169. Some Three Valued Logics and its Algebraic Representations

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In his papers [3], [4], A. Rose formulated a three valued logic given by the following matrices:

x	0	1	2
Nx	2	1	0

$\vee$	0	1	2
0	0	0	0
1	0	0	1
2	0	1	2

$\wedge$	0	1	2
0	0	1	2
1	1	2	2
2	2	2	2

and for the implication $x\rightarrow y$,

$\rightarrow$	0	1	2
0	0	1	2
1	0	0	1
2	0	0	0 .

Where 0 is the designated value, and from $N1=1$, 1 is the center of this calculus.

Let {0, 1, 2} be a ring with characteristic 3 (see Gr. C. Moisil [1], [2]). Then these primitive functors are algebraically denoted by

$$Nx=2(x+1),$$
$$x\vee y=x^2y^2+xy(x+y),$$
$$x\wedge y=2x^2y^2+2xy(x+y)+(x+y),$$

and

$$x\rightarrow y=x^2y^2+xy(x+y)+2xy+y.$$

Further, two functors μ and ν defined by

x	0	1	2
μx	0	2	2
νx	0	0	2

are represented by $2x^2$ and x^2+2x respectively. These results are obtained by a similar way of Gr. C. Moisil [1].

B. Sobociński introduced an interesting partial system of three valued calculus of propositions in his paper [5]. In his calculus, the negation and the implication are defined by the following matrices:

x	0	1	2
Nx	1	0	2

$\rightarrow$	0	1	2
0	1	1	1
1	0	1	0
2	0	1	2

These primitive functors are represented by a ring of characteristic 3 mentioned above as follows:

$$Nx = 2x + 1,$$
$$x \rightarrow y = 2x^2y + 2xy^2 + 2x^2 + 1.$$

If we define two functors $\wedge$ and $\vee$ by the usual way, then these functors are given by the following matrices:

$\wedge$	0	1	2
0	0	0	0
1	0	1	1
2	0	1	2

$\vee$	0	1	2
0	0	1	0
1	1	1	1
2	0	1	2

Therefore we have these algebraic expressions as follows:

$$x \wedge y = 2x^2y^2 + 2xy(x+y) + xy,$$
$$x \vee y = x^2y^2 + 2(x^2+y^2) + xy + 2(x+y).$$

References

[1] Gr. C. Moisil: Sur les anneaux de caractéristique 2 ou 3 et leurs applications. Bull. de l'Ecole Poly. de Bucarest, **12**, 1-25 (1941).

[2] ——: Teoria Algebrica a Mecanismelor Automate. Bucarest (1959).

[3] A. Rose: An axiom system for three valued logic. Methodos, 233-239 (1951).

[4] ——: Axiom systems for three valued logic. Jour. of London Math. Soc., **26**, 50-58 (1951).

[5] B. Sobociński: Axiomatization of a partial system of three valued calculus of propositions. Jour. of Computing systems, **1**, 23-55 (1952).