HOLOMORPHIC FUNCTIONS WITH BOUNDED REAL PARTS

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Recently, Stout has shown that the following theorem is valid for bounded strictly pseudoconvex domains in the complex n-space $\mathbb{C}^n$. In this note we shall show by very elementary means that his theorem is valid for any complex analytic space, if we understand that a pluriharmonic function is the real part of a holomorphic function.

THEOREM. *Let* D *be a complex analytic space. If* $f = u + iv$ *is a holomorphic function on* D *with* u *bounded, then for each* $p > 0$, $|f|^p$ *has a pluriharmonic majorant.*

Proof. We may assume $|u| < 1$ without loss of generality. It is also clear that we have only to treat the cases $p = 4k$, $k = 1, 2, \cdots$. Now let $p = 4k$, $k = 1, 2, \cdots$, $A_p = (\sin \pi/3p)^{-1}$, $S = \{w \in \mathbb{C}: -1 < \operatorname{Re} w < 1\}$,

$$S_1 = \{w \in S: |w| \geq A_p\},$$

and $S_2 = S \setminus S_1$. Then we have $|\arg w^p| \leq \pi/3 \pmod{2\pi}$ for $w \in S_1$, and

$$-A_p^p \leq \operatorname{Re} w^p \leq |w|^p \leq A_p^p, \qquad \text{for } w \in S_2.$$

Hence we get $|w|^p \leq 2\operatorname{Re} w^p$ for $w \in S_1$, and $|w|^p \leq 2\operatorname{Re} w^p + 3A_p^p$, for $w \in S_2$. Therefore, we have $|w|^p \leq 2\operatorname{Re} w^p + 3A_p^p$, for $w \in S$. Now, since $f(z) \in S$ for all $z \in D$, we have $|f(z)|^p \leq 2\operatorname{Re} f^p(z) + 3A_p^p$, for $z \in D$. Since p is an even integer, the above inequality yields the theorem.

Finally, we would like to take this opportunity to point out that Theorem IV.1 in Stout [2] is valid with constant $C_p = \tan \pi/2p$, $1 < p \leq 2$, and $C_p = \cot \pi/2p$, $2 < p < \infty$, and (as Stout communicated to us) for any domain in $\mathbb{C}^n$. To prove it, one can use Theorem 1 in Yabuta [3] for $1 < p \leq 2$ and Theorem 5.7 in Barbey-König [1] for $2 < p < \infty$.

REFERENCES

1. K. Barbey and H. König, *Abstract analytic function theory and Hardy algebras.* Lecture Notes in Mathematics, Vol. 593, Springer, Berlin, 1977.

2. E. L. Stout, *H^p-functions on strictly pseudoconvex domains.* Amer. J. Math., 98 (1976), 821-852.

3. K. Yabuta, *M. Riesz's theorem in the abstract Hardy space theory.* Arch. Math. 29 (1977), 308-312.

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