On λ-pseudo bi-starlike and λ-pseudo bi-convex functions with respect to symmetrical points

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Abstract

In this paper, defining new interesting classes, λ-pseudo bi-starlike functions with respect to symmetrical points and λ-pseudo bi-convex functions with respect to symmetrical points in the open unit disk $\mathbb{U}$, we obtain upper bounds for the initial coefficients of functions belonging to these new classes.

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1 Introduction

Let $\mathcal{A}$ denote the class of functions $f(z)$ which are analytic in the open unit disk $\mathbb{U} = \{z : z \in \mathbb{C} \quad \text{and} \quad |z| < 1\}$ and normalized by the conditions $f(0) = f'(0) - 1 = 0$ and having the following form:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n. \tag{1.1}$$

Also let $\mathcal{S}$ denote the subclass of functions in $\mathcal{A}$ which are univalent in $\mathbb{U}$.

By the Koebe One-Quarter Theorem, we know that the range of every function of class $\mathcal{S}$ contains the disk $\{w : |w| < \frac{1}{4}\}$ (see, for example, [5]). Therefore, every univalent function f has an inverse f^{-1} satisfying the following conditions:

$$f^{-1}(f(z)) = z \qquad (z \in \mathbb{U})$$

and

$$f(f^{-1}(w)) = w \qquad \left(|w| < r_0(f);\ r_0(f) \geq \frac{1}{4}\right).$$

In fact, the inverse function f^{-1} is given by

$$g(w) = f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \cdots \tag{1.2}$$

A function $f \in \mathcal{A}$ is said to be *bi-univalent* in $\mathbb{U}$ if both $f(z)$ and $f^{-1}(z)$ are univalent in $\mathbb{U}$. The class of all bi-univalent functions in $\mathbb{U}$ having the Taylor-Maclaurin series expansion (1.1) is denoted by Σ.

For a brief history of functions in the class Σ, see [8] (see also [3], [6], and [16]). In fact, judging by the remarkable flood of papers on the subject (see, for example, [1, 4, 9, 10, 11, 12, 13, 14, 15,

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17, 18, 19]), the recent pioneering work of Srivastava *et al.*[8] appears to have revived the study of analytic and bi-univalent functions in recent years.

We denote by $\mathcal{S}^*$ and $\mathcal{C}$ the class of starlike functions and the class of convex functions, respectively, where

$$\mathcal{S}^* = \left\{ f \in \mathcal{A} \ : \ Re\left\{ \frac{zf'(z)}{f(z)} \right\} \geq 0, \ z \in \mathbb{U} \right\}$$

$$\mathcal{C} = \left\{ f \in \mathcal{A} \ : \ Re\left\{ 1 + \frac{zf''(z)}{f'(z)} \right\} \geq 0, \ z \in \mathbb{U} \right\}.$$

We note that $f(z) \in \mathcal{C} \Leftrightarrow zf'(z) \in \mathcal{S}^*$.

A function $f(z)$ of the form (1.1) is said to be *starlike functions with respect to symmetrical points* if

$$Re\left\{ \frac{2zf'(z)}{f(z) - f(-z)} \right\} > 0, \qquad z \in \mathbb{U}.$$

We let $\mathcal{S}^*_s$ denote the set of all such functions. Sakaguchi [7] proved that if $f(z)$ is in $\mathcal{S}^*_s$ and has the form (1.1), then $|a_n| \leq 1$, for $n = 2, 3, ...$

The class of starlike functions with respect to symmetrical points obviously includes the class of *convex functions with respect to symmetrical points*, $\mathcal{C}_s$, satisfying the following condition:

$$Re\left\{ \frac{(zf'(z))'}{(f(z) - f(-z))'} \right\} \geq 0, \qquad z \in \mathbb{U}.$$

It is easily seen that for the classes $\mathcal{S}^*_s$ and $\mathcal{C}_s$, the Alexander relation is holds, namely $f(z) \in \mathcal{C}_s \Leftrightarrow zf'(z) \in \mathcal{S}^*_s$.

Recently, Babalola [2] defined the class $\mathcal{L}_\lambda$ of λ-pseudo-starlike functions as follows:

Let $f \in \mathcal{A}$ and $\lambda \geq 1$ is real. Then $f(z)$ belongs to the class $\mathcal{L}_\lambda$ of λ-pseudo-starlike functions in the unit disc $\mathbb{U}$ if and only if

$$Re \frac{z(f'(z))^\lambda}{f(z)} > 0, \qquad \big(z \in \mathbb{U}\big).$$

It is clear that, for $\lambda = 1$, we have the class of starlike functions. In the aforementioned work, the author showed that all pseudo starlike functions are univalent in $\mathbb{U}$.

In this paper we have define two new and interesting function classes of $\mathcal{LS}^{*,\lambda}_{s,\Sigma}$ and $\mathcal{NS}^{\lambda}_{s,\Sigma}$, λ-pseudo bi-starlike functions with respect to symmetrical points and λ-pseudo bi-convex functions with respect to symmetrical points, respectively. Furthermore, we have found estimates for the initial coefficients $|a_2|$ and $|a_3|$ of functions belonging these classes.

In the sequel, it is assumed that φ is an analytic function with positive real part in $\mathbb{U}$, satisfying $\varphi(0) = 1$, $\varphi'(0) > 0$ and $\varphi(\mathbb{U})$ is symmetric with respect to real axis. Such a function has a following series expansion

$$\varphi(z) = 1 + B_1 z + B_2 z^2 + B_3 z^3 + \cdots, (B_1 > 0). \tag{1.3}$$

2 Main results

Definition 1. A function $f(z) \in \Sigma$ is said to be in the class $\mathcal{LS}^{*,\lambda}_{s,\Sigma}(\alpha)$, ($\lambda \geq 1$ is real, $0 \leq \alpha \leq 1$) if the following subordinations hold:

$$(1-\alpha)\frac{2z[f'(z)]^{\lambda}}{f(z)-f(-z)} + \alpha\frac{2[(zf'(z))']^{\lambda}}{(f(z)-f(-z))'} \prec \varphi(z) \tag{2.1}$$

and

$$(1-\alpha)\frac{2w[g'(w)]^{\lambda}}{g(w)-g(-w)} + \alpha\frac{2[(wg'(w))']^{\lambda}}{(g(w)-g(-w))'} \prec \varphi(w) \tag{2.2}$$

where the function g is inverse of the function f given by (1.2).

For functions in the class $\mathcal{LS}^{*,\lambda}_{s,\Sigma}(\alpha)$, we obtain the following coefficient inequalities.

Theorem 2.1 If $f(z)$ given by (1.1) be in the class $\mathcal{LS}^{*,\lambda}_{s,\Sigma}(\alpha)$, then

$$|a_2| \leqq \frac{B_1\sqrt{B_1}}{\sqrt{|[(2\lambda^2+\lambda-1)+2\alpha(3\lambda^2-1)]B_1^2 - 4\lambda^2(1+\alpha)^2(B_2-B_1)|}} \tag{2.3}$$

and

$$|a_3| \leq \frac{B_1^2}{4\lambda^2(1+\alpha)^2} + \frac{B_1}{(3\lambda-1)(1+2\alpha)}. \tag{2.4}$$

Proof. Let $f \in \mathcal{LS}^{*,\lambda}_{s,\Sigma}(\alpha)$ and $g = f^{-1}$. Then there are analytic functions $u, v : \mathbb{U} \to \mathbb{U}$, with $u(0) = v(0) = 0$, satisfying

$$(1-\alpha)\frac{2z[f'(z)]^{\lambda}}{f(z)-f(-z)} + \alpha\frac{2[(zf'(z))']^{\lambda}}{(f(z)-f(-z))'} = \varphi(u(z))$$

and

$$(1-\alpha)\frac{2w[g'(w)]^{\lambda}}{g(w)-g(-w)} + \alpha\frac{2[(wg'(w))']^{\lambda}}{(g(w)-g(-w))'} = \varphi(v(w)).$$

Define the functions p_1 and p_2 by

$$p_1(z) = \frac{1+u(z)}{1-u(z)} = 1 + c_1 z + c_2 z^2 + \cdots$$

and

$$p_2(z) = \frac{1+v(z)}{1-v(z)} = 1 + b_1 z + b_2 z^2 + \cdots$$

or, equivalently

$$u(z) = \frac{p_1(z)-1}{p_1(z)+1} = \frac{1}{2}\left(c_1 z + \left(c_2 - \frac{c_1^2}{2}\right) z^2 + \cdots\right) \tag{2.5}$$

and

$$v(z)=\frac{p_2(z)-1}{p_2(z)+1}=\frac{1}{2}\left(b_1z+\left(b_2-\frac{b_1^2}{2}\right)z^2+\cdots\right) \tag{2.6}$$

It is clear that p_1 and p_2 are analytic in $\mathbb{U}$ and $p_1(0)=p_2(0)=1$. Since $u,v:\mathbb{U}\to\mathbb{U}$, the functions p_1 and p_2 have positive real part in $\mathbb{U}$, and hence $|b_i|\le 2$ and $|c_i|\le 2$. By virtue of (2.1), (2.2) (2.5) and (2.6) we have

$$(1-\alpha)\frac{2z[f'(z)]^\lambda}{f(z)-f(-z)}+\alpha\frac{2[(zf'(z))']^\lambda}{(f(z)-f(-z))'}=\varphi\left(\frac{p_1(z)-1}{p_1(z)+1}\right) \tag{2.7}$$

and

$$(1-\alpha)\frac{2w[g'(w)]^\lambda}{g(w)-g(-w)}+\alpha\frac{2[(wg'(w))']^\lambda}{(g(w)-g(-w))'}=\varphi\left(\frac{p_2(w)-1}{p_2(w)+1}\right). \tag{2.8}$$

Using (2.5) and (2.6) together with (1.3), we easily obtain

$$\varphi\left(\frac{p_1(z)-1}{p_1(z)+1}\right)=1+\frac{1}{2}B_1c_1z+\left(\frac{1}{2}B_1\left(c_2-\frac{c_1^2}{2}\right)+\frac{1}{4}B_2c_1^2\right)z^2+\cdots \tag{2.9}$$

and

$$\varphi\left(\frac{p_2(w)-1}{p_2(w)+1}\right)=1+\frac{1}{2}B_1b_1w+\left(\frac{1}{2}B_1\left(b_2-\frac{b_1^2}{2}\right)+\frac{1}{4}B_2b_1^2\right)w^2+\cdots. \tag{2.10}$$

Since

$$(1-\alpha)\frac{2z[f'(z)]^\lambda}{f(z)-f(-z)}+\alpha\frac{2[(zf'(z))']^\lambda}{(f(z)-f(-z))'}$$

$$=1+2\lambda(1+\alpha)a_2z+[2\lambda(\lambda-1)(1+3\alpha)a_2^2+(3\lambda-1)(1+2\alpha)a_3]z^2+\cdots \tag{2.11}$$

and

$$(1-\alpha)\frac{2w[g'(w)]^\lambda}{g(w)-g(-w)}+\alpha\frac{2[(wg'(w))']^\lambda}{(g(w)-g(-w))'}$$

$$=1-2\lambda(1+\alpha)a_2w+\left\{[2(\lambda^2+2\lambda-1)+2\alpha(3\lambda^2+3\lambda-2)]a_2^2-(3\lambda-1)(1+2\alpha)a_3]\right\}w^2+\cdots, \tag{2.12}$$

it follows from (2.7)-(2.12) that

$$2\lambda(1+\alpha)a_2=\frac{1}{2}B_1c_1, \tag{2.13}$$

$$2\lambda(\lambda-1)(1+3\alpha)a_2^2+(3\lambda-1)(1+2\alpha)a_3=\frac{1}{2}B_1\left(c_2-\frac{c_1^2}{2}\right)+\frac{1}{4}B_2c_1^2, \tag{2.14}$$

$$-2\lambda(1+\alpha)a_2=\frac{1}{2}B_1b_1, \tag{2.15}$$

and

$$[2(\lambda^2+2\lambda-1)+2\alpha(3\lambda^2+3\lambda-2)]a_2^2-(3\lambda-1)(1+2\alpha)a_3=\frac{1}{2}B_1\left(b_2-\frac{b_1^2}{2}\right)+\frac{1}{4}B_2b_1^2, \quad (2.16)$$

From (2.13) and (2.15), we get

$$c_1=-b_1 \qquad \text{and} \qquad 8\lambda^2(1+\alpha)^2a_2^2=\frac{1}{4}B_1^2(b_1^2+c_1^2) \quad (2.17)$$

Also, from (2.14) and (2.16), we obtain

$$\left[2(2\lambda^2+\lambda-1)+4\alpha(3\lambda^2-1)\right]a_2^2=\frac{1}{2}B_1(b_2+c_2)+\frac{1}{4}(b_1^2+c_1^2)(B_2-B_1). \quad (2.18)$$

Using (2.17) in (2.18), we obtain

$$a_2^2=\frac{B_1^3(b_2+c_2)}{4\left[(2\lambda^2+\lambda-1)+2\alpha(3\lambda^2-1)\right]B_1^2-16\lambda^2(1+\alpha)^2(B_2-B_1)}$$

Since $|b_i|\le 2$ and $|c_i|\le 2$ $(i=1,2)$, for functions with positive real part, this gives us estimate on $|a_2|$ as asserted in (2.3).

Next, in order to find the bound on $|a_3|$, by subtracting (2.16) from (2.14) and using (2.17) we get

$$a_3=\frac{B_1^2c_1^2}{16\lambda^2(1+\alpha)^2}+\frac{B_1(c_2-b_2)}{4(3\lambda-1)(1+2\alpha)}$$

and applying $|b_i|\le 2$ and $|c_i|\le 2$ $(i=1,2)$ again, we get

$$|a_3|\le\frac{B_1^2}{4\lambda^2(1+\alpha)^2}+\frac{B_1}{(3\lambda-1)(1+2\alpha)}.$$

This completes the proof of Theorem. Q.E.D.

For $\alpha=0$ the class $\mathcal{LS}^{*,\lambda}_{s,\Sigma}(\alpha)$ reduced to the class of $\lambda-$ pseudo bi-starlike functions with respect to symmetrical points. For functions belong to this class we have the following corollary:

Corollary 2.2 If $f(z)$ given by (1.1) be in the class $\mathcal{LS}^{*,\lambda}_{s,\Sigma}$, then

$$|a_2|\leqq\frac{B_1\sqrt{B_1}}{\sqrt{|(2\lambda^2+\lambda-1)B_1^2-4\lambda^2(B_2-B_1)|}}$$

and

$$|a_3|\le\frac{B_1^2}{4\lambda^2}+\frac{B_1}{(3\lambda-1)}.$$

Also, for $\alpha=1$ the class $\mathcal{LS}^{*,\lambda}_{s,\Sigma}(\alpha)$ reduced to the class of $\lambda-$ pseudo bi-convex functions with respect to symmetrical points. For functions belong to this class we have the following corollary :

Corollary 2.3 If $f(z)$ given by (1.1) be in the class $\mathcal{LS}^{*,\lambda}_{s,\Sigma}(1)$, then

$$\left|a_2\right| \leqq \frac{B_1\sqrt{B_1}}{\sqrt{\left|(8\lambda^2+\lambda-3)B_1^2-16\lambda^2(B_2-B_1)\right|}}$$

and

$$\left|a_3\right| \leq \frac{B_1^2}{16\lambda^2}+\frac{B_1}{3(3\lambda-1)}.$$

Definition 2. A function $f(z) \in \Sigma$ is said to be in the class $\mathcal{NS}^{*,\lambda}_{s,\Sigma}(\alpha)$, ($\lambda \geq 1$ is real, $\alpha \geq 0$) if the following subordinations hold:

$$\left(\frac{2z[f'(z)]^\lambda}{f(z)-f(-z)}\right)^\alpha \left(\frac{2[(zf'(z))']^\lambda}{(f(z)-f(-z))'}\right)^{1-\alpha} \prec \varphi(z) \tag{2.19}$$

and

$$\left(\frac{2w[g'(w)]^\lambda}{g(w)-g(-w)}\right)^\alpha \left(\frac{2[(wg'(w))']^\lambda}{(g(w)-g(-w))'}\right)^{1-\alpha} \prec \varphi(w) \tag{2.20}$$

where the function g is inverse of the function f given by (1.2).

For functions in the class $\mathcal{NS}^{*,\lambda}_{s,\Sigma}(\alpha)$, we obtain the following coefficient inequalities.

Theorem 2.4 If $f(z)$ given by (1.1) be in the class $\mathcal{NS}^{*,\lambda}_{s,\Sigma}(\alpha)$, then

$$\left|a_2\right| \leqq \frac{B_1\sqrt{B_1}}{\sqrt{\left|[2\lambda^2(\alpha-2)^2+(\lambda+2\alpha-3)]B_1^2-4\lambda^2(\alpha-2)^2(B_2-B_1)\right|}} \tag{2.21}$$

and

$$\left|a_3\right| \leq \frac{B_1^2}{4\lambda^2(\alpha-2)^2}+\frac{B_1}{(3\lambda-1)|3-2\alpha|}. \tag{2.22}$$

Proof. Let $f \in \mathcal{NS}^{*,\lambda}_{s,\Sigma}(\alpha)$ and $g = f^{-1}$. Then there are analytic functions $u, v : \mathbb{U} \to \mathbb{U}$, with $u(0) = v(0) = 0$, such that

$$\left(\frac{2z[f'(z)]^\lambda}{f(z)-f(-z)}\right)^\alpha \left(\frac{2[(zf'(z))']^\lambda}{(f(z)-f(-z))'}\right)^{1-\alpha} = \varphi(u(z))$$

and

$$\left(\frac{2w[g'(w)]^\lambda}{g(w)-g(-w)}\right)^\alpha \left(\frac{2[(wg'(w))']^\lambda}{(g(w)-g(-w))'}\right)^{1-\alpha} = \varphi(v(w)).$$

Since

$$\left(\frac{2z[f'(z)]^\lambda}{f(z)-f(-z)}\right)^\alpha \left(\frac{2[(zf'(z))']^\lambda}{(f(z)-f(-z))'}\right)^{1-\alpha}$$

$$= 1 - 2\lambda(\alpha-2)a_2 z + \left\{[2\lambda^2(\alpha-2)^2 + 2\lambda(3\alpha-4)]a_2^2 + (3\lambda-1)(3-2\alpha)a_3\right\} z^2 + \cdots \tag{2.23}$$

and

$$\left(\frac{2w[g'(w)]^\lambda}{g(w)-g(-w)}\right)^\alpha \left(\frac{2[(wg'(w))']^\lambda}{(g(w)-g(-w))'}\right)^{1-\alpha}$$

$$= 1 + 2\lambda(\alpha-2)a_2 w + \left\{[2\lambda^2(\alpha-2)^2 + 2\lambda(5-3\alpha) + 2(2\alpha-3)]a_2^2 + (3\lambda-1)(2\alpha-3)a_3\right\} w^2 + \cdots \tag{2.24}$$

from (2.9), (2.10), (2.23) and (2.24), it follows that

$$-2\lambda(\alpha-2)a_2 = \frac{1}{2}B_1 c_1, \tag{2.25}$$

$$[2\lambda^2(\alpha-2)^2 + 2\lambda(3\alpha-4)]a_2^2 + (3\lambda-1)(3-2\alpha)a_3 = \frac{1}{2}B_1\left(c_2 - \frac{c_1^2}{2}\right) + \frac{1}{4}B_2 c_1^2, \tag{2.26}$$

$$2\lambda(\alpha-2)a_2 = \frac{1}{2}B_1 b_1, \tag{2.27}$$

and

$$[2\lambda^2(\alpha-2)^2 + 2\lambda(5-3\alpha) + 2(2\alpha-3)]a_2^2 + (3\lambda-1)(2\alpha-3)a_3 = \frac{1}{2}B_1\left(b_2 - \frac{b_1^2}{2}\right) + \frac{1}{4}B_2 b_1^2, \tag{2.28}$$

From (2.25) and (2.27), we get

$$c_1 = -b_1 \qquad \text{and} \qquad 8\lambda^2(\alpha-2)^2 a_2^2 = \frac{1}{4}B_1^2(b_1^2 + c_1^2) \tag{2.29}$$

Also, from (2.26) and (2.28), we obtain

$$\left[4\lambda^2(\alpha-2)^2 + 2(\lambda+2\alpha-3)\right] a_2^2 = \frac{1}{2}B_1(b_2+c_2) + \frac{1}{4}(b_1^2+c_1^2)(B_2-B_1). \tag{2.30}$$

Using (2.29) in (2.30), we obtain

$$a_2^2 = \frac{B_1^3(b_2+c_2)}{4\left\{[2\lambda^2(\alpha-2)^2 + (\lambda+2\alpha-3)]B_1^2 - 4\lambda^2(\alpha-2)^2(B_2-B_1)\right\}}$$

Since $|b_i| \leq 2$ and $|c_i| \leq 2$ $(i = 1, 2)$, for functions with positive real part, this gives us estimate on $|a_2|$ as asserted in (2.21).

Next, in order to find the bound on $|a_3|$, by subtracting (2.28) from (2.26) and using (2.29) we get

$$a_3 = \frac{B_1^2 c_1^2}{32\lambda^2(\alpha-2)^2} + \frac{B_1(c_2-b_2)}{4(3\lambda-1)(3-2\alpha)}$$

and applying $|b_i| \leq 2$ and $|c_i| \leq 2$ $(i = 1, 2)$ again, we get

$$|a_3| \leq \frac{B_1^2}{4\lambda^2(\alpha-2)^2} + \frac{B_1}{(3\lambda-1)|3-2\alpha|}.$$

This completes the proof of Theorem. Q.E.D.

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