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SCORZA-DRAGONI PROPERTY OF FILIPPOV MAPPINGS

Dedicated to Professor Taro Yoshizawa on his sixtieth birthday

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The notion of Filippov's generalized solution of differential equation [1] can be introduced in the following way. A function $x(t)$ is a solution in Filippov's sense of $\dot{x} = f(t, x)$ if $x(t)$ is a solution of differential relation $\dot{x} \in F(t, x)$ where the Filippov mapping F is defined by

$$F(t, x) = \bigcap_{d>0} \bigcap_{l(N)=0} \overline{\mathrm{Conv}}\, f(t, U(x, d) - N)$$

where $U(x, d) = \{y: \|y - x\| < d\}$ are regions on an n-dimensional linear normed space R_n, $N \subset U(x, d)$ and l is the Lebesgue measure in R_n.

To the author's best knowledge, the first who pointed out a certain minimality property of the Filippov mapping was Kurzweil who formulated this property in [2] for the autonomous case. In particular, it is proved there that the Filippov mapping $F(x)$ has the smallest possible value for every x among all upper semi-continuous mappings having compact, convex values and containing values of f almost everywhere. The precise formulation of this property will be given later.

It was shown in [3] that the construction of the Filippov mapping can be based solely on the minimal property. Further, it was shown there that in the frame of this new construction we can easily modify the definition in the way that the values of the modified Filippov mapping need not be convex. Generally we assume that $F(t, x)$ takes its values from a given family $\mathscr{E}$ of sets. The family $\mathscr{E}$ may be the family of all compact sets. If, for example, $\mathscr{E}$ is the family of all Cartesian products of intervals, the solutions of the corresponding differential relation are precisely the generalized solutions in Viktorovskii's sense of the original equation [4]. Note that the definition of Viktorovskii is a pure analytical one and the above geometrical characterization of Viktorovskii's solutions is due to Pelant [5], [6].

The aim of the present paper is the investigation of measurability properties of the modified Filippov mappings. Roughly speaking a mapping has the Scorza-Dragoni property if there exists a sequence of cylindrical sets covering the domain of definition up to a set of measure zero such that the mapping is upper semi-continuous on each of them. The precise definition is given below.

The theory of differential relations can be limited to those which have right hand sides with the Scorza-Dragoni property. This follows from [7]. It is proved there that the right hand side of a differential relation (provided it is upper semi-continuous in x for every t and in a certain sense measurable) can be replaced by a mapping having the Scorza-Dragoni property without changing the system of solutions. Since the Scorza-Dragoni property implies measurability, the question naturally arises whether the modified Filippov mappings have automatically the Scorza-Dragoni property. This question has an affirmative answer in the case of Filippov mappings [2]. The result about measurability proved in [3] is now a consequence of the stronger result expressed in Theorem 3 of the present paper.

Definitions and notation. Let G be a region in R_{n+1}. Denote the points of R_{n+1} by $[t, x]$ where $t \in R_1$ and $x \in R_n$. $\bar{A}$ denotes the closure of the set A. We shall use the notation $A_t = \{x: [t, x] \in A\}$ for $A \subset R_{n+1}$. Let h be a mapping $h: G \to \mathscr{A}_0$ where $\mathscr{A}_0$ is the class of all subsets of R_m. We can define mappings h_t on G_t for every t by $h_t(x) = h(t, x)$ for $x \in G_t$. In addition to $\mathscr{A}_0$ we shall need the class $\mathscr{C}_0$ of all compact subsets of R_m. By $\mathscr{E}_0$ we denote a class fulfilling

(a) to every $A \in \mathscr{C}_0$ there exists a set $B \in \mathscr{E}_0$, $A \subset B$;

(b) if $B_p \in \mathscr{E}_0$ then $\bigcap_p B_p \in \mathscr{E}_0$;

(c) if $A \in \mathscr{E}_0$ then $A \in \mathscr{C}_0$.

Let $\mathscr{C}$, $\mathscr{E}$ and $\mathscr{A}$ be classes originating respectively from $\mathscr{C}_0$, $\mathscr{E}_0$ and $\mathscr{A}_0$ by excluding the empty set.

DEFINITION. If A is a bounded set in R_m, then $\mathscr{E}(A) = \bigcap_{B \supset A, B \in \mathscr{E}} B$ is called the $\mathscr{E}$-closure of A. The $\mathscr{E}$-closure is called continuous if $\bigcap_n \mathscr{E}(A_n) = \mathscr{E}(\bigcap_n A_n)$ for every sequence of nonempty compact sets $A_1 \supset A_2 \supset \cdots$.

Note that $\mathscr{E}(A)$ is the ordinary closure if $\mathscr{E}$ is the class of all nonempty compact sets and $\mathscr{E}(A)$ is the closed convex hull if $\mathscr{E}$ is the class of all nonempty compact convex sets. In both the cases the $\mathscr{E}$-closure is continuous.

Recall that a mapping $h: D \to \mathscr{A}_0$ (D is a nonempty subset of R_{n+1}) is upper semi-continuous if to every $d > 0$ and $[t, x] \in D$ there exists $r > 0$ such that $h(s, y) \subset U(h(t, x), d)$ for $[s, y] \in U([t, x], r) \cap D$ where $U(A, d)$ is the d-neighborhood of the set A with $U(\varnothing, d) = \varnothing$.

DEFINITION. The mapping $h: G \to \mathscr{A}_0$ has the Scorza-Dragoni property if to every $d > 0$ there exists a measurable set T, $T \subset R_1$ such that $l(T) < d$ and h is upper semi-continuous on $G - (T \times R_n)$. We say briefly that the mapping h is SD.

DEFINITION. A vector function $f: G \to R_m$ is t-locally essentially bounded if to every point $[t_0, x_0] \in G$ there exist $d > 0$ and a function $c(t)$ defined on the interval $\langle t_0 - d, t_0 + d\rangle$ such that $l\{x: x \in U(x_0, d), f(t, x) \notin U(0, c(t))\} = 0$ for all t.

This definition is slightly different from the corresponding one in [3].

Further, let $\{h_z, z \in Z\}$, $Z \neq \emptyset$ be a family of mappings $h_z: G \to \mathscr{A}_0$. The greatest lower bound $h: G \to \mathscr{A}_0$ of the family is the mapping defined by $h(t, x) = \bigcap_{z \in Z} h_z(t, x)$. We shall write $h = \bigwedge_{z \in Z} h_z$. The mapping h_1 is before h_2 ($h_1 \leqq h_2$) if $h_1(t, x) \subset h_2(t, x)$ for all $[t, x] \in G$.

Construction of modified mappings. Since the construction is given in [3] we repeat only the basic steps here.

DEFINITION. Let f be a vector function $f: G \to R_m$ and $\mathscr{E}$ a class such that $\mathscr{E}_0$ fulfills (a) to (c). Denote by $R(f, \mathscr{E})$ the family of all mappings h fulfilling

(i) $h(t, x) \in \mathscr{E}$ for all $[t, x] \in G$;
(ii) $h_t(x)$ is upper semi-continuous on G_t for every t;
(iii) $f_t(x) \in h_t(x)$ for almost all $x \in G_t$ and all t.

A condition under which the set $R(f, \mathscr{E})$ is nonempty is given in

THEOREM 1. *Let a class $\mathscr{E}_0$ fulfill* (a) *to* (c). *The set $R(f, \mathscr{E})$ is nonempty if and only if the vector function f is t-locally essentially bounded.*

Given a class $\mathscr{E}$ and a vector function f, Theorem 1 enables us to construct the greatest lower bound $S = \bigwedge_{h \in R(f, \mathscr{E})} h$. We shall write $S(t, x)$, $S(f)$, $S(t, x; f)$, $S(t, x; f; \mathscr{E})$ etc. if we need to emphasize, the dependence of S on parameters. Basic properties of S are given in

THEOREM 2. *Let a class $\mathscr{E}_0$ fulfill* (a) *to* (c). *If the vector function $f: G \to R_m$ is t-locally essentially bounded then $S \in R(f, \mathscr{E})$.*

If the class $\mathscr{E}$ is the class of all nonempty compact convex subsets of R_n then S is the Filippov mapping defined at the beginning of the article. Theorem 2 yields that $F(t, x) \subset h(t, x)$ for all $h \in R(f, \mathscr{E})$ which is the minimum property mentioned above. If $\mathscr{E}$ is the class of all Cartesian products $\prod_i J_i$ of compact nonempty intervals, then the solutions of the corresponding differential relation $\dot{x} \in S(t, x)$ are the generalized solutions of $\dot{x} = f(t, x)$ in the sense of Viktorovskii [4] as was mentioned in introduction.

Scorza-Dragoni property of S. In this section we use the definition of S as the greatest lower bound to prove that S is SD.

THEOREM 3. *Let $\mathscr{E}_0$ fulfill* (a) *to* (c) *and let the $\mathscr{E}$-closure be continuous. If the vector function $f: G \to R_m$ is measurable and t-locally essentially bounded then the corresponding mapping $S(f)$ is* SD.

The proof will be divided into several lemmas. First we prove Theorem 3 if $m = 1$, $\mathscr{E} = \mathscr{C}$ and f is an indicator.

LEMMA 1. *Let $\mathscr{E} = \mathscr{C}$. If f is a measurable function $f: G \to R_1$ assuming only values* 0 *or* 1, *then $S = S(t, x; f; \mathscr{C})$ is* SD.

PROOF. According to Theorem 2 there exists a mapping $S(t, x; f; \mathscr{C})$ fulfilling (i) to (iii). We have $S(t, x) \subset \{0, 1\}$ for all $[t, x] \in G$. Certainly the constant mapping $h(t, x) = \{0, 1\}$ fulfills (i) to (iii) so that $S \leqq h \in R(f, \mathscr{C})$. We pass to the proof that S is SD. Denote $T(\varepsilon) = \{t: \exists x, \varepsilon \in S(t, x)\}$ where $\varepsilon = 0$ or $\varepsilon = 1$. First we shall prove that $T(\varepsilon)$ are measurable, and for that we need to prove that $T(\varepsilon) = \{t: l_e\{y: f(t, y) = \varepsilon\} > 0\}$ where l_e is the Lebesgue outer measure. Let a point $[t, x] \in G$ fulfill

$$\varepsilon \in S(t, x) \,, \tag{1}$$

then

$$l_e\{y: f(t, y) = \varepsilon\} > 0 \,. \tag{2}$$

If (2) did not hold, we could define $S_0(\tau, z) = S(\tau, z)$ for $\tau \neq t$ and $S_0(t, z) = \{1 - \varepsilon\}$ for $z \in G_t$. The mapping S_0 belongs evidently to $R(f, \mathscr{C})$ and at the same time $S_0(\tau, z) \subset S(\tau, z)$ for all $[\tau, z] \in G$, $S_0(t, z) \neq S(t, z)$ due to (1). This would contradict Theorem 2. Conversely let (2) be fulfilled for some t. Assume for the moment that $S(t, x) = \{1 - \varepsilon\}$ for all $x \in G_t$. By (iii) we obtain $f(t, x) = 1 - \varepsilon$ for almost all $x \in G_t$ and this contradicts (2). Thus inequality (2) implies that there exists x such that (1) is fulfilled. We have proved $T(\varepsilon) = \{t: l_e\{y: f(t, y) = \varepsilon\} > 0\}$.

Denote $g(t, x; \varepsilon) = \varepsilon f(t, x) + (1 - \varepsilon)(1 - f(t, x))$. Since $g(t, x; \varepsilon)$ is measurable in t, x for both $\varepsilon = 0$ and $\varepsilon = 1$, $0 \leqq g(t, x; \varepsilon) \leqq 1$, the integrals $\int_G g(t, y; \varepsilon) dt dy$ exist. Fubini's theorem yields that the integrals $\int_{G_t} g(t, y; \varepsilon) dy$ exist for almost all t, i.e., for $t \in R_1 - N$ where $l(N) = 0$. (For simplicity we assume that the last integrals exist and equal zero even if $G_t = \varnothing$.) Define

$$T_0(\varepsilon) = \left\{t\colon \int_{G_t} g(t, y; \varepsilon)dy \text{ exists and is positive}\right\} .$$

First we prove $T(\varepsilon) - T_0(\varepsilon) \subset N$. Let $t \in T(\varepsilon) - T_0(\varepsilon)$. Since $t \notin T_0(\varepsilon)$ we have either $t \in N$ or $\int_{G_t} g(t, y; \varepsilon)dy = 0$. The last equality would imply $l_\varepsilon\{y\colon f(t, y) = \varepsilon\} = 0$ and so $t \notin T(\varepsilon)$, a contradiction. Secondly we prove $T_0(\varepsilon) \subset T(\varepsilon)$. Let $t \notin T(\varepsilon)$ then $l\{y\colon f(t, y) = \varepsilon\} = 0$ and hence $l\{y\colon g(t, y; \varepsilon) \neq 0\} = 0$. For such t, $\int_{G_t} g(t, y; \varepsilon)dy = 0$ and thus $t \notin T_0(\varepsilon)$.

Since Fubini's theorem implies that $T_0(\varepsilon)$ is measurable we have proved the measurability of $T(\varepsilon)$. Let P_i be a countable basis of open sets in R_n. Denote $G_i = G \cap (R_1 \times P_i)$, f_i the function f restricted to G_i. $S(t, x; f_i; \mathscr{C})$ is evidently the mapping $S(t, x; f; \mathscr{C})$ (i.e., S) restricted to G_i and $T_i(\varepsilon) = \{t\colon \exists x, [t, x] \in G_i, \varepsilon \in S(t, x; f_i; \mathscr{C})\}$ equals

$$T_i(\varepsilon) = \{t\colon \exists x, [t, x] \in G \cap (R_1 \times P_i), \varepsilon \in S(t, x)\} . \tag{3}$$

We proved that $T_i(\varepsilon)$ are measurable. Choose a number $d > 0$. Let $T_i^{(1)}(\varepsilon)$ be a closed set $T_i^{(1)}(\varepsilon) \subset T_i(\varepsilon)$ and $l(T_i(\varepsilon) - T_i^{(1)}(\varepsilon)) < d2^{-i-3}$; $T_i^{(2)}(\varepsilon)$ be an open set $T_i(\varepsilon) \subset T_i^{(2)}(\varepsilon)$, $l(T_i^{(2)}(\varepsilon) - T_i(\varepsilon)) < d2^{-i-3}$. Then the set $Q_i(\varepsilon) = T_i^{(2)}(\varepsilon) - T_i^{(1)}(\varepsilon)$ is open, $l(Q_i(\varepsilon)) < d2^{-i-2}$ and $\hat{T}_i(\varepsilon) = T_i(\varepsilon) - Q_i(\varepsilon) = T_i^{(1)}(\varepsilon)$ is closed. Denote $Q = \bigcup_{i,\varepsilon} Q_i(\varepsilon)$. The set Q is open, $l(Q) < d$ and sets $T_i^*(\varepsilon) = T_i(\varepsilon) - Q = \hat{T}_i(\varepsilon) - Q$ are closed.

We shall prove that S is upper semi-continuous on $D = G - (Q \times R_n)$. Let $[t_p, x_p]$ be a sequence of points $p = 1, 2, \cdots$ converging to $[t_0, x_0]$ and $[t_p, x_p] \in D$ for $p = 0, 1, 2, \cdots$.

It is sufficient to prove that if $\varepsilon \in S(t_p, x_p)$ then $\varepsilon \in S(t_0, x_0)$. Let P_j be a sequence of elements of the basis such that $\{x_0\} = \bigcap_j P_j$. Since $\varepsilon \in S(t_p, x_p)$ and $x_p \to x_0$ the relation (3) implies $t_p \in T_j(\varepsilon)$ for sufficiently great p. Since $[t_p, x_p] \in D$ we have $t_p \notin Q$ and $t_p \in T_j^*(\varepsilon)$ for sufficiently great p. Since $T_j^*(\varepsilon)$ are closed $t_0 \in T_j^*(\varepsilon)$ i.e., $t_0 \in T_j(\varepsilon)$ which means there exists $y_j \in G_{t_0} \cap P_j$, $\varepsilon \in S(t_0, y_j)$. Due to $\{x_0\} = \bigcap P_j$ the sequence of points y_j converges to x_0 and since $S(t_0, y)$ is upper semi-continuous in y (see (ii)) we conclude $\varepsilon \in S(t_0, x_0)$.

The next step is to prove Theorem 3 for vector simple functions.

LEMMA 2. *Let $\mathscr{E} = \mathscr{C}$. If f is a measurable vector function $f\colon G \to R_m$ having only a finite number of values, then $S = S(t, x; f; \mathscr{C})$ is* SD.

PROOF. Denote by f_i, $i = 1, \cdots, s$, the values of f. Obviously $S(t, x) \subset \{f_i\colon i = 1, \cdots, s\}$ for $[t, x] \in G$ for the same reason as previously. Define real functions F^i:

$$F^i(t, x) = 1 \quad \text{if} \quad f(t, x) = f_i ,$$
$$F^i(t, x) = 0 \quad \text{if} \quad f(t, x) \neq f_i .$$

We shall prove

$$(4) \qquad f_i \notin S(t, x) \quad \textit{if and only if} \quad 1 \notin S(t, x; F^i; \mathscr{C}) .$$

The assumption $f_i \notin S(t, x)$ together with (ii) and with the fact that $S \in R(f, \mathscr{C})$ (see Theorem 2) imply that there exists $d > 0$ such that $f_i \notin S(t, y)$ for $y \in U(x, d)$ and (iii) yields $f \neq f_i$ for almost all $y \in U(x, d)$ which means $F^i(t, y) = 0$ for almost all $y \in U(x, d)$ and by (iii) again and by the minimality of $S(t, x; F^i; \mathscr{C})$ we conclude $1 \notin S(t, x; F^i; \mathscr{C})$. The proof of the converse implication is similar. The mappings $S(t, x; F^i; \mathscr{C})$ are SD due to Lemma 1 and the relation (4) yields that S is SD as well.

Further, we shall prove a lemma dealing with the dependence of $S(f)$ on f.

LEMMA 3. *Let f, g be vector functions $G \to R_m$ which are t-locally essentially bounded and fulfil $\|f(t, x) - g(t, x)\| < d$ for all $[t, x] \in G$. Then $S(t, x; f; \mathscr{C}) \subset U(S(t, x; g; \mathscr{C}), 2d)$ for all $[t, x] \in G$.*

PROOF. Denote $h(t, x) = \bar{U}(S(t, x; g; \mathscr{C}), d)$. This mapping fulfills (i) to (iii) with respect to f, i.e., $h \in R(f, \mathscr{C})$. The statement of Lemma 3 follows from the definition of $S(t, x; f; \mathscr{C})$.

The next lemma states that the family of SD mappings is closed with respect to the uniform convergence.

LEMMA 4. *Let mappings $h^{(s)}: G \to \mathscr{A}_0$ be* SD *for $s = 1, 2, \cdots$. If $h^{(s)}$ converges uniformly to a mapping $h: G \to \mathscr{A}_0$ (i.e., to every $d > 0$ there exists s_0 such that $h^{(s)}(t, x) \subset U(h(t, x), d)$ and $h(t, x) \subset U(h^{(s)}(t, x), d)$ for every $s \geqq s_0$ and $[t, x] \in G$), then the mapping h is* SD *as well.*

PROOF. To a given $d > 0$ and s there exists a measurable set T_d^s such that $l(T_d^s) < d2^{-s}$ and the mapping $h^{(s)}$ is upper semi-continuous on $G - (T_d^s \times R_n)$. Denote $T_d = \bigcup_s T_d^s$ then $l(T_d) < d$ and all mappings $h^{(s)}$ are upper semi-continuous on $G - (T_d \times R_n)$. Let a point $[t, x]$ from $G - (T_d \times R_n)$ be given. Choose $r > 0$. Certainly there exists s such that $\rho(h^{(s)}(\tau, y), h(\tau, y)) < r/3$ for $[\tau, y] \in G$ where ρ is the Hausdorff distance of sets. Since $h^{(s)}$ is upper semi-continuous on the domain considered there exists $q > 0$ so that $h^{(s)}(\tau, y) \subset U(h^{(s)}(t, x), r/3)$ for $[\tau, y] \in U([t, x], q) - (T_d \times R_n)$. Finally, we have

$$h(\tau, y) \subset U(h^{(s)}(\tau, y), r/3) \subset U(h^{(s)}(t, x), 2r/3) \subset U(h(t, x), r)$$

for $[\tau, y] \in U([t, x], q) - (T_d \times R_n)$.

Before formulating the last lemma we mention certain properties of the $\mathscr{E}$-closure. The closure $\mathscr{E}(A)$ belongs to $\mathscr{E}$ if $\mathscr{E}(A) \neq \emptyset$. If $A \subset B$, then $\mathscr{E}(A) \subset \mathscr{E}(B)$. If the $\mathscr{E}$-closure is continuous then to every $d > 0$ and to every bounded set A there exists $r > 0$ so that

$$\mathscr{E}(\bar{U}(A, r)) \subset U(\mathscr{E}(A), d) . \tag{5}$$

LEMMA 5. *Let a family $\mathscr{E}_0$ fulfill* (a) *to* (c) *and let the $\mathscr{E}$-closure be continuous. If a mapping $h: G \to \mathscr{A}_0$ is* SD, *then the mapping $h^+(t, x) = \mathscr{E}(h(t, x))$ is* SD *as well.*

It is sufficient to prove that h^+ is upper semi-continuous if h is upper semi-continuous. Choose an arbitrary $d > 0$ and a point $[t, x]$. Due to (5) there exists $r > 0$ such that $\mathscr{E}(\bar{U}(h(t, x)), r) \subset U(\mathscr{E}(h(t, x)), d)$ and to this r there exists $q > 0$ such that $h(\tau, y) \subset U(h(t, x), r)$ for $[\tau, y] \in U([t, x], q)$. These inclusions yield

$$\mathscr{E}(h(\tau, y)) \subset \mathscr{E}(\bar{U}(h(t, x), r)) \subset U(\mathscr{E}(h(t, x)), d)$$

for $[\tau, y] \in U([t, x], q)$.

Now we return to the proof of Theorem 3. Let f, $\mathscr{E}$ fulfill the assumptions of Theorem 3. First we put $\mathscr{E} = \mathscr{C}$ and prove that $S(f; \mathscr{C})$ is SD. Define $f^+(t, x) = f(t, x)(1 + \|f(t, x)\|)^{-1}$. We have $\|f^+(t, x)\| < 1$ and from the t-locally essential boundedness we obtain $\|f^+(t, y)\| < 1 - \delta$ for almost all $y \in U(x, \delta)$ and some $\delta > 0$. This implies

$$\rho(S(t, x; f^+; \mathscr{C}), 0) < 1 \quad \text{for} \quad [t, x] \in G . \tag{6}$$

f^+ can be approximated by measurable functions f_n^+ having a finite number of values such that $\|f_n^+(t, x)\| < 1$ and $\|f^+(t, x) - f_n^+(t, x)\| < 1/n$. Lemma 2 states that $S(f_n^+, \mathscr{C})$ are SD. Since by Lemma 3 $S(f_n^+)$ uniformly converges to $S(f^+)$, Lemma 4 implies that $S(f^+)$ is SD. Evidently $S(t, x; f; \mathscr{C}) = \{z(1 - \|z\|)^{-1}, z \in S(t, x; f^+; \mathscr{C})\}$ (see (6)). This transformation cannot affect the SD property. Due to Lemma 5 [3] we have $S(t, x; f; \mathscr{E}) = \mathscr{E} S(t, x; f; \mathscr{C})$ and hence the application of Lemma 5 implies that $S(t, x; f; \mathscr{E})$ is SD as well.

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