on the ideal boundary. IV, Proc. Japan Acad., **38**, 200–203 (1962).

2) $w(\Omega_{1-\varepsilon}\cap C(r,p)\cap\underline{B},w)=\lim\limits_n w(\Omega_{1-\varepsilon}\cap\underline{B}_n\cap C(r,p),w)$, where $w(A\cap\underline{B}_n,w)$ is the least positive superharmonic function in $\underline{R}$ such that $w(A\cap\underline{B}_n,z)\geqq 1$ on $A\cap\underline{B}_n$ and $A=\Omega_{1-\varepsilon}\cap C(r,p)$.

3) $\omega(G\cap\underline{B},w)=\lim\limits_n\omega(G\cap\underline{B}_n,w)$, where $\omega(G\cap B_n,w)$ is harmonic function in $R-R_0-(G\cap\underline{B}_n)$ such that $\omega(G\cap\underline{B}_n,w)=1$ on $G\cap B_n$, $=0$ on $\partial\underline{R}_0$ and has M.D.I. and $G=C(r,p)\cap CV_n$.

4) Z. Kuramochi: On the behaviour of analytic functions on the ideal boundary. II and III, Proc. Japan Acad., **38**, 188–198 (1962).

are D.S.[4] Clearly Martin's topologies and Stoilow's topology are *compact*. If R satisfies the following conditions, we said that R is almost finitely sheeted. 1) If we take sufficiently large compact set $\underline{R}'$, $n(w) \leqq M < \infty$ in $\underline{R} - \underline{R}'$, where $n(w)$ is the number of times when w is covered by R. 2) For any point p of $\underline{R}$, there exists a compact circle $C(r, p) \subset \underline{R}$ such that $C(r, p)$ is mapped onto a compact domain D_ζ in the ζ-plane by a local parameter at p and that the area of any connected piece over $C(r, p)$ has finite area.

Let $w = f(z)$ be an analytic function from R into $\underline{R}$. Suppose α-Martin's topology is defined on $R + B$. Put $M^\alpha(p) = \bigcap_n \overline{f(G_n)}: G_n \ni^\alpha p$ (G contains p α-approximately)[5] and put $\delta M^\alpha(p) = \text{dia}\, M^\alpha(p)$ relative to the topology defined on $\underline{R}$, where $\alpha = N$ or K. Then

Theorem 4. b).[6] *Let R be a covering surface with positive boundary and with N-Martin's topology over a basic surface $\underline{R}$ with* D.S. *topology (R has null or positive boundary). Suppose R is a covering surface of almost finitely sheeted. Then $M^N(p): p \in B_1^N$ is defined except an F_σ set of capacity zero and $S^N = E[p \in B_1^N : \delta M(p) > 0]$ is a $G_{\delta\sigma}$ set of inner capacity zero.*

This theorem is false. C. Constantinescu showed the following example: Let $\underline{R}$ be a closed Riemann surface of genus $\geqq 3$. Let R be the universal covering surface of $\underline{R}$ and let $w = f(z)$: $z \in R$, $w \in \underline{R}$. Then $f(z)$ is almost finitely sheeted in my sense, but Theorem 4, b) does not hold. I find the error which is the part (from 2nd column from bottom of p. 201 to 1st column from top of p. 202) in which I asserted by $\omega(F \cap \sum G_i, z) > 0$ that there exists at least one G_i such that $\omega(F \cap G_i, z) > 0$, where F is a closed set in B (boundary of R). This is false. The mistake is a wrong application of P.C. 5:

$$\sum_i^k w(F \cap G_i, z, G') \geqq w(F \cap \sum_i^k G_i, z, G'): \quad G_i \subset G', \tag{1}$$

$$\sum_i^k \omega(F \cap G_i, z, G') \geqq \omega(F \cap \sum_i^k G_i, z, G'): \quad G_i \subset G', \tag{2}$$

where $w(F \cap G_i, z, G')(\omega(F \cap G_i, z, G'))$ is harmonic measure (capacity) of $F \cap G_i$ relative to G' and G_i and G' may consist of an infinite number of components. We proved the above inequalities for finite number k[6] but we applied them for infinite k. This is my mistake. Hence we must change the definition of finitely sheeted and assume another condition for the validity of Theorem 4, b). If R satisfies the following two conditions 1′) (this is the same as 1) and 2′)). For any point of R, there exists a compact circle $C(r, p) \subset \underline{R}$ such that the sum of area of all connected pieces over $C(r, p)$ is finite. Then we say that R is almost finitely sheeted.

5) See 4).
6) See 1).

Theorem 4. b′). *Let R be a covering surface with positive boundary and with N-Martin's topology over $\underline{R}$ with* D.S. *topology and suppose $\underline{R}+\underline{B}$ is compact with respect to the* H.D. *topology (this is the newly added condition). Suppose R is a covering surface of almost finitely sheeted in new sense. Then $M^N(p)$ is defined except an F_σ set of capacity zero and $S=E[p\in B_1^N : \delta M(p)>0]$ is a $G_{\delta\sigma}$ set of inner capacity zero.*

It is sufficient to correct the part of mistake but we shall prove for convenience. We cover $\underline{R}+\underline{B}$ by a system of a finite number of circles $C_{n,i}$ with radius $\frac{1}{n}$ so that any circle with radius $\frac{1}{3n}$ is contained in a certain $C_{n,i}$. Put $T_n=E[p\in B_1^N$: $p^N\notin$ any component of $f^{-1}(C_{n,i})]$. Assume T_n has a closed subset F of B of positive capacity. Then $\omega(F,z)>0$ (we write $\omega(F,z)$ simply for $\omega(F,z,R-R_0)$, where R_0 is a fixed compact disc in R.

Case 1. $0<\omega(F\cap\underline{B}',z)=\lim_n\lim_m\omega(F_m\cap\underline{B}'_n,z)$: $F_m=E\left[z\in\overline{R}:\text{dist}(F,z)\leqq\frac{1}{m}\right]$, $\underline{B}'_n=f^{-1}(\underline{B}_n)$ and $\underline{B}_n=E\left[w\in R:\text{dist}(w,\underline{B})\leqq\frac{1}{n}\right]$. Let $\{\underline{R}_n\}$ be an exhaustion of $\underline{R}$ with compact relative boundary ∂R_n. Since R_0 is compact, there exist numbers n_0 and m_0 such that $f(R_0)\cap(\underline{R}-\underline{R}_{n_0})=0$ and $n(w)\leqq M$ in $\underline{R}-\underline{R}_{n_0}\supset\underline{B}_{m_0}$. Since $\underline{R}+\underline{B}$ is compact, we can find a finite number of circles $\{C(r,p_i)\}$ and $\{C(r',p_i)\}$ such that $\underline{B}_{2m_0}\subset\sum_i^k C(r,p_i)\subset\sum_i^k C(r',p_i)\subset\underline{B}_{m_0}$: $r<r'<\frac{1}{3n}$. By $\omega(F\cap\underline{B}',z)\leqq\sum_i^k\omega(F\cap f^{-1}(C(r,p_i))\cap\underline{B}',z)$, there exists a circle $C(r,p)$ such that $\omega(F\cap\underline{B}'\cap f^{-1}(C(r,p)),z)>0$, where $\omega(F\cap\underline{B}'\cap f^{-1}(C(r,p),z)=\lim_m\omega(F\cap\underline{B}'_m\cap f^{-1}(C(r,p)),z)$. Next since the topology is D.S., there exists a sequence $V_n\uparrow$ such that $\omega(CV_n\cap C(r,p)\cap\underline{B},w)\downarrow 0$ as $n\to\infty$ and $D(\omega(C(r,p)\cap V_n,w,C(r',p))<L_n<\infty$. Now since $\underline{R}_{n_0}$ is compact we have $\omega(CV_n\cap C(r,p)\cap\underline{B},w,R-R_{n_0})\downarrow 0$ by $\omega(CV_n\cap C(r,p)\cap\underline{B},w)(=\omega(CV_n\cap C(r,p)\cap\underline{B},w,\underline{R}-\underline{R}_0)\downarrow 0$ as $n\to\infty$. On the other hand, $n(w)\leqq M$ in $C(r',p)\subset B_{m_0}\subset(\underline{R}-\underline{R}_{n_0})$. Put $U_{n,m}(z)=\omega(CV_n\cap C(r,p)\cap\underline{B}_m,w,\underline{R}-\underline{R}_{n_0})$ in $f^{-1}(\underline{R}-\underline{R}_{n_0})$ and $U_{n,m}(z)=0$ in $R-f^{-1}(\underline{R}-\underline{R}_{n_0})\supset R-R_0$. Then $U_{n,m}(z)=1$ on $(G\cap f^{-1}(CV_n)\cap\underline{B}'_m)\supset(F\cap G\cap f^{-1}(CV_n)\cap\underline{B}'_m)$ and by the Dirichlet principle

$$\begin{aligned}&D(\omega(F\cap G\cap f^{-1}(CV_n)\cap\underline{B}'_m,z))\leqq D(U_{n,m}(z))\\&\qquad\leqq M\,D(\omega(CV_n\cap C(r,p)\cap\underline{B}_m,w,\underline{R}-\underline{R}_{n_0})).\end{aligned}$$

Let $m\to\infty$. Then

$$\begin{aligned}&D(\omega(F\cap f^{-1}(C(r,p))\cap CV_n\cap\underline{B}',z))\\&\qquad\leqq M\,D(\omega(CV_n\cap C(r,p)\cap\underline{B},w,R-R_{n_0})\downarrow 0\text{ as }n\to\infty.\qquad(3)\end{aligned}$$

Consider $\omega(C(r,p)\cap V_n,w,C(r',p))$ in R. Then by $n(w)\leqq M$ in $C(r',p)$ we see that there exists a harmonic function $V(z)$ in $G'-(f^{-1}(V_n)\cap G)$

such that $V(z)=1$ on $f^{-1}(V_n)\cap G$, $=0$ on $\partial G'$ and $D(V(z))\leqq MD(\omega(C(r,p)\cap V_n, w, C(r',p))\leqq ML_n<\infty$, where $G=f^{-1}(C(r,p))$ and $G'=f^{-1}(C(r',p))$ and G and G' may consist of infinite number of components. Hence by the Dirichlet principle

$$D(\omega(F\cap G\cap f^{-1}(V_n), z, G')\leqq D(V(z))\leqq ML_n.$$

On the other hand, by $\omega(F\cap G\cap \underline{B}'\cap f^{-1}(V_n), z)+\omega(F\cap G\cap \underline{B}'\cap f^{-1}(CV_n), z)\geqq \omega(F\cap G\cap \underline{B}', z)>0$ we have by (3) $\omega(F\cap G\cap \underline{B}'\cap f^{-1}(V_{n'}), z)>0$ for a number n'. Whence clearly for any number m' $0<D(\omega(F\cap G\cap \underline{B}'_{m'}\cap f^{-1}(V_{n'}), z))\leqq ML_{n'}$. Next also by the Dirichlet principle (by $R-R_0\supset G'$)

$$0<D(\omega(F\cap G\cap \underline{B}_{m'}\cap f^{-1}(V_{n'}), z))$$
$$\leqq D(\omega(F\cap G\cap \underline{B}'_{m'}\cap f^{-1}(V_{n'}), z, G')\leqq ML_{n'}<\infty.$$

Put $g=G\cap \underline{B}'_{m'}\cap f^{-1}(V_{n'})$. Then $\omega(F\cap g, z, G')>0$. Let $\{g'_i\}$ be components of G'. Then clearly $\omega(F\cap g, z, G')=\omega(F\cap g, z, g'_i)$ in g'_i. By $\sum_{g'_i} D(\omega(F\cap g, z, G'))>0$, there exists at least one component g' of G' such that

$$\omega(F\cap g, z, g')>0. \tag{4}$$

Case 2. $\omega(F\cap \underline{B}', z)=0$. In this case $\omega(F\cap C\underline{B}'_{n_0}, z)>0$ for a number n_0. Let n' be a number such that $\underline{R}_{n'}\supset C\underline{B}_{n_0}$. Since $\underline{R}_{n'}$ is compact, we can cover $\underline{R}_{n'}$ by $\sum_i^k C(r, p_i)$, where $\sum^k C(r', p_i)$ is compact and $r<r'<\dfrac{1}{3n}$. Hence as case 1) there exists at least one compact circle $C(r,p)$ such that $\omega(F\cap G, z)>0$: $G=f^{-1}(C(r,p))$.

We suppose $C(r',p)$ is mapped onto a circle Γ' on the ζ-plane by a local parameter at p. Let Γ be a circle in Γ' such that $\Gamma\supset$image of $C(r,p)$. Let $U(\zeta)$ be a continuous function in Γ' such that $U(\zeta)=1$ on Γ, $=0$ on $\partial\Gamma'$ and is harmonic in $\Gamma'-\Gamma$. Then $\left|\dfrac{\partial U(\zeta)}{\partial\xi}\right|\leqq M$, $\left|\dfrac{\partial U(\zeta)}{\partial\eta}\right|\leqq M$: $M<\infty$ and $\zeta=\xi+i\eta$. Put $U(z)=U(\zeta)$ in $G'=f^{-1}(C(r',p))$, $=0$ in $R-G'$. Then $D(U(z))\leqq M^2\times$area of G'. Put $V(z)=\min(U(z), \omega(F_m, z))$: $F_m=E\left[z\in\overline{R}: \text{dist}(z,F)\leqq\dfrac{1}{m}\right]$. Then $V(z)$ is continuous in $R-R_0$, $=0$ on $\partial G'+\partial R_0$ and $=1$ on $F_m\cap G$: $G=f^{-1}(C(r,p))$ and $D(V(z))\leqq D(U(z))+D(\omega(F_m\cap G, z))<\infty$. Next by the Dirichlet principle $D(V(z))\geqq D(\omega(F_m\cap G, z, G'))\geqq D(\omega(F_m\cap G, z))\geqq D(\omega(F\cap G, z))>0$. Let $m\to\infty$. Then $\omega(F\cap G, z, G')>0$. Hence there exists at least one component g' of G' such that

$$\omega(F\cap G, z, g')>0. \tag{5}$$

By (5) and (4) there exists at least one point $p\in(F\cap B_1^N)$ such

7) Z. Kuramochi: On the behaviour of analytic function on the ideal boundary. I, Proc. Japan Acad., **38**, 150–155 (1962).

that $p \overset{N}{\in} g'$ by Lemma 2.[7] But g' is a component of G' and $\delta f(g') \leqq \frac{1}{3n}$ and $f(g')$ is contained in a certain $C_{n,i}$. This contradicts the definition of T_n. Hence T_n has not a closed set of positive capacity and by Lemma 5[7] $S=\bigcup_n T_n$ is a $G_{\delta\sigma}$ of inner capacity zero.

We applied the inequality (1) for infinite number k. Hence also we must add another condition for Theorem 4, a).

Theorem 4. a). *Let $w=f(z)$ be an analytic function from R into $\underline{R}$, where R is a Riemann surface with K-Martin's topology and $\underline{R}$ is a surface with positive boundary with* H.S. *topology. Suppose $\underline{R}+\underline{B}$ is compact (this is the condition newly added) with respect to the topology. Then $M^k(p)$ is defined except an F_σ set of harmonic easure zero and $S=E[p\in B_1^k: \delta M(p)>0]$ is a $G_{\delta\sigma}$ set of harmonic measure zero.*

We cover $\underline{R}+\underline{B}$ by $\{C_{n,i}\}$ and put $T_n=E[p\in B_1^k: p \overset{k}{\notin}$ any component of $f(C_{n,i})]$. Assume T_n has a closed set F of positive harmonic measure. Then $w(F,z)>0$. Now since $\underline{R}+\underline{B}$ is compact, as theorem 4, b) two case occur.

Case 1. $w(F\cap \underline{B}', z)=\lim_{n=\infty} w(F\cap \underline{B}'_n, z)>0$. Also $\underline{B}$ is compact, we can find circles $C(r,p)\subset C(r',p)$ such that $r<r'<\frac{1}{3n}$ and $w(F\cap \underline{B}' \cap G, z)>0$: $G=f^{-1}(C(r,p))$, $G'=f(C(r',p))$ and G and G' may consist of infinite number of components. Let ${}_{CG'}w(F\cap G\cap \underline{B}',z)$ be the least positive superharmonic function in R such that ${}_{CG'}w(F\cap G\cap \underline{B}', z) \geqq w(F\cap G\cap \underline{B}', z)$ on CG'. Then clearly ${}_{CG'}w(F\cap G\cap \underline{B}', z)={}_{Cg'}w(F\cap G \cap \underline{B}', z)$ in any component g' of G'. If g' is compact ${}_{CG'}w(F\cap G\cap \underline{B}', z) =w(F\cap G\cap \underline{B}', z)$ in g' and if g' is non compact $w(F\cap G\cap \underline{B}', z) -{}_{CG'}w(F\cap G\cap B', z)=w(F\cap G\cap \underline{B}', z, g')$.[8] Since $w(\partial C(r',p), w)$ can be considered in R and since ${}_{CG'}w(F\cap G\cap \underline{B}', z)\leqq w(\partial C(r',p), w)=1$ on $\partial G'$, we have $f(G\cap \Omega^*_{1-\varepsilon})\subset C(r,p)\cap \Omega^{W*}_{1-\varepsilon}$, where $\Omega^*_{1-\varepsilon}=E[z: {}_{CG'}w(F\cap G\cap \underline{B}', z) >1-\varepsilon]$ and $\Omega^{W*}_{1-\varepsilon}=E[w: w(\partial C(r',p), w)>1-\varepsilon]$. Now the topology on $\underline{R}$ is H.S. and $\underline{R}$ has a positive boundary and $(w(G\cap \underline{B}'\cap \Omega^*_{1-\varepsilon}, z) \leqq w(C(r,p)\cap \Omega^{W*}_{1-\varepsilon}\cap \underline{B}, w)\downarrow 0$ as $\varepsilon\to 0$. On the other hand, by $w(\Omega_{1-\frac{\varepsilon}{2}} \cap G\cap F\cap \underline{B}' - \Omega^*_{1-\varepsilon}\cap G\cap F\cap \underline{B}', z) + w(\Omega^*_{1-\varepsilon}\cap G\cap F\cap \underline{B}', z)\geqq w(\Omega_{1-\frac{\varepsilon}{2}}\cap G \cap F\cap \underline{B}', z)>0$ and $w(\Omega_{1-\frac{\varepsilon}{2}}\cap G\cap F\cap \underline{B}', z)=w(G\cap F\cap \underline{B}', z)$[8] is obtained by P.H. 3 by $w(G\cap C\Omega_{1-\frac{\varepsilon}{2}}\cap F\cap \underline{B}', z)=0$, where $\Omega_{1-\frac{\varepsilon}{2}}=E\left[z: w(G\cap F \cap \underline{B}', z)>1-\frac{\varepsilon}{2}\right]$. Now $w(G\cap \underline{B}'\cap \Omega^*_{1-\varepsilon}, z)\downarrow 0$ as $\varepsilon\to 0$. Fix a point z_0 at present. Choose ε such that $w(G\cap \Omega^*_{1-\varepsilon}\cap F\cap \underline{B}', z_0)\leqq w(G\cap \Omega^*_{1-\varepsilon}$

8) Z. Kuramochi: Potentials on Riemann surface, Journ. Sci. Hokkaido Univ., **14** (1962).

$\cap B', z_0) \leqq \frac{1}{2} w(G \cap F \cap \underline{B}', z_0)$. Then $w(\Omega_{1-\frac{\varepsilon}{2}} \cap G \cap F \cap \underline{B}' - \Omega^*_{1-\varepsilon} \cap G \cap F \cap \underline{B}', z_0) > \frac{1}{2} w(G \cap F \cap \underline{B}, z_0) > 0$. Hence $(\Omega_{1-\frac{\varepsilon}{2}} - \Omega^*_{1-\varepsilon}) \cap G$ is non void and in which $w(G \cap F \cap \underline{B}', z) - {}_{CG'}w(G \cap F \cap \underline{B}', z) > \frac{\varepsilon}{2} > 0$. Hence there exists at least one component g' of G' such that $w(G \cap F \cap \underline{B}', z, g') > 0$.

Case 2. $w(F \cap \underline{B}', z) = 0$. In this case we can find a number n_0 such that $w(F \cap f^{-1}(R_{n_0}), z) > 0$ and compact circles $C(r, p)$ and $C(r', p)$ such that $r < r' < \frac{1}{3n}$ and $\omega(F \cap G, z) > 0$, where $G = f^{-1}(C(r, p))$. Since $\underline{R}$ is of positive boundary and $C(r', p)$ is compact, $w(C(r, p), w) \leqq M$ in $\underline{R} - C(r', p)$: $M = \max w(C(r, p), w)$ on $\partial C(r', p)$ and $M < 1$. Since $w(C(r, p), w)$ can be considered on R, we have ${}_{CG'}w(F \cap G, z) \leqq w(F \cap G, z) \leqq w(G, z) \leqq w(C(r, p), w) \leqq M$ in $R - G'$ and ${}_{CG'}w(G \cap F, z) \leqq M$ in $G' = f^{-1}(C(r', p))$ by $w(G \cap F, z) \leqq w(C(r, p), w) \leqq M$ on $\partial G' \subset f^{-1}(\partial C(r', p))$. Thus ${}_{CG'}w(G \cap F, z) \leqq M$ in R. On the other hand, $w(G \cap F, z) > 0$ implies $\sup w(G \cap F, z) = 1$ and $w(G \cap F, z) - {}_{CG'}w(G \cap F, z) > 0$. Hence as in case 1 there exists at least one component g' of G' such that $w(G \cap F, z, g') > 0$. Thus as Theorem 4, b) T_n has not a closed set of positive harmonic measure. Now Borel sets in B[9] is harmonically measurable by Theorem 3.[10] Hence by Lemma 4 S is a $G_{\delta\sigma}$ set of harmonic measure zero.

Remark. Since w-plane is compact with Stoilow's topology, proofs of Theorem a'), original Fatou's and Beurling's theorems remain valid without any change.

Correction to the paper "Singular points of Riemann surface."[11] In the proofs of Theorem 12, b) and c) of the above paper, we used the inequalities 1) and 2) for infinite number k. To avoid this misapplication we use the spherical metric instead of Euclidean metric. Then the w-plane is compact. Next since $w(p \cap \sum \Delta_{ij} \cap D, z, G) > 0$ for b) $(\omega(p \cap \sum \Delta_{ij} \cap D, z, G) > 0$ for c)) does not imply that there exists at least one component Δ of $\{\Delta_{ij}\}$ such that $w(p \cap \Delta \cap D, z, G) > 0 (\omega(p \cap D \cap \Delta, z, G) > 0)$, we must read Δ_{ij} instead of component of Δ_{ij} i.e. $w(p \cap \Delta_{ij} \cap D, z, G) > 0 (\omega(p \cap \Delta_{ij} \cap D, z, G) > 0)$. Then proofs of the Theorem b) and c) are valid without any other revision.

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9) See 4).

10) See 4).

11) Z. Kuramochi: Singular points of Riemann surfaces, Journ. Sci. Hokkaido Univ., **14** (1962).