

88. A New Characterization of Regular Duo Semigroups

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Let S be a semigroup. Following the notation and terminology of A. H. Clifford and G. B. Preston [1] we say that S is regular if, to every element a in S , there exists at least one element x in S such that $axa = a$. For the sake of brevity we shall say that S is a duo semigroup if every one-sided ideal of S is two-sided. The author proved several ideal-theoretic characterizations of regular duo semigroups (cf. [2]-[4]).

In this short note some new criteria for a semigroup to be a regular duo semigroup will be proved.

Theorem 1. *A semigroup S is a regular duo semigroup if and only if the conditions*

$$(1) \quad (L \cup LS)^2 = L,$$

and

$$(2) \quad (R \cup SR)^2 = R$$

hold for every left and every right ideal of S , respectively.

Proof. Let S be a regular duo semigroup. Then every one-sided ideal of S is two-sided, and

$$(3) \quad I \cap J = IJ$$

holds for any couple of ideals of S . (3) implies that every ideal I of S is globally idempotent, i.e.

$$(4) \quad I^2 = I$$

for any ideal I of S . This implies both (1) and (2).

Conversely, suppose that S is a semigroup with properties (1) and (2) for every left and right ideal, respectively. Then (2) implies that each right ideal R of S is also a left ideal, and (1) implies that every left ideal L of S is two-sided. Therefore S is a duo semigroup. Finally (1) implies (4) for any ideal I of S , which is equivalent to the regularity of a duo semigroup. (See [5].)

Theorem 1 is completely proved.

Next utilizing the author's recent results concerning the (m, n) -ideals of regular duo semigroups (see [6]), one can prove the following result.

Theorem 2. *A semigroup S is a regular duo semigroup if and only if the relation*

$$(5) \quad (B \cup SB)^2 = B = (B \cup BS)^2$$

holds for any bi-ideal B of S .

It is easy to see that the result of Theorem 2 remains true with quasi-ideal instead of bi-ideal.

Theorem 3. *A semigroup S is a regular duo semigroup if and only if the condition*

$$(6) \quad (Q \cup QS)^2 = Q = (Q \cup SQ)^2$$

holds for every quasi-ideal Q of S .

References

- [1] A. H. Clifford and G. B. Preston: *The Algebraic Theory of Semigroups*, Vol. I (2nd edition). American Mathematical Society, Providence, R. I. (1964).
- [2] S. Lajos: Note on semigroups which are semilattices of groups. *Proc. Japan Acad.*, **44**, 805–806 (1968).
- [3] —: On semilattices of groups. *Proc. Japan Acad.*, **45**, 383–384 (1969).
- [4] —: Notes on semilattices of groups. *Proc. Japan Acad.*, **46**, 151–152 (1970).
- [5] —: A remark on regular semigroups. *Proc. Japan Acad.*, **37**, 29–30 (1961).
- [6] —: On (m, n) -ideals in regular duo semigroups. *Acta Sci. Math.* (Széged), **31**, 179–180 (1970).