SINGULAR SETS OF IDEAL INSTANTONS AND POINCARÉ DUALITY

By

Yasuyuki NAGATOMO

Abstract. A point in the boundary of a moduli space of quaternion ASD connections can be regarded as a singular ASD connection with a particular singular set. In the case of generalized 1 instantons on HP^n and $Gr_2(C^{n+2})$, it is proved that Poincaré dual of the singular set is the Chern class of the vector bundle.

1. Introduction

The purpose of the present paper is to show that Poincaré dual of the homology class represented by the singular set of an ideal instanton is the even degree Chern class of the vector bundle on which the ideal instanton is defined (Theorem 2.3 and the Table).

It is well known that the moduli space of 1 instantons on the 4 dimensional sphere S^4 is identified with the 5 dimensional open ball. From the viewpoint of Uhlenbeck compactification, a point in the boundary of the moduli space can be considered as an ASD connection with a point singularity. Since the Chern class of the bundle is equal to 1, Poincaré dual of the singular set is the second Chern class c_2.

The quaternion projective space HP^n and the complex Grassmannian manifold $Gr_2(C^{n+2})$ are quaternion-Kähler manifolds and in particular, in the case $n = 1$, these manifolds are S^4 and the complex projective plane CP^2, respectively. By definition, a quaternion-Kähler manifold is a $4n$-dimensional Riemannian manifold for which the linear holonomy group can be reduced to $Sp(1) \cdot Sp(n)$. Anti-self-duality can be defined over quaternion-Kähler manifolds in the same way as in the 4-dimensional case (see, for example, Mamone Capria and Salamon [3]).

From this point of view, 1 instantons on S^4 and CP^2 are generalized to objects on HP^n and $Gr_2(C^{n+2})$ (Definition 2.1). The moduli spaces of these

generalized 1 instantons can be described by the theory of monads on the Salamon twistor space (see §2). This theory indicates a natural compactification of moduli spaces and a point in the boundary of the moduli can also be regarded as an ASD connection which is defined only on the complementary set of a closed subset of the base manifold. We call such a closed subset the singular set. In the case of generalized 1 instantons, the singular sets are described in Theorem 2.2, and all of them are quaternion submanifolds.

As for the method of our proof, see §3. In particular, we use holomorphic vector bundles and sections of them on the twistor spaces. All the holomorphic vector bundles stated here have common properties in terms of ASD bundles and line bundles on the twistor spaces. These properties will be formulated in the forthcoming paper.

2. Preliminaries

The symmetric spaces HP^n and $Gr_2(C^{n+2})$ are quaternion-Kähler manifolds with positive scalar curvature. A connection on a vector bundle on a quaternion-Kähler manifold is called an *ASD connection* if its curvature 2-form is invariant under the action of I, J, K (see, for example, [3]). A vector bundle with an ASD connection is also called *instanton* (bundle). On the other hand, every quaternion-Kähler manifold has a twistor space with a natural complex structure [9]. The twistor spaces of HP^n and $Gr_2(C^{n+2})$ are the odd dimensional complex projective spaces CP^{2n+1} and the generalized flag manifolds $F^{2n+1} = SU(n+2)/S(U(1) \times U(n) \times U(1))$, respectively. The pull-back bundle with ASD connection on the twistor space has a holomorphic structure induced by the pull-back connection. Hence we do not distinguish ASD bundles on quaternion-Kähler manifolds from the pull-back bundles on the twistor space, and we use the same symbol for both.

To specify vector bundles, we need to describe the cohomology rings on the twistor spaces. As for the complex projective spaces, x is defined as the standard positive generator of $H^2(CP^{2n+1}, Z)$. Since the twistor space F^{2n+1} can be expressed as the projective bundle of the holomorphic cotangent bundle of CP^{n+1}, the Leray-Hirsch theorem implies that the cohomology ring $H^*(F^{2n+1}, Z)$ is isomorphic to the quotient ring of $Z[x, y]$ by the ideal generated by x^{n+2} and

$x^{n+1} - x^n y + \cdots + (-1)^{n+1} y^{n+1}$, where x is the pull-back of the standard positive generator of $H^2(CP^{n+1}, Z)$. The twistor space F^{2n+1} has another fibration over the dual complex projective space CP^{n+1*}. Then y is the pull-back of the standard positive generator of $H^2(CP^{n+1*}, Z)$.

We now define the ASD bundles which we wish to consider in the present paper.

DEFINITION 2.1. (0) Let E_0 be an ASD bundle on HP^n with structure group $Sp(n)$. The total Chern class $c(E_0)$ of E_0 is assumed to be equal to $(1 - x^2)^{-1}$. More precisely, the Chern classes are expressed as $c_2(E_0) = x^2$, $c_4(E_0) = x^4, \ldots, c_{2n}(E_0) = x^{2n}$.

(1) Let E_1 be an ASD bundle on $Gr_2(C^{n+2})$ with structure group $Sp(n)$. The total Chern class $c(E_1)$ of E_1 is assumed to be equal to $(1 - x^2)^{-1}(1 - y^2)^{-1}$. More precisely, the Chern classes are expressed as $c_{2i}(E_1) = \sum_{k=0}^{i} x^{2i-2k} y^{2k}$, where $i = 1, 2, \ldots, n$.

(2) Let E_2 be an ASD bundle on $Gr_2(C^{n+2})$ with structure group $SU(n+1)$. The total Chern class $c(E_2)$ of E_2 is assumed to be equal to $\{1 + (-x + y)\} \cdot (1 - x)^{-1}(1 + y)^{-1}$. More precisely, the Chern classes are expressed as $c_i(E_2) = -\sum_{k=1}^{i-1} x^{i-k}(-y)^k$, where $i = 1, 2, \ldots, n + 1$.

REMARK. In the case $n = 1$, E_0 is nothing but a 1 instanton on S^4 and E_1 and E_2 are isomorphic to each other, because $Sp(1) \cong SU(2)$, and our relations for generators of the cohomology yield that $c(E_1) = c(E_2)$. The last two bundles are 1 instantons on CP^2.

Let $\mathcal{M}_0$, $\mathcal{M}_1$ and $\mathcal{M}_2$ be the moduli spaces of ASD connections on E_0, E_1 and E_2, respectively. From the viewpoint of [6],

(0) $\mathcal{M}_0$ is identified with an open ball in $\wedge^2 C^{2n+2}$, where $\wedge^2 C^{2n+2}$ is the corresponding representation of $Sp(n + 1)$.

(1) $\mathcal{M}_1$ is identified with an open cone over $P(\wedge^2 C^{n+2})$, where $\wedge^2 C^{n+2}$ is the corresponding representation of $SU(n + 2)$.

(2) $\mathcal{M}_2$ is identified with an open cone over $P(C^{n+2})$, where C^{n+2} is the standard representation space of $SU(n + 2)$.

The completeness of these moduli spaces is proved in [1], [2], [8] and [4]. In particular, the theory of monads is available for proving the completeness of the moduli ([3], [2], [8] and [4]). From the viewpoint of the theory of monads, the boundary point of the moduli spaces represents a singular ASD connection with a singular set, which we denote by S. These singular sets are described in [1], [7] and [4].

THEOREM 2.2. *In each case, the singular set is one of the following quaternion submanifolds of HP^n or $Gr_2(C^{n+2})$:*

(0) HP^i, where $i = 0, 1, \ldots, n+1$,

(1) HP^i, where $i = 0, 1, \ldots, [(n+2)/2]$ *and*

(2) $Gr_2(C^{n+1})$,

where $[m]$ is the greatest integer not greater than m.

THEOREM 2.3 (Main Theorem). *The Poincaré dual of the homology class represented by the (real) $4(n - i)$-dimensional singular set is the $2i$th Chern class of the ASD bundle on which the singular ASD connection is defined.*

● Table (Singular sets)

Bundles	Singular sets	Poincaré dual
E_0	1 point, $HP^1, \ldots, HP^{n-1}$	$c_{2n}(E_0), c_{2n-2}(E_0), \ldots, c_2(E_0)$
E_1	1 point, $HP^1, \ldots, HP^{[n/2]}$	$c_{2n}(E_1), c_{2n-2}(E_1), \ldots,$
		$c_n(E_1)(n: \text{ even}), c_{n+1}(E_1)(n: \text{ odd})$
E_2	$Gr_2(C^{n+1})$	$c_2(E_2)$

3. Zero Loci of Sections

The following two sections are devoted to the proof of the Main Theorem. Since our strategy is common to all three cases, we shall now describe it.

First of all, we determine a holomorphic vector bundle V on the twistor space which has a (holomorphic) section s. It is shown that the zero locus of the section s is the inverse image $\tilde{S}$ by the twistor fibration of the singular set S and is transverse to the zero section. Hence, Poincaré dual of $\tilde{S}$ equals the top Chern class of V.

Next, it is proved that the top Chern class of V equals the pull-back of the appropriate Chern class of the ASD bundle.

Finally, the Gysin sequence or fibre integration yields the desired results and we omit the details of this procedure.

3.1. THE CASE OF E_0. We may choose V to be the direct sum of line bundles of degree 1. To put it more accurately, we denote by $[z_0; z_1; \ldots; z_{2n+1}]$ the homogeneous coordinates on CP^{2n+1} and by $\mathcal{O}(d)$ the line bundle of degree d on CP^{2n+1}. Now V is the direct sum $\mathcal{O}(1)^{\oplus 2i}$, the top Chern class of V is x^{2i}, and so it equals $c_{2i}(E_0)$. The section s of V which we should choose is

$$\left(z_{2(n-i)+1}, z_{2(n-i)+2}, \ldots, z_{2n}, z_{2n+1}\right).$$

3.2. THE CASE OF E_1 AND E_2. Since F^{2n+1} is a Fano manifold, the Picard group is $H^2(F^{2n+1}, \mathbb{Z})$. We denote by $\mathcal{O}(p,q)$ the line bundle of which the first Chern class is $px + qy$. As mentioned in the previous section, F^{2n+1} has two holomorphic fibrations:

$$p_1 : F^{2n+1} \to CP^{n+1} \quad \text{and} \quad p_2 : F^{2n+1} \to CP^{n+1^*}.$$

From our definition of the cohomology classes x and y, we obtain $p_1^*\mathcal{O}(1) \cong \mathcal{O}(1,0)$ and $p_2^*\mathcal{O}(1) \cong \mathcal{O}(0,1)$. For brevity, we fix a unitary basis $\{e_1, \ldots, e_{n+2}\}$ of the standard representation space C^{n+2} of $SU(n+2)$ and the dual basis $\{e^1, \ldots, e^{n+2}\}$ of C^{n+2^*}. The corresponding homogeneous coordinates on CP^{n+1} and CP^{n+1^*} are denoted by $[z_1; \ldots; z_{n+2}]$ and $[w_1; \ldots; w_{n+2}]$, respectively. Then F^{2n+1} can be identified with the divisor on $CP^{n+1} \times CP^{n+1^*}$ defined by the equation $\sum_{i=1}^{n+2} z_i w_i = 0$.

Let t be a holomorphic section of $\mathcal{O}(1,0)$. If t is the pull-back section of $\mathcal{O}(1)$ on CP^{n+1} which corresponds to $\sum a^i z_i$, we shall say that t is the section corresponding to $\sum a^i z_i$. We also use a similar terminology for a section of $\mathcal{O}(0,1)$.

LEMMA 3.1. Let s be a section of the direct sum $\mathcal{O}(1,0) \oplus \mathcal{O}(0,1)$. If s corresponds to (z_{n+2}, w_{n+2}), then the zero locus of s is F^{2n-1} which is the twistor space of the quaternion submanifold $Gr_2(C^{n+1})$ of $Gr_2(C^{n+2})$.

PROOF. Using our notation, we define divisors $CP^n = \{z_{n+2} = 0\}$ and $CP^{n^*} = \{w_{n+2} = 0\}$ on CP^{n+1} and CP^{n+1^*}, respectively. Then we have $s^{-1}(0) = p_1^{-1}(CP^n) \cap p_2^{-1}(CP^{n^*})$. $\qquad\square$

In this case, it is clear that $s^{-1}(0)$ is transverse to the zero section. The second Chern class of $\mathcal{O}(1,0) \oplus \mathcal{O}(0,1)$ is xy, and so it is equal to $c_2(E_2)$. Thereby, in the case of E_2, the proof is completed.

Pulling back the Euler sequence on CP^{n+1^*}, we obtain the exact sequence:

$$0 \to \mathcal{O}(1,-1) \to \mathcal{O}(1,0)^{\oplus n+2} \to Q \to 0.$$

Using weights for example, we also obtain the exact sequence:

$$0 \to V \to Q \to \mathcal{O}(2,0) \to 0.$$

(The vector bundle V is the "twisted tautological bundle".) By the Bott-Borel-Weil theorem, $H^0(F^{2n+1}, V)$ is identified with $\wedge^2 C^{n+2^*}$ as representations of $SU(n+2)$.

For brevity, we replace $SU(n + 2)$ by G and the isotropy subgroup $S(U(1) \times U(n) \times U(1))$ by K_Z. As a C^∞ vector bundle, V is identified with the associated vector bundle with G of which the typical fibre is the representation space $Ce^1 \otimes C^{n^*}$ of K_Z, where C^{n^*} is spanned by $e^2, \ldots, e^{n+1}$. We denote the orthogonal projection by $\pi : \wedge^2 C^{n+2^*} \to Ce^1 \otimes C^{n^*}$, which is explicitly given by

$$\pi(\phi) = e^1 \wedge \iota(e_1)\phi - h(\phi, e^1 \wedge e^{n+2})e^1 \wedge e^{n+2},$$

where $\phi \in \wedge^2 C^{n+2^*}$, ι is the interior product and h is the induced Hermitian inner product on $\wedge^2 C^{n+2^*}$. Using π, the isomorphism between $\wedge^2 C^{n+2^*}$ and $H^0(F^{2n+1}, V)$ is expressed as

$$s_\phi[g] = [g, \pi(g^{-1}\phi)],$$

where $[g]$ is the point of F^{2n+1} represented by $g \in G$, and $[g, \pi(g^{-1}\phi)]$ is the element of V represented by $(g, \pi(g^{-1}\phi))$.

PROPOSITION 3.2. *We assume that n is even. If $\phi \in \wedge^2 C^{n+2^*}$ is non-degenerate, then the zero locus of s_ϕ is expressed as*

$$s_\phi^{-1}(0) = \{([v], [\iota(v)\phi]) \in CP^{n+1} \times CP^{n+1^*}\} \cong CP^{n+1}.$$

PROOF. With our notation, $s_\phi^{-1}(0) = \{[g] \in F^{2n+1} \mid \pi(g^{-1}\phi) = 0\}$. From the definition of π, $\pi(g^{-1}\phi) = 0$ if and only if $\phi(ge_1, ge_l) = 0$ for an arbitrary $l = 2, \ldots, n + 1$ and so, there exists a non-zero constant α such that $\iota(g^{-1}e_1)\phi = \alpha ge^{n+2}$, because ϕ is non-degenerate. $\square$

If n is odd, then any $\phi \in \wedge^2 C^{n+2^*}$ is degenerate, but in the generic case, the rank of ϕ is equal to $n + 1$. Note that if ϕ is of rank $n + 1$, then there exists a non-zero vector $v_0 \in C^{n+2}$ such that $\iota(v_0)\phi = 0$.

PROPOSITION 3.3. *We assume that n is odd. If $\phi \in \wedge^2 C^{n+2^*}$ is of rank $n + 1$ and satisfies $\iota(v_0)\phi = 0$ for non-zero vector $v_0 \in C^{n+2}$, then the zero locus of s_ϕ is isomorphic to the blow up of CP^{n+1} at one point. More explicitly, the zero locus is expressed as*

$$s_\phi^{-1}(0) = \{([v], [\iota(v)\phi]) \in CP^{n+1} \times CP^{n+1^*} \mid [v] \neq [v_0].\}$$

$$\cup \{([v_0], [\psi]) \in CP^{n+1} \times CP^{n+1^*} \mid \psi(v_0) = 0.\}$$

PROOF. Under the same notation as in the proof of Proposition 3.2, we

obtain the same condition $\phi(ge_1, ge_l) = 0$ for an arbitrary $l = 2, \ldots, n + 1$. This implies the explicit expression of the zero locus in the Proposition.

To see that this set is identified with the blow up of CP^{n+1} at $[v_0]$, we may assume that $v_0 = e_{n+2}$ and $\phi = e^1 \wedge e^2 + e^3 \wedge e^4 + \cdots + e^n \wedge e^{n+1}$. Under the assumption $z_{n+2} \neq 0$, we introduce the inhomogeneous coordinate $\xi_m = z_m/z_{n+2}$ for $m = 1, \ldots, n + 1$. After a "compatible" transformation on CP^{n+1} ($e_1 \mapsto e_2, e_2 \mapsto -e_1, \ldots, e_{n+1} \mapsto -e_n$, and $e^{n+2} \mapsto e^{n+2}$) related to ϕ, on a neighbourhood of $([v_0], [e^1]) \in F^{2n+1} \subset CP^{n+1} \times CP^{n+1*}$, the zero locus can be expressed as $([\xi_1; \ldots; \xi_{n+1}; 1], [w_1; \ldots; w_{n+1}; 0])$, where there exists $\lambda \in C$ such that $(\xi_1, \ldots, \xi_{n+1}) = \lambda(w_1, \ldots, w_{n+1})$. $\qquad\square$

LEMMA 3.4. *If $\phi \in \wedge^2 C^{n+2*}$ is of full rank, then the corresponding section $s_\phi \in H^0(F^{2n+1}, V)$ is transverse to the zero section.*

PROOF. It is sufficient to show that there exists $X \in \mathfrak{su}(n + 2)$ such that $\phi(Xge_1, ge_l) + \phi(ge_1, Xge_l) \neq 0$ for an arbitrary $l = 2, \ldots, n + 1$ under the assumption $\phi(ge_1, ge_l) = 0$ for an arbitrary $l = 2, \ldots, n + 1$. In the case that n is even, we may choose X_l such that $X_l ge_1 = \sqrt{-1} ge_1$ and $X_l ge_l = ge_{n+2}$.

If n is odd and $ge_1 = v_0$, then X_l may be chosen such that $\phi(X_l ge_1, ge_l) \neq 0$, because ϕ is non-degenerate on the orthogonal complement of Cv_0 in C^{n+2}. $\qquad\square$

PROPOSITION 3.5. *We now assume that n is even. If $\phi \in \wedge^2 C^{n+2*}$ defines a compatible quaternion structure with the Hermitian inner product h on C^{n+2}, then the zero locus of the section s_ϕ of V is the twistor space of a quaternion submanifold $HP^{(n+2)/2}$ of $Gr_2(C^{n+2})$.*

PROOF. From the assumption, there exists a unitary basis $\{e_1, \ldots, e_{n+2}\}$ of C^{n+2} such that $\phi = e^1 \wedge e^2 + e^3 \wedge e^4 + \cdots + e^{n+1} \wedge e^{n+2}$. The corresponding quaternion structure is denoted by j.

We denote by σ_F the real structure of F^{2n+1} [9]. If $([v], [\iota(v)\phi])$ is in the zero locus of the section s_ϕ, we obtain $\sigma_F([v], [\iota(v)\phi]) = ([-jv], [\iota(jv)\phi])$. This means that the real structure σ_F can be restricted to the zero locus and corresponds to the real structure of CP^{n+1} as the twistor space of $HP^{(n+2)/2}$. $\qquad\square$

In the case that n is odd, we define a vector bundle V' as $V' = V \oplus \mathcal{O}(1, 0)$. Using Lemma 3.1 and Proposition 3.3, we can find a section whose zero locus is holomorphically isomorphic to CP^n. Then, the following proposition is obtained in a similar way.

PROPOSITION 3.6. *In the case that n is odd, it is assumed that $\phi \in \wedge^2 C^{n+2^*}$ defines a compatible quaternion structure with the Hermitian inner product h on C^{n+1}, where C^{n+1} is the orthogonal complement of Cv_0 and $v_0 \in C^{n+1}\backslash\{0\}$ satisfies $\iota(v_0)\phi = 0$. The corresponding section (s_ϕ, s_{v^0}) of V' is defined in the following way. The section s_ϕ is the corresponding section of V. The linear form v^0 is defined as $\mathrm{Ker}\, v^0 = C^{n+1}$ and $v^0(v_0) = 1$. The section s_{v^0} is the section of $\mathcal{O}(1,0)$ corresponding to v^0.*

Then the zero locus of the section (s_ϕ, s_{v^0}) of V' is the twistor space of a quaternion submanifold $HP^{(n+1)/2}$ of $Gr_2(C^{n+2})$.

Using Lemma 3.1 and Propositions 3.5 and 3.6, we know that the vector bundles $V \oplus (\mathcal{O}(1,0) \oplus \mathcal{O}(0,1))^{\oplus k}$ has a section of which the zero locus is the twistor space of $HP^{(n+2)/2-k}$ in the case that n is even and the vector bundle $V' \oplus (\mathcal{O}(1,0) \oplus \mathcal{O}(0,1))^{\oplus k}$ has a section whose zero locus is the twistor space of $HP^{(n+1)/2-k}$ in the case that n is odd. Lemma 3.4 yields that each section is transverse to the zero section.

4. The Chern Classes

Under our notation, V is isomorphic to $G \times_{K_z} Ce^1 \otimes C^{n^*}$. Theorem 3.4 in [6] assures that the vector bundle $G \times_{K_z} C^{n^*}$ is isomorphic to the pull-back of an instanton bundle E on $Gr_2(C^{n+2})$. The pull-back of E can be obtained by the monad [5]:

$$\mathcal{O}(0, -1) \to \underline{C^{n+2^*}} \to \mathcal{O}(1, 0).$$

Hence we have $c_p(E) = \sum_{i=0}^{p}(-x)^i y^{p-i}$.

LEMMA 4.1. *The top Chern class $c_n(V)$ of V on F^{2n+1} equals $\sum_{p=0}^{[n/2]} x^{2p} y^{n-2p}$.*

PROOF. Since $V \cong E \otimes \mathcal{O}(1,0)$ and $c_1(\mathcal{O}(1,0)) = x$, it follows that $c_n(V) = \sum_{p=0}^{n} c_p(E) x^{n-p} = \sum_{p=0}^{n} \sum_{i=0}^{p}(-1)^i x^{n-p+i} y^{p-i}$. Consequently, we obtain $c_{n+1}(V^{n+1}) = c_n(V)x + \sum_{i=0}^{n+1}(-x)^i y^{n+1-i}$, where V^{n+1} is defined on $F^{2(n+1)+1}$. The induction yields our result. $\qquad\square$

4.1. *n: even.* The relation $x^{n+1} - x^n y + \cdots + (-1)^{n+1} y^{n+1}$, the definition of E_1 and Lemma 4.1 imply that $c_n(V) = c_n(E_1)$ and $x c_n(V) = y c_n(V)$. On the other hand,

$$c_{n+2k}(V \oplus (\mathcal{O}(1,0) \oplus \mathcal{O}(0,1))^{\oplus k}) = x^k y^k c_n(V),$$

$$c_{n+2k}(E_1) = c_n(E_1)y^{2k} + x^{2m}\sum_{q=1}^{k}x^{2q}y^{2k-2q}.$$

The relation $x^{n+2} = 0$ yields $c_{n+2k}(E_1) = c_n(E_1)y^{2k} = c_n(V)y^{2k} = x^k y^k c_n(V)$. Thereby, $c_{n+2k}(V \oplus (\mathcal{O}(1,0) \oplus \mathcal{O}(0,1))^{\oplus k}) = c_{n+2k}(E_1)$.

4.2. n: odd. Lemma 4.1 yields $c_{n+1}(V') = xy\sum_{p=0}^{(n-1)/2}x^{2p}y^{n-1-2p}$. This equation, the relation $x^{n+1} - x^n y + \cdots + (-1)^{n+1}y^{n+1}$ and the definition of E_1 imply $c_{n+1}(V') = c_{n+1}(E_1)$. The relation $y^{n+2} = 0$ yields $yc_{n+1}(E_1) = xc_{n+1}(V') = xc_{n+1}(E_1)$ and $c_{n+1+2k}(E_1) = x^{2k}c_{n+1}(E_1)$. Hence

$$c_{n+1+2k}(V' \oplus (\mathcal{O}(1,0) \oplus \mathcal{O}(0,1))^{\oplus k}) = x^k y^k c_{n+1}(V') = x^{2k}c_{n+1}(E_1)$$

$$= c_{n+1+2k}(E_1).$$

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Institute of Mathematics
University of Tsukuba
Tsukuba-shi, Ibaraki 305, Japan

Mathmatical Institute, 24-29 St. Giles'
Oxford, OX1 3LB, UK

Current Address: Faculty of Mathematics
Kyushu University, Ropponmatsu, Fukuoka, 810-8560, Japan
E-mail address: nagatomo@math.kyushu-u.ac.jp