FINITE GROUPS OF R-AUTOMORPHISMS OF R[[X]]

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Let R be an integral domain with identity, let X be an indeterminate over R, let S be the formal power series ring R[[X]], and let G be a finite group of R-automorphisms of S. If $S^G = \{h \in S \mid \phi(h) = h \ \forall \ \phi \in G\}$, then we call S^G the *ring of invariants* of G. In [10], P. Samuel shows that if R is a local domain (that is, a Noetherian integral domain with unique maximal ideal M) and R is complete in the M-adic topology, then there exists a $f \in S$ such that $S^G = R[[f]]$.

In recent papers, O'Malley [7] and J.-B. Castillon [1], [2] have considered the same problem. O'Malley shows that the same conclusion holds if R is a Noetherian integral domain with identity whose integral closure is a finite R-module. In [1], using simpler techniques than either Samuel or O'Malley, Castillon extends Samuel's result to the case when R is a quasi-local domain that is a complete Hausdorff space in its maximal ideal-adic topology. In [2], using the results of this author [6], [7], and [8], Castillon proves that $S^G = R[[f]]$ if R is a Noetherian integral domain with identity. The specific results of [2] are contained in Theorem 5 and the corollary of this paper.

In this paper, we prove the following more general result.

THEOREM 1. *Let* R *be an integral domain with identity, let* X *be an indeterminate over* R, *let* S *be the formal power series ring* R[[X]], *and let* G *be a finite group of* R*-automorphisms of* S. *If* $f = \prod_{\phi \in G} \phi(x)$ *and* S^G *denotes the ring of invariants of* G, *then* $S^G = R[[f]]$.

We make strong use of Theorem 2.6 of [8] and Corollary 5.8 of [6]. In Section 2, observing an easy extension of a proof given in [1], we derive a result (Theorem 4) of prime importance in our proof of Theorem 1. In Section 3, we prove Theorem 1.

1. NOTATION AND TERMINOLOGY

All rings considered in this paper are assumed to be commutative and to contain an identity element. We use the symbols ω and ω_0 to denote the sets of positive and nonnegative integers, respectively, and the symbols $\subseteq$ and $\subset$ to denote containment and proper containment, respectively. If R is a ring, then J(R) will denote the Jacobson radical of R, and S will denote the formal power series ring R[[X]]. If $g = \sum_{i=0}^{\infty} c_i X^i$ is a nonzero element of S such that the first nonzero coefficient of g is c_k, then we say g *has order* k, and we write $O(g) = k$. If d is an element of R, then (d) will denote the ideal of R generated by d.

If A is an ideal of R, then the collection $\{A^k\}_{k \in \omega}$ of ideals of R induces a topology, called the A-*adic topology*, on R. We write (R, A) to denote the topological ring R under this topology. It is well known that (R, A) is a Hausdorff space if

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and only if $\bigcap_{k \in \omega} A^k = (0)$. We say that (R, A) is *complete* if each Cauchy sequence of (R, A) converges to an element of R.

2. THE CONDITION $S = R[[f]] \cdot 1 + R[[f]] \cdot X + \cdots + R[[f]] \cdot X^{n-1}$

Throughout the paper, $G = \{\phi_i\}_{i=1}^n$ will denote a finite group of R-automorphisms of S; by f we denote the product $\prod_{i=1}^n \phi_i(X)$, and by S^G the ring of invariants of G. Since $R[[f]] \subseteq S^G$, even when R contains proper zero divisors [7, Corollary 3.5], the difficulty in any attempt to show that $S^G = R[[f]]$ lies in showing that $S^G \subseteq R[[f]]$.

The following theorem, while not explicitly stated in [1], is essentially proved there. It should be noted that a crucial requirement in the proof of the theorem is that $\phi_i(X) \neq \phi_j(X)$ if $i \neq j$. This is true because an R-automorphism of S is uniquely determined by $\phi(X)$ [9, Theorem 4.5].

THEOREM 2. *Let* R *be an integral domain, and suppose that* T *is a subring of* S *such that* $\phi(t) = t$ *for each* $\phi \in G$ *and each* $t \in T$. *If*

$$S^G \subseteq T \cdot 1 + T \cdot X + \cdots + T \cdot X^{n-1},$$

where n *is the cardinality of* G, *then* $S^G = T$. *In particular, if* $T = R[[f]]$, *then* $S^G = R[[f]]$.

Recently, O'Malley [8] has proved the following result.

THEOREM 3. *Let* R *be a ring, and let* $h = \sum_{i=0}^{\infty} a_i X^i \in S$. *Suppose that for some* $n \geq 1$, a_n *is a unit of* R, *and that the ideal* $A = (a_0, a_1, \cdots, a_{n-1})$ *of* R *generates a complete Hausdorff topology on* R. *Then* $\{1, X, \cdots, X^{n-1}\}$ *is a free-module basis for* S *over* R[[h]].

Thus, if R is an integral domain, and if the coefficients of $f = \prod_{i=1}^n \phi_i(X)$, f being written as a power series $\sum_{i=0}^{\infty} a_i X^i$, satisfy the hypothesis of Theorem 3, then

$$S = R[[f]] \cdot 1 + R[[f]] \cdot X + \cdots + R[[f]] \cdot X^{n-1}.$$

Hence, by Theorem 2, we see that $S^G = R[[f]]$.

Now, if R is an integral domain, and if

$$\phi_i(X) = \sum_{j=0}^{\infty} b_j^{(i)} X^j \quad \text{for } i = 1, \cdots, n,$$

then, for each i, $(R, (b_0^{(i)}))$ is a complete Hausdorff space and $b_1^{(i)}$ is a unit of R [6, Corollary 5.8]. It follows that if $f = \sum_{k=0}^{\infty} a_k X^k$ and $B = (b_0^{(1)}, \cdots, b_0^{(n)})$, then a_n is a unit of R and $A = (a_0, a_1, \cdots, a_{n-1}) \subseteq B$. Moreover, since $(R, (b_0^{(i)}))$ is complete for each i, then (R, B) is complete [4, Proposition 2]; hence, if (R, B) is also a Hausdorff space, then it follows that (R, A) is a complete Hausdorff space [7, Corollary 2.2]. In particular, if B is a principal ideal, generated by $b_0^{(i)}$ for some i, then (R, A) is a complete Hausdorff space. Therefore, we have proved the following theorem.

THEOREM 4. *Let* R *be an integral domain, and suppose that* $G = \{\phi_i\}_{i=1}^{n}$ *is a finite group of* R*-automorphisms of* S, *where* $\phi_i(X) = \sum_{j=0}^{\infty} b_j^{(i)} X^j$ *for each* i. *If* $B = (b_0^{(1)}, \cdots, b_0^{(n)})$ *and if* $\bigcap_{k\in\omega} B^k = (0)$, *then* $S^G = R[[f]]$. *In particular, if* $B = (b_0^{(i)})$ *for some* i *such that* $1 \leq i \leq n$, *then* $S^G = R[[f]]$.

Since $b_0^{(i)} \in J(R)$ for each i [6, Lemma 5.1], it follows that if

$$\bigcap_{k\in\omega} (J(R))^k = (0)\,,$$

then (R, B) is a Hausdorff space. In particular, if R is Noetherian, then $\bigcap_{k\in\omega} (J(R))^k = (0)$ [5, p. 12]; thus, we have the following corollary.

COROLLARY. *Under the hypothesis of Theorem* 4, *if* $\bigcap_{k\in\omega} (J(R))^k = (0)$, *then* $S^G = R[[f]]$. *In particular, if* R *is Noetherian, then* $S^G = R[[f]]$.

Remark. In general, if R is a ring and $C = (c_1, \cdots, c_p)$ is an ideal of R such that $(R, (c_i))$ is a complete Hausdorff space for each i, it is not true that (R, C) must be a Hausdorff space. However, the only examples of which this author is aware involve rings T containing zero divisors. Moreover, for each such ring T, the ideal C satisfying the condition $\bigcap_{k\in\omega} C^k \neq (0)$ has the property that there exists $d \in C$ such that $\bigcap_{k\in\omega} (d)^k \neq (0)$. Specifically, R. Gilmer [3] has constructed a ring R with zero divisors for which the ring S = R[[X]] admits an R-automorphism sending X onto $a_0 - X$, where $\bigcap_{n=1}^{\infty} (a_0^n) \neq (0)$. Therefore S is a complete Hausdorff space in its $(a_0 - X)$-adic topology and its (X)-adic topology (by Theorem 4.5 of [9]), but is not a Hausdorff space in its $(a_0 - X, X)$-adic topology. However, in the case of a domain R, if $C = (c_1, \cdots, c_p)$ is an ideal of R such that $(R, (c_i))$ is a complete Hausdorff space for each i, then it can be shown that for each $d \in C$ the topological ring (R, (d)) is a complete Hausdorff space.

It is now clear that if R is a domain and if the condition that for each $i = 1, \cdots, p$ the topological ring $(R, (c_i))$ is a complete Hausdorff space guarantees that $(R, (c_1, \cdots, c_p))$ is a Hausdorff space, then Theorem 4 proves the main result of the paper. However, since we do not know the validity of this implication, we develop another approach to the problem, in the next section. We make strong use of the special case of Theorem 4 in which B is generated by $b_0^{(i)}$ for some i.

We conclude this section with a theorem that may be of some interest in itself. A special case of the result is needed for the proof of Theorem 1. The more general result is closely related to Theorems 2.1 and 2.6 of [8].

THEOREM 5. *Let* R *be a ring, and let* $h \in S$ *be such that* (S, h) *is a complete Hausdorff space. By Theorem* 2.2 *of* [9], *there exists a unique* R*-endomorphism* ψ_h *of* S *that maps* X *onto* h. *Let* R[[h]] *denote the range of* ψ_h. *If* T *is a subring of* S *containing* R[[h]], *and if* M *denotes the* R*-submodule of* S *with basis* $\{1, X, \cdots, X^{n-1}\}$, *then the following are equivalent.*

(i) $T = hT \oplus M$ *and* h *is regular in* T.

(ii) $\{1, X, \cdots, X^{n-1}\}$ *is a free-module basis for* T *over* R[[h]], *and* ψ_h *is one-to-one.*

Proof. (i) $\rightarrow$ (ii). Since $hT \cap M = (0)$, we see in particular that $hT \cap R = (0)$, and hence, by Lemma 2.5 of [8], ψ_h is one-to-one. The remainder of the proof follows that of Theorem 2.6 of [8], and we omit the details.

(ii) $\rightarrow$ (i). If $g = \sum_{j=0}^{n-1} \left(\sum_{i=0}^{\infty} r_i^{(j)} h^i \right) X^j \in T$, then

$$g = (r_0^{(0)} + r_0^{(1)} X + \cdots + r_0^{(n-1)} X^{n-1}) + h\left(\sum_{j=0}^{n-1} \left(\sum_{i=1}^{\infty} r_i^{(j)} h^{i-1} \right) X^j \right)$$

is an element of $hT + M$, and therefore $T = hT + M$. Suppose now that

$$hg = r_0 + r_1 X + \cdots + r_{n-1} X^{n-1} \in hT \cap M,$$

where $g = \sum_{j=0}^{n-1} \left(\sum_{i=0}^{\infty} c_i^{(j)} h^i \right) X^j$. Then

$$0 = hg - (r_0 + r_1 X + \cdots + r_{n-1} X^{n-1}) = \sum_{j=0}^{n-1} \left(-r_j + \sum_{i=0}^{\infty} c_i^{(j)} h^{i+1} \right) X^j,$$

and since $\{1, X, \cdots, X^{n-1}\}$ is a free-module basis for T over $R[[h]]$, we see that $-r_j + \sum_{i=0}^{\infty} c_i^{(j)} h^{i+1} = 0$ for each j. But, since

$$\psi_h\left(-r_j + \sum_{i=0}^{\infty} c_i^{(j)} X^{i+1} \right) = -r_j + \sum_{i=0}^{\infty} c_i^{(j)} h^{i+1},$$

and since ψ_h is one-to-one, it follows that $r_j = c_i^{(j)} = 0$ for all i and j. Therefore, $hg = 0$ and $hT \cap M = (0)$. Finally, we observe that the proof shows that if $hg = 0$ and $g \in T$, then $g = 0$, so that h is regular in T.

3. FINITE GROUPS OF R-AUTOMORPHISMS OF R[[X]]

Throughout this section, $G = \{\phi_i\}_{i=1}^{n}$ will denote a finite group of R-automorphisms of S, where, for each $i = 1, \cdots, n$, we let $\phi_i(X) = \sum_{j=0}^{\infty} b_j^{(i)} X^j = \beta_i$.

We begin with two lemmas.

LEMMA 1. *Let* R *be a ring, and suppose that* ψ_1 *and* ψ_2 *are* R-*automorphisms of* S, *where* $\psi_i(X) = \sum_{j=0}^{\infty} a_j^{(i)} X^j = \alpha_i$ *for* $i = 1, 2$. *Then the following statements are true.*

(i) *If* $(R, (a_0^{(1)}))$ *is a Hausdorff space, then the constant term of* $\psi_1 \circ \psi_2(X)$ *is an element of the ideal* $(a_0^{(1)}, a_0^{(2)})$ *of* R.

(ii) *If* $\alpha_1 \in (\alpha_2)$, *then* $\alpha_1 = u\alpha_2$, *where* u *is a unit of* S, *and* $a_0^{(1)} = ca_0^{(2)}$, *where* c *is the constant term of* u *and hence is a unit of* R.

Proof. (i). Clearly, $\psi_1 \circ \psi_2(X) = \psi_1(\alpha_2) = \psi_1 \left(\sum_{j=0}^{\infty} a_j^{(2)} X^j \right)$. Since $(R, (a_0^{(1)}))$ is a Hausdorff space, $(R, (a_0^{(1)}))$ is a complete Hausdorff space [6, Theorem 4.10], and it follows from [6, pp. 65-66] that the constant term of $\psi_1(\alpha_2)$ is equal to

$\lim_n \sum_{j=0}^{n} a_j^{(2)}(a_0^{(1)})^j$, where the limit is taken in $(R, (a_0^{(1)}))$. But

$$\sum_{j=0}^{n} a_j^{(2)}(a_0^{(1)})^j = a_0^{(2)} + a_0^{(1)} \cdot \sum_{j=1}^{n} a_j^{(2)}(a_0^{(1)})^{j-1},$$

and hence

$$\lim_n \sum_{j=0}^{n} a_j^{(2)}(a_0^{(1)})^j = a_0^{(2)} + a_0^{(1)} \lim_n \sum_{j=1}^{n} a_j^{(2)}(a_0^{(1)})^{j-1}.$$

Thus, the constant term of $\psi_1 \circ \psi_2(X)$ is of the form $a_0^{(2)} + a_0^{(1)} \cdot r$, where $r \in R$; hence (i) is true.

(ii) Suppose that $\alpha_1 \in (\alpha_2)$. Then there exists $h = \sum_{p=0}^{\infty} h_p X^p \in S$ such that

$$\alpha_1 = \sum_{j=0}^{\infty} a_j^{(1)} X^j = \sum_{p=0}^{\infty} h_p X^p \cdot \sum_{q=0}^{\infty} a_q^{(2)} X^q = \sum_{k=0}^{\infty} d_k X^k,$$

where $d_k = \sum_{p+q=k} h_p a_q^{(2)}$ for each $k \in \omega_0$. Therefore, equating coefficients of like powers of X, we see that

$$a_0^{(1)} = d_0 = h_0 a_0^{(2)}, \quad a_1^{(1)} = d_1 = h_0 a_1^{(2)} + h_1 a_0^{(2)}.$$

Thus, since $a_0^{(2)} \in J(R)$ [6, Lemma 5.1], and since $a_1^{(1)}$ and $a_1^{(2)}$ are units of R [9, Theorem 4.5], it follows that the element $h_0 a_1^{(2)} = a_1^{(1)} - h_1 a_0^{(2)}$ is a unit of R [11, p. 206], and hence h_0 is a unit of R. Therefore, h is a unit of R [12, p. 131], and (ii) is proved.

In particular, we note that if R is an integral domain and ψ is an R-automorphism of S such that $\psi(X) = \alpha = \sum_{j=0}^{\infty} a_j X^j$, then $(R, (a_0))$ is a complete Hausdorff space [6, Corollary 5.8]; hence Lemma 1 is applicable to the elements of G, if R is a domain.

LEMMA 2. *Let* R *be a domain, and suppose that* ψ *is an* R-*automorphism of* S *such that* $\psi(X) = \alpha$. *Then, for each* $k \in \omega$, *the ideal* (α^k) *is* (α)-*primary. In particular, for each* $i = 1, \cdots, n$ *and each* $k \in \omega$, *the ideal* (β_i^k) *is* (β_i)-*primary.*

Proof. Since R is a domain, (X^k) is (X)-primary for each $k \in \omega$; because ψ is an isomorphism of S onto S, the result follows.

We can now prove the main result of the paper.

Proof of Theorem 1. First we observe that $(S, (f))$ is a complete Hausdorff space [7, p. 253]; as a special case of Theorem 5, it follows that if $S^G = fS^G \oplus R$, then $S^G = R[[f]]$. We propose to show that $S^G = fS^G \oplus R$. Since $R \subseteq S^G$ and $f \in S^G$, where $O(f) \geq 1$, it suffices to show that $S^G \subseteq fS^G + R$. Moreover, if $g = \sum_{i=0}^{\infty} g_i X^i \in S^G$, then, since $g_0 \in S^G$, we see that $g - g_0 \in S^G$. Therefore, in order to establish the relation $S^G \subseteq fS^G + R$, it suffices to show that if $g \in S^G$ and $O(g) \geq 1$, then $g \in fS^G$. The remainder of the proof deals with this last statement.

Let $T = \{\phi_i(X)\}_{i=1}^n = \{\beta_i\}_{i=1}^n$, where $\beta_i = \sum_{j=0}^\infty b_j^{(i)} X^j$ for each i. We define an equivalence relation $\sim$ on T by saying that $\beta_i \sim \beta_j$ if and only if β_i and β_j are associates (that is, if and only if there exists a unit u_{ij} of S such that $\beta_i = u_{ij}\beta_j$; note that since R is a domain, this is equivalent to the condition $(\beta_i) = (\beta_j)$). The relation $\sim$ partitions T into k disjoint equivalence classes $E_1, \cdots, E_k$ $(1 \leq k \leq n)$. After suitable renumbering of the mappings ϕ_i, we may suppose that $\beta_1, \cdots, \beta_k$ are representatives for the equivalence classes $E_1, \cdots, E_k$, respectively (in other words, that $E_j = \{\beta \in T \mid (\beta) = (\beta_j)\}$ for each $j = 1, \cdots, k$). Then $T = \bigcup_{j=1}^k E_j$; it follows from Lemma 1 (ii) that if $u \neq v$ and $\beta_r \in E_u$ and $\beta_s \in E_v$, then $\beta_r \notin (\beta_s)$ and $\beta_s \notin (\beta_r)$.

For $j = 1, \cdots, k$, let

$$U_j = \{\beta_r \in T \mid b_0^{(r)} \in (b_0^{(j)})\}, \qquad H_j = \{\phi_r \in G \mid \phi_r(X) = \beta_r \in U_j\}.$$

Note that if $O(\beta_i) = 1$, then β_i belongs to the equivalence class $\{\beta \in T \mid (\beta) = (X)\}$. It follows then that $E_j \subseteq U_j$ for each j; but the equivalence class determined by X is contained in each U_j; in particular, $X \in U_j$ for each j.

We show that H_j is a subgroup of G for each j. Since $X \in U_j$, the identity automorphism belongs to H_j for each j. Let $\phi_r, \phi_s \in H_j$. By Lemma 1 (i), the constant term of $\phi_r \circ \phi_s(X)$ is an element of the ideal $(b_0^{(r)}, b_0^{(s)})$. But by definition of H_j and U_j, both $b_0^{(r)}$ and $b_0^{(s)}$ are elements of $(b_0^{(j)})$, and hence, $\phi_r \circ \phi_s(X) \in U_j$. Therefore $\phi_r \circ \phi_s \in H_j$. Finally, if $\phi_r \in H_j$, where ϕ_r is not the identity automorphism, then $\phi_r \in G$ and hence ϕ_r has finite order, say q. Therefore, $\phi_r^{q-1} = \phi_r^{-1}$, and, extending Lemma 1 (i) by induction, we see that $\phi_r^{-1} \in H_j$.

For $j = 1, \cdots, k$, let $f_j = \prod_{\phi \in H_j} \phi(X) = \prod_{\beta \in U_j} \beta$. Then, by definition of U_j, it follows from the special case of Theorem 4 that

$$S^{H_j} = \{\alpha \in S \mid \phi(\alpha) = \alpha \text{ for each } \phi \in H_j\} = R[[f_j]].$$

Moreover, since $G = \bigcup_{j=1}^k H_j$, we see that

$$S^G = \bigcap_{j=1}^k S^{H_j} = \bigcap_{j=1}^k R[[f_j]].$$

Since $O(f_j) \geq 1$, it follows that if $g \in S^G$ and $O(g) \geq 1$, then $g \in f_j R[[f_j]]$. For each j, let $g = t_j f_j$, where $t_j \in S$.

Since $E_j \subseteq U_j$, we see that $f_j = \prod_{\beta \in U_j} \beta \in \left(\prod_{\beta \in E_j} \beta\right)$; let $f_j = h_j\left(\prod_{\beta \in E_j} \beta\right)$, where $h_j \in S$. By definition of E_j, if $\beta \in E_j$, then $(\beta) = (\beta_j)$; hence, it follows from Lemma 1 (ii) that for each j there exists a unit u_j of S such that $\prod_{\beta \in E_j} \beta = u_j \beta_j^{e_j}$, where e_j is the cardinality of the set E_j. Therefore, for $j = 1, \cdots, k$,

$$g = t_j h_j u_j \beta_j^{e_j} = w_j \beta_j^{e_j},$$

where $w_j \in S$.

Now, in particular, $g = w_1 \beta_1^{e_1} = w_2 \beta_2^{e_2} \in (\beta_2^{e_2})$. Since $\beta_1^{e_1} \notin (\beta_2)$, and since $(\beta_2^{e_2})$ is (β_2)-primary (Lemma 2), it follows that $w_1 \in (\beta_2^{e_2})$. Let $w_1 = v_2 \beta_2^{e_2}$, where $v_2 \in S$; hence $g = w_1 \beta_1^{e_1} = v_2 \beta_2^{e_2} \beta_1^{e_1}$. By an easy induction argument, it follows that $g = w \prod_{j=1}^{k} \beta_j^{e_j}$, where $w \in S$. Moreover, from the definition of the sets E_j and by Lemma 1 (ii), it follows that there exists a unit $v \in S$ such that

$$g = v^{-1} w v \prod_{j=1}^{k} \beta_j^{e_j} = v^{-1} w \left(\prod_{\beta \in E_1} \beta \right) \cdots \left(\prod_{\beta \in E_k} \beta \right) = tf,$$

where $t = v^{-1} w \in S$. Finally, we observe that if $\phi_i \in G$, then $tf = g = \phi_i(g) = \phi_i(t) f$. Thus, since R is a domain, it follows that $\phi_i(t) = t$, so that $t \in S^G$. Therefore $g \in fS^G$, and our proof is complete.

Remark. In general, Theorem 1 is not true if R contains zero divisors (see [10]). However, as was observed in [10] for the case of a local ring, there is a partial result if R is Noetherian and contains no nonzero nilpotent elements. In particular, if R is a Noetherian ring, if G is a finite group of R-automorphisms of S, if $\{P_i\}_{i=1}^{k}$ is the set of minimal prime ideals of R, and if $(0) = \bigcap_{i=1}^{k} P_i$, then each $\phi \in G$ induces an R/P_i-automorphism ϕ_i^* of $R/P_i[[X]]$, for $i = 1, \cdots, k$. If, for each $i = 1, \cdots, k$, the group G has the additional property that $\phi_i^* \neq \psi_i^*$ whenever $\phi, \psi \in G$ and $\phi \neq \psi$, then, following the proof indicated in [10] and using Lemmas 3.7 and 3.8 of [7], we can show that $S^G = R[[f]]$, where $f = \prod_{\phi \in G} \phi(X)$. The crucial requirement is the condition that $\phi, \psi \in G$ together with $\phi \neq \psi$ implies that $\phi_i^* \neq \psi_i^*$ for each i; an example given in [10] shows that this does not hold in general.

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