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# ON VALUE DISTRIBUTION OF NONDEGENERATE HOLOMORPHIC MAPS OF A TWO-DIMENSIONAL STEIN MANIFOLD $M$ TO $\mathbf{C}^2$ AND CLASSIFICATION OF $M$

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**Abstract**

We classify nondegenerate holomorphic maps of a two-dimensional Stein manifold $M$ to $\mathbf{C}^2$ by study about the value distribution of them.

## Introduction

In 1941, R. Nevanlinna [Ne] who had established the value distribution theory of entire functions of one complex variable, studied “nullberandeten Flächen” which formed a class of open Riemann surfaces having the value distribution property similar to $\mathbf{C}$, and made an epoch in the classification theory of open Riemann surfaces.

The author studied value distribution of the nondegenerate entire maps of $\mathbf{C}^2$ to $\mathbf{C}^2$ in [A1, 2], which was based on the value distribution theory of entire functions of two complex variables studied by Nishino [Ni1, 2, 3], Yamaguchi [Y1, 2] and others, not on the Nevanlinna theory of higher dimension.

Such value distribution theory of two complex variables was extended to the value distribution theory of holomorphic functions on a two-dimensional Stein manifold by Suzuki [Su1, 2] and Nishino [Ni3].

In this article, we study nondegenerate holomorphic maps of a two-dimensional Stein manifold to $\mathbf{C}^2$ using above theory and classify the two-dimensional Stein manifolds by the criterion of existence or nonexistence of certain maps.

We lay down a new paradigm that a generalization of a holomorphic function of one complex variable is an equi-dimensional nondegenerate holomorphic map of several complex variables.

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## Chapter 1. Definitions of types of nondegenerate holomorphic maps of a two-dimensional Stein manifold $M$ to $\mathbf{C}^2$

### §1. Open Riemann surfaces

Let $R$ and $R'$ be abstract open Riemann surfaces.

Definition 1.1. We call that $R$ is hyperboric, if there is a Green function on it. We call that $R$ is parabolic, that is, $R \in O_G$, if there is no Green function on it. According to Nishino, we say that $R$ is specially parabolic if it is parabolic and its genus is finite, and we say that $R$ is of algebraic type, if its genus $g$ is finite and its boundary consists of $n(<\infty)$ punctures. We say that such an algebraic type Riemann surface is of type $(g, n)$. If there is no nonconstant bounded holomorphic function on $R$, we denote that $R \in O_{AB}$.

The following proposition is well known.

Proposition 1.2. $O_G \subsetneqq O_{AB}$.

It is easy to see the following

Proposition 1.3. *Let $R$ and $R'$ be hyperbolic Riemann surfaces which do not belong to $O_{AB}$. Then there is no analytic curve in $R \times R'$ whose normalization is holomorphically isomorphic to a Riemann surface belonging to $O_{AB}$. It is similar for a bounded domain of $\mathbf{C}^2$, that is, there is no analytic curve in it whose normalization is holomorphically isomorphic to a Riemann surface belonging to $O_{AB}$.*

The following proposition is well known.

Proposition 1.4 (cf. [Ni3]). *Let $R$ and $R'$ be open Rieamann surfaces. If there is a nonconstant holomorphic map $R \to R'$, then $R'$ is parabolic (specially parabolic, of algebraic type) in case $R$ is parabolic (resp. specially parabolic, of algebraic type).*

### §2. Type of nonconstant holomorphic functions on $M$

We assume that $f \in \mathcal{O}(M)$, the set of the holomorphic functions on $M$, is nonconstant and we put $D = f(M) \subset \mathbf{C}$, the image of $M$.

Definition 2.1 (cf. [Ni3]).

(1) We say that $f$ is a hyperbolic type function if there exists at least one value $\alpha \in D$ such that the normalization of one of the irreducible components of $\{f = \alpha\}$ is holomorphically isomorphic to a hyperbolic Riemann surface, in short, a surface of hyperbolic type. Then we denote $f \in \mathcal{O}_H(M)$.

(2) We say that $f$ is a parabolic (specially parabolic, algebraic) type function if for every $\alpha \in D$, $\{f = \alpha\}$ consists of surfaces of parabolic (resp. specially parabolic, algebraic) type. Then we denote $f \in \mathcal{O}_P(M)$ (resp. $\mathcal{O}_{SP}(M)$, $\mathcal{O}_A(M)$).

The following theorem is known as a principle of uniformity or resonance.

Theorem 2.2 ([Y1, 2], [Ni3], [Su1, 2]). *If there is a set $E \subset D$ whose capacity is positive such that $\{f = \alpha\}$ for any $\alpha \in E$ contains a surface of parabolic (specially parabolic, algebraic) type, then $f \in \mathcal{O}_P(M)$ (resp. $\mathcal{O}_{SP}(M)$, $\mathcal{O}_A(M)$).*

### §3. Type of nondegenerate holomorphic maps of $M$ to $\mathbf{C}^2$

We call a holomorphic map $F : M \to \mathbf{C}^2$ is nondegenerate if $F(M)$ contains an open set in $\mathbf{C}^2$. Then we denote $F \in E(M)$.

Definition 3.1. Let $F \in E(M)$ and $P(x, y)$ be a nonconstant polynomial in $\mathbf{C}^2$.

(1) We say that $F$ is of genuinly hyperbolic type if $P \circ F \in \mathcal{O}_H(M)$ for every $P$, and denote $F \in GH(M)$.

(2) We say that $F$ is of hyperbolic type if $P \circ F \in \mathcal{O}_H(M)$ for some $P$, and denote $F \in H(M)$.

(3) We say that $F$ is of parabolic (specially parabolic, algebraic) type if $P \circ F \in \mathcal{O}_P(M)$ (resp. $\mathcal{O}_{SP}(M)$, $\mathcal{O}_A(M)$) for every $P$, and denote $F \in P(M)$ (resp. $SP(M)$, $A(M)$).

(4) We say that $F$ is of quasi-parabolic type if there are polynomials $P_1$ and $P_2$ such that $(P_1, P_2) \circ F \in E(M)$ and $P_i \circ F \in \mathcal{O}_P(M)$ $(i = 1, 2)$, and denote $F \in QP(M)$.

*Remark* 3.2. If $M = \mathbf{C}^2$, the map $F : z = e^x$, $w = e^y$ is contained in $QP(\mathbf{C}^2) - P(\mathbf{C}^2)$. Because $F \in H(\mathbf{C}^2)$ (see Proposition 6.4 in [A2]) and if we set $P_1 = z$, $P_2 = w$, then $P_i \circ F \in \mathcal{O}_A(\mathbf{C}^2) \subset \mathcal{O}_P(\mathbf{C}^2)$ $(i = 1, 2)$.

## Chapter 2. Value distribution of nondegenerate holomorphic maps of $M$ to $\mathbf{C}^2$

### §4. BL(Blaschke)-type map

Let $R$ be an open Rieamann surface. Heins [H] (cf. [S-N] and [K] p. 280) introduced the notion $SO_{HB}$ for a domain in $R$. Roughly speaking, it is a non-relatively compact subdomain $G$ in $R$, whose relative boundary $\partial G$ consits of at most countable Jordan curves which may not necessarily be closed and do not accumulate in $R$, and it is called of $SO_{HB}$ type, if its terminal domain has some parabolical property. Conventionally, a relatively compact subdomain in $R$ is assumed to belong to $SO_{HB}$ type.

Let $R$ and $R'$ be open Riemann surfaces and $\varphi : R \to R'$ be a nonconstant holomorphic map.

Definition 4.1 ([H], [K] p. 291). We call that $\varphi$ is locally of BL-type at $p' \in R'$ if there is a neiborhood $U'$ of $p'$ such that every connected component of $\varphi^{-1}(U')$ is of $SO_{HB}$ type. We say that $\varphi$ is of BL-type if $\varphi$ is locally of BL-type for every point of $R'$.

It is easy to see the following

Proposition 4.2 ([K] p. 292). *If $R \in O_G$, then for every $R'$, every nonconstant holomorphic map $\varphi : R \to R'$ is of BL-type.*

Definition 4.3. We denote by $n_\varphi(p')$ the number of $\{\varphi^{-1}(p'); p' \in R'\}$ counted with multiplicity, and set $n_\varphi = \sup_{p' \in R'} n_\varphi(p')$ $(\leq \infty)$.

Theorem 4.4 (Heins in [H], [K] p. 292). *If $\varphi$ is a BL-type map of $R$ to $R'$, then $n_\varphi = n_\varphi(p')$ for every $p' \in R'$, except for a set of capacity zero.*

## §5. Value distribution of nondegenerate holomorphic maps of $M$ to $\mathbf{C}^2$

The class $QP(M)$ includes $P(M)$ and a part of $H(M) - GH(M)$, and it has a value distribution property similar to $QP(\mathbf{C}^2)$. In [A2] we proved a generalization of the little Picard theorem for $QP(\mathbf{C}^2)$ and we will prove it for $QP(M)$ by the same method.

Let $F \in E(M)$ and $E_0$ be the set of points $p \in \mathbf{C}^2$ such that $\{F^{-1}(p)\}$ contains a curve of $M$. It is easy to see that $E_0$ consists of at most countable points.

Theorem 5.1. *Let $F \in QP(M)$. We denote that $N_F = \sup_{p \in \mathbf{C}^2 - E_0} N_F(p)$, where $N_F(p)$ is the number of $\{F^{-1}(p)\}$ counted with multiplicity $(0 \leq N_F(p) \leq \infty)$. Then there is a set $E : E_0 \subset E \subset \mathbf{C}^2$ with four-dimensional Lebesgue measure $0$ such that $N_F(p) = N_F$ for every point $p \in \mathbf{C}^2 - E$ and $N_F(p) < N_F$ for every point $p \in E - E_0$.*

*Proof.* Since $F \in QP(M)$, there are polynomials $P_1$ and $P_2$ such that $(P_1 \circ F, P_2 \circ F) \in E(M)$ and $P_i \circ F \in \mathcal{O}_P(M)$ $(i = 1, 2)$. We set $F = (f, g) = (P_1 \circ F, P_2 \circ F)$ anew.

We will separate the proof into two cases.

(1) There is a point $p_0'$ such that $N_F(p_0') = N_F$. If $N_F < \infty$, there is always such a point. Let $p_0' = (\alpha, \beta)$ and $L = \{x' = \alpha\}$. Since $f$ is a parabolic type function on $M$, $F^{-1}(L) = S_1 \cup S_2 \cup \cdots \cup T_1 \cup T_2 \cup \cdots$ where $S_i$ and $T_j$ are surfaces of parabolic type such that the holomorphic map $\varphi_i = F|_{S_i} : S_i \to L$ is nonconstant and the map $F|_{T_j} : T_j \to p_j' \in L \cap E_0$ is constant. By Proposition 4.2, $\varphi_i$ is a BL-type map. Then $N_F = n_{\varphi_1} + n_{\varphi_2} + \cdots$ and there is a set $e \subset L$ whose

capacity is zero such that for every point $p' \in L - e$, $N_F(p') = N_F$ by Theorem 4.4. We have used the fact that the capacity of the union of countable zero capacity sets is zero.

Let $L' = \{y' = \beta'\}$ where $\beta'$ is an arbitrary number such that $(\alpha, \beta') \in L - e$. Since $g$ is a parabolic type function on $M$, $F^{-1}(L') = S_1' \cup S_2' \cup \cdots \cup T_1' \cup T_2' \cup \cdots$ where $S_i'$ and $T_j'$ are surfaces of parabolic type such that $\varphi_i' = F|_{S_i'} : S_i' \to L'$ is a BL-type map and $F|_{T_j'}$ is a constant map. Then $N_F = n_{\varphi_1'} + n_{\varphi_2'} + \cdots$ and there is a set $e' \subset L'$ whose capacity is zero for every point $p' \in L' - e'$ and $N_F(p') = N_F$.

If we set $E = E_0 \cup \{p' \in M - E_0; N_F(p') < N_F\}$, the four-dimensional Lebesgue measure of $E$ is zero by Fubini's thorem.

(2) There are points $p_1', p_2', \ldots$ such that $N_F(p_i') \to \infty$ $(i \to \infty)$. From the proof of case (1), there is a set $E_i$ whose Lebesgue measure is zero such that, for every point $p' \in \mathbf{C}^2 - E_i$, we have $N_F(p') \geq N_i = N_F(p_i')$. Then for every point $p' \in \mathbf{C}^2 - \bigcup_{i=1}^{\infty} E_i$, we have $N_F(p') = \infty$. Since the Lebesgue measure of $\bigcup_{i=1}^{\infty} E_i$ is zero, we proved Theorem 5.1. □

Corollary 5.2. *If $F \in E(M)$ has an exceptional set of positive four-dimensional Lebesgue measure, then $F \in H(M) - QP(M)$.*

Corollary 5.3 (A generalization of the little Picard thorem). *If the map $F \in QP(M)$ and $N_F = \infty$, then $N_F(p) = \infty$ for $p \in \mathbf{C}^2 - E$ where $E$ is a set of four-dimensional Lebegue measure zero.*

## Chapter 3. Classification of two-dimensional Stein manifold $M$

### §6. Classification of $M$

Definition 6.1.

(1) $M$ is called of hyperbolic type $(M \in \mathscr{H})$ when $P(M) = \emptyset$.

(2) $M$ is called of parabolic type $(M \in \mathscr{P})$ when $P(M) \neq \emptyset$.

(3) $M$ is called of special parabolic type $(M \in \mathscr{SP})$ when $SP(M) \neq \emptyset$.

(4) $M$ is called of algebraic type $(M \in \mathscr{A})$ when $A(M) \neq \emptyset$.

(5) $M$ is called of quasi-parabolic type $(M \in \mathscr{QP})$ when $QP(M) \neq \emptyset$.

(6) $M$ is called of genuinly hyperbolic type $(M \in \mathscr{GH})$ when $E(M) = GH(M)$.

By proposition 1.4 it is easy to see the following

Proposition 6.2. *If there is a biholomorphic map $\Phi : M \to M'$, the type of $M$ and $M'$ are coincident.*

Proposition 6.3. *For every Stein manifold $M$, $GH(M) \neq \emptyset$.*

*Proof.* Let $\Phi$ be a Fatou-Bieberbach map of $\mathbf{C}^2$ to $\mathbf{C}^2$, that is, $\mathbf{C}^2 - \Phi(\mathbf{C}^2)$ has an inner point. Since $M$ is assumed to be a Stein manifold, it follows from the well known Grauert's theorem that there is a scattered inverse holomorphic map $\Psi : M \to \mathbf{C}^2$. It is easy to see that $F = \Phi \circ \Psi \in GH(M)$. $\square$

THEOREM 6.4. $\emptyset \neq \mathscr{GH} \subsetneqq \mathscr{H}$. $\mathscr{H} \cap \mathscr{P} = \emptyset$. $\emptyset \neq \mathscr{A} \subsetneqq \mathscr{SP} \subsetneqq \mathscr{P} \subset \mathscr{QP} \subsetneqq (\mathscr{P} \cup \mathscr{H}) - \mathscr{GH}$.

*Proof.*

(1) ($\emptyset \neq \mathscr{GH}$) By Proposition 1.3, a bounded Stein domain in $\mathbf{C}^2$ is included in $\mathscr{GH}$. And let $R$ and $R'$ be hyperbolic non-$O_{AB}$ Riemann surfaces and set $M_1 = R \times R'$. Then by the same reason above, $M_1 \in \mathscr{GH}$.

(2) ($\mathscr{GH} \subsetneqq \mathscr{H}, \mathscr{QP} \subsetneqq (\mathscr{P} \cup \mathscr{H}) - \mathscr{GH}$) Let $M_2$ be a connected component of $\{(x, y) \in \mathbf{C}^2; |f(x, y)| < 1\}$ where $f \in \mathscr{O}_P(\mathbf{C}^2)$. It is easy to see that $M_2 \notin \mathscr{GH}$. By Theorem 7.1 $M_2 \notin \mathscr{QP}$. Let $R \in O_G$ and $R'$ be hyperbolic non-$O_{AB}$ Riemann surface and set $M_3 = R \times R'$. By the same reason above, $M_3 \in (\mathscr{H} - \mathscr{GH}) - \mathscr{QP}$.

(3) ($\emptyset \neq \mathscr{A}$) If $M_4$ has a compactification $(M_4, \Phi, \hat{M})$ where $\hat{M}$ is a compact complex manifold, $\Phi : M_4 \to M_0 = \Phi(M_4)$ is a biholomorphic map and $C = \hat{M} - M_0$ is an analytic curve of $\hat{M}$, and if there is a meromorphic extension on $\hat{M}$ such that $F|_{M_0} \in E(M_0)$, then $M_4 \in \mathscr{A}$ because $F \circ \Phi \in A(M_4)$. For example $\mathbf{C}^2 \in \mathscr{A}$.

(4) ($\mathscr{A} \subsetneqq \mathscr{SP}$) Set $M_5 = \mathbf{C}^2(x, y) - \{x = a_1, a_2, \dots\} - \{y = b_1, b_2, \dots\}$ where $\{a_i\}$ and $\{b_j\}$ are infinite sequences of complex numbers which do not accumulate in inner points of $\mathbf{C}$. It is easy to see that $M_5 \in \mathscr{SP}$. By Corollary 7.4, $M_5 \notin \mathscr{A}$.

(5) ($\mathscr{SP} \subsetneqq \mathscr{P}$) Let $R$ be a Riemann surface of $\sqrt{(e^z - 1)(e^z + 1)}$ which is a parabolic Riemann surface of the genus $\infty$ and set $M_6 = \mathbf{C}(w) \times R$. At first, we will prove that $M_6 \in \mathscr{P}$. Set $F : x = w, \ y = \sqrt{(e^z - 1)(e^z + 1)}$. We will show that $F \in P(M_6)$. Let $P(x, y)$ be a nonconstant arbitrary polynomial. If $P(x, y)$ is a polynomial such that $\dfrac{\partial P}{\partial x}$ and $\dfrac{\partial P}{\partial y}$ are not identically zero, then for every complex value $\alpha$, $\{P(x, y) \circ F = \alpha\}$ be a covering space of $\mathbf{C}(z)$ except for at most countable values $\alpha$ and the genus $= \infty$ because it is expressed $w = \xi(\sqrt{(e^z - 1)(e^z + 1)})$, where $\xi(y)$ is an algebraic function defined by $\{P(x, y) = \alpha\}$. If $\dfrac{\partial P}{\partial y} \equiv 0$, then $P(x, y) = P(x)$ and $\{P(x) \circ F = \alpha\}$ consists of the set such as $\{w_i\} \times R$, where $w_i$ is a solution of $P(w) = 0$. If $\dfrac{\partial P}{\partial x} \equiv 0$, then $P(x, y) = P(y)$ and $\{P(y) \circ F = \alpha\}$ consists of the set such as $\mathbf{C}(w) \times \{p_j\}$ where $p_j \in R$. Therefore $F \in P(M_6)$.

At the second, we will prove that $M_6 \notin \mathscr{SP}$. For this, we will show that for every $F \in P(M_6)$ there is a polynomial $P(x, y)$ such that $P \circ F \notin \mathscr{O}_{SP}(M_6)$. Since $F = (\varphi(w, p), \psi(w, p))$, where $w \in \mathbf{C}(w)$ and $p \in R$, is nondegenerate, at least one of $\varphi$ or $\psi$ includes the variable $w$. So, we may assume that $\varphi$ is such a function. Then for $P(x, y) = x$, every level curve of $P(x, y) \circ F$ is a level curve of $\varphi(w, p)$ and it has a nonconstant projection to $R$, except for at most countable level curves. Since every level curve of $\varphi$ consits of surfaces of specially parabolic type, it is a contradiction by Proposition 1.4. Then $SP(M_6) = \emptyset$ and $M_6 \notin \mathscr{SP}$. $\square$

*Remark* 6.5. Unfortunately, we have no example of $M$ such that $M \in \mathscr{Q}\mathscr{P} - \mathscr{P}$, but we can not prove that $\mathscr{Q}\mathscr{P} = \mathscr{P}$.

## §7. Property of some class of $M$

THEOREM 7.1. *There is no nonconstant bounded holomorphic function on $M \in \mathscr{Q}\mathscr{P}$.*

*Proof.* Assume that there is a bounded function $g \in \mathscr{O}(M)$. Since $M \in \mathscr{Q}\mathscr{P}$, there is a map $F$ where $P_1 \circ F$, $P_2 \circ F \in \mathscr{O}_P(M)$, $(P_1 \circ F, P_2 \circ F) \in E(M)$ and $P_i$ are polynomials $(i = 1, 2)$. Since $P_i \circ F \in \mathscr{O}_P(M)$ and $O_G \subset O_{AB}$, $g$ is constant on each level curve of $P_i \circ F$. As, on each level curve of $P_2 \circ F$, almost all level curves of $P_1 \circ F$ intersect transversally, $g$ is constant on $M$. $\square$

*Remark* 7.2. Let $R, R' \in O_{AB} - O_G$ and set $M_7 = R \times R'$. Then by the same reason of the above theorem, there is no nonconstant bounded holomorphic function on $M_7$. On the other hand, from Proposition 1.4 it is easy to see that $M_7 \in \mathscr{G}\mathscr{H}$.

By virtue of Nishino [Ni2, 3] and Suzuki [Su2] following theorem is proved.

THEOREM N–S (Theorem IV in [Ni3]). *Let $M$ be topologically finite, that is,* $\dim H_i(M, \mathbf{Z}) < \infty$, $i \geq 0$, *and $M \in \mathscr{A}$. Let $F$ be an arbitrary map in $A(M)$. Then there is a compactification $(M, \Phi, \hat{M})$ and $F \circ \Phi^{-1}$ is a rational holomorphic map of* $\Phi(M)$ to $\mathbf{C}^2$. *Generally $(M, \Phi, \hat{M})$ depends on $F$.*

*Remark* 7.3. If $M = \mathbf{C}^2$, we proved elementarily in [A1] that $\Phi$ is an element of $\mathrm{Aut}(\mathbf{C}^2)$. In this case the compactification is independent of $F$.

COROLLARY 7.4. *Let $M$ be topologically finite and $M \in \mathscr{A}$. Then $M$ is limited a sort of $M_4$ in the proof of Theorem* 6.4.

PROBLEM 7.5. According to the properties of topological compactifications of the elements of $\mathscr{SP}$ and $\mathscr{P}$, can we clarify the difference between $\mathscr{SP}$ and $\mathscr{P}$?.

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