PROBLEMS

Problems, solutions, and any comments on the problems or solutions should be sent to Curtis Cooper, Department of Mathematics and Computer Science, University of Central Missouri, Warrensburg, MO 64093 or via email to cooper@ucmo.edu.

Problems which are new or interesting old problems which are not well–known may be submitted. They may range from challenging high school math problems to problems from advanced undergraduate or graduate mathematics courses. It is hoped that a wide variety of topics and difficulty levels will encourage a number of readers to actively participate in problems and solutions. An asterisk ($*$) after a number indicates a problem submitted without a solution.

Problems and solutions should be typed or neatly printed on separate sheets of paper. They should include the name of the contributor and the affiliation. Solutions to problems in this issue should be mailed no later than September 1, 2009, although solutions received after that date will also be considered until the time when a solution is published.

169. *Proposed by Dorin Marghidanu, Colegiul National "A. I. Cuza", Corabia, Romania.*

Let $0 < a, b, c < 1$. Prove that

$$2^a(b+c)^{1-a} + 2^b(c+a)^{1-b} + 2^c(a+b)^{1-c} < 4(a+b+c).$$

170. *Proposed by Don Redmond, Southern Illinois University, Carbondale, Illinois.*

Let a, b, and n be integers with a and n positive. Suppose that

$$a^n + b = p_1 \cdots p_r,$$

where $r \geq 1$ and the p's are primes. Let $f = (p_1 - 1) \cdots (p_j - 1)m$, where $1 \leq j \leq r$ and m is a natural number. Show that $a^{n+fk} + b$ is composite for all natural numbers k.

171. *Proposed by José Luis Díaz-Barrero, Universidad Politècnica de Cataluña, Barcelona, Spain.*

Let P be a point in the plane of triangle ABC with sides a, b, and c, respectively. Prove that

$$\left(PA^4 + PB^4 + PC^4\right)^3 \geq \frac{a^4 b^4 c^4}{27}.$$

When does equality occur?

172. *Proposed by Ovidiu Furdui, University of Toledo, Toledo, Ohio.*

Find all integer solutions to the Diophantine equation

$$x^4 - x^3 + 1 = y^2.$$