

Book Review

Charles S. Chihara. *A Structural Account of Mathematics*. The Clarendon Press, Oxford University Press, New York, 2004. xiv + 380 pages.

1 Overview

This book provides a thought-provoking and incisive tour by one of the leading contemporary philosophers of mathematics through many of the major issues that preoccupy practitioners of the subject today. With the exception of neo-Fregeanism, most of the active areas of current debate are covered: structuralism, fictionalism, indispensability arguments, the issue of applied mathematics. Chihara's own positive view of mathematics is a fusion of a technical nominalistic program ("the Constructibility Theory") with a broader philosophical picture of the nature and role of mathematics (the "Structural Account" of the book's title). Chihara has already given a detailed exposition of the Constructibility Theory in his 1990 book, *Constructibility and Mathematical Existence*. The reader is often referred to this earlier book for the finer technical points of the analysis, and only one of the eleven chapters of the new book is devoted to the Constructibility Theory per se. The bulk of Chihara's attention is directed elsewhere, and in particular to developing the structural account that forms the backbone of the book.

Chihara begins the book with five "puzzles" which he thinks any adequate philosophy of mathematics ought to be able to illuminate. He uses these puzzles to motivate the idea that the crucial feature of any mathematical claim is its structural content. Unlike some other structuralists, however, Chihara wants to avoid the postulation of abstract structures; in other words, he wants to combine structuralism with nominalism. The role of his Constructibility Theory (CT) is to show one way in which this can be done. In particular, he argues that CT is powerful enough to capture the structural content of all of the mathematics that is applied in science. Very briefly, CT is a reduction of mathematics to the simple theory of types. Each existence claim, for example, that there exists a set of objects with property F , is replaced by the claim that it is possible to construct a corresponding open sentence, ' Fx '. What are said

to be constructible by CT are open sentence tokens; hence all reference to abstract objects is avoided.

The Constructibility Theory has close affinities to mathematical fictionalism, the structural account has strong parallels with mathematical structuralism, and the combination of the two shares certain features with deductivism (or “if-thenism”). Thus Chihara spends significant portions of the book distinguishing his views from those in each of the above categories and to defending the superiority of his own version where the respective positions differ. In many ways, the critical analysis of these alternative views is the most satisfying element of the book. Chihara exhibits excellent philosophical instincts and, while the broader direction of the dialectic can sometimes meander, the individual arguments and analyses he provides are nearly always effective and illuminating. The chapter he devotes to critiquing the structuralist accounts of Shapiro and of Resnik is particularly effective in this regard.

In view of the multitude of topics, authors, and strands of argument covered in the book, I shall be highly selective in what I discuss in this review. I shall be concerned here with some of the broader philosophical issues raised by Chihara’s work and how these issues intersect with the more technical aspects of his program.

2 The Aims of Nominalism

In an influential paper dating back to 1983, John Burgess proposed a twofold division of possible goals for nominalistic constructions of science, presuming that science as a whole is presumed to be aiming at truth. A *hermeneutic nominalist* claims that a proper understanding of the meaning of mathematical statements reveals that they do not assert the existence of any abstract objects. A *revolutionary nominalist* claims that some proposed nominalistic version of science is preferable to existing scientific theories.

Chihara argues that his own brand of nominalism does not fit comfortably into either half of Burgess’s disjunction, and this seems right. The book is peppered with reminders to the reader that Chihara does not take himself to be in the business of interpreting mathematical claims as, for example, when he writes that he has “taken pains, throughout this work, not to presuppose any specific analysis of the meaning of any mathematical theorem” (p. 340, footnote 24). So the goal is not hermeneutic in any straightforward sense. Nor is Chihara offering his Constructibility Theory as something “scientists should actually adopt . . . in place of classical mathematics for the purposes of formulating their laws and expressing their scientific theories” (p. 165). So the goal is not revolutionary either. The question, then, is whether there is some third kind of goal for the nominalist to aim at here, and—if so—what precisely the value and significance of such a goal might be.

Chihara’s own answer as to what the goal is of his nominalistic project is that he is trying to answer the following question: “can our contemporary scientific theories be reformulated in a way that will not require the assertion or the presupposition of abstract mathematical objects?” (p. 165). He sees this not as a practical question but as a “highly theoretical and deeply philosophical” one. However it is unclear, on this conception of the nominalist goal, just how one is supposed to measure success. What is the “criterion of adequacy” for a putative nominalistic reconstruction of this sort? I take it that one motivation for Burgess’s twofold classification is that each

horn has some kind of standard against which success may be measured. Hermeneutic nominalists must convince the experts in linguistics that their proposed interpretations of mathematical statements are correct. And revolutionary nominalists must convince scientists that their carefully constructed abstract-object-free replacements are worth adopting. It can certainly be objected (as Chihara does) that, for various reasons, these criteria are in practice overly stringent and hence unfair to the nominalist. But at least they have the virtue of indicating the sorts of theoretical virtues that nominalists should be aiming to preserve in their reconstructions of mathematics and of science.

Chihara cites with approval an argument of Susan Vineberg to the effect that confirmation of a particular scientific theory requires eliminating (or at least revealing as highly improbable) alternative theories. Chihara claims that the nominalistic views he describes are “fitting and reasonable” and he concludes from this that “in the absence of genuine evidence that allows scientists to eliminate these alternatives, the nominalistic reconstruction provides us with rational grounds for being skeptical about the existence of mathematical objects” (p. 167). But it is quite unclear here just what a theory has to do to count as “fitting and reasonable.” Moreover, drawing skeptical conclusions from the mere presence of an alternative theory which cannot be refuted outright by the empirical evidence raises troubling parallels with Cartesian skepticism. Does the fact that we can construct an “evil demon” scenario in which the material world does not exist give rational grounds for being skeptical about the existence of ordinary, concrete objects? Perhaps the evil demon theory is not “fitting and reasonable,” but in the absence of further analysis it is difficult to come to a decision on this matter.

At several points during his discussion of the various possible goals of nominalist reconstructions, Chihara draws parallels with the work of set theorists in the foundations of mathematics (see, e.g., p. 165). He takes these parallels to bolster his own “third way” because the work of set theorists seems to fit neither Burgess’s hermeneutic nor his revolutionary label. A major goal of set theory is to reformulate theories from other parts of mathematics in the language of sets. Yet theorists are not in the business, presumably, of showing what mathematicians “really mean” when they assert propositions of number theory or of complex analysis. Nor do they urge that other mathematicians use exclusively set theoretic apparatus in presenting their theories and proving their results. Chihara is certainly correct about the difficulty of fitting set-theoretic practice into Burgess’s twofold division (although Burgess himself has never claimed that it does or should). But his own position can only draw strength from the example of set theory if the set theory/mathematics relation is sufficiently similar to the nominalism/science relation, and this is far from clear. For a start, set theorists (typically) *are* mathematicians, whereas nominalists (typically) are not scientists. This difference is also reflected in the respective methodologies. As Penelope Maddy has pointed out, set theoretic reduction aims to realize methodological goals, goals such as UNIFY and MAXIMIZE, which are pursued in other areas of mathematics. By contrast, it seems highly unlikely that the motivations for most contemporary nominalist reconstructions, motivations such as a desire for ontological economy of abstract objects or concerns about epistemological access to acausal mathematical entities, are ones which are recognized—either explicitly or implicitly by working scientists. (See Maddy [5].)

The stark contrast between the situation of the set theorist and that of the nominalist only shows that the goals of set theory cannot be co-opted for the nominalist project; it does not show that there is no “third way” out of Burgess’s disjunction for Chihara. What we are left with is another version of our earlier worry about Chihara’s position. Chihara may be right that there is room for a nominalism about mathematics which is neither hermeneutic nor revolutionary. But he has not said enough in specific terms about what the alternative goal of nominalist reconstruction might be, about why this goal is of significance for our beliefs about mathematics and about science, and about how success in achieving this goal is to be determined.

3 Modality

Almost all of the various putative “nominalistic reconstructions” of mathematics that have been proposed in the literature make appeal at some point to *modal* notions. Sometimes this is explicit in the very name of the theory, as with Hellman’s modal structuralism. Other times the modality is further from the surface, as with Field’s use of a primitive “logical possibility” operator to ground the use of consistency and conservativeness results in his fictionalist approach to mathematics. Chihara’s position falls somewhere between these two extremes, but the modal dimension is not difficult to see, especially in his constructibility theory—a theory about what it is (and is not) possible to construct.

Nominalists need modality in order to accommodate mathematics because there are not guaranteed to be enough actual, concrete things in the universe to serve as surrogates for the objects of mathematics. So one standard way to put pressure on a proposed nominalistic reconstruction is to question the legitimacy of such appeals to modality in general and to the theorist’s explication of modality in particular. In keeping with his strategy of clearing the field of potential nominalist competitors, the issue of modality is one which Chihara makes extensive use of to attack the approaches of Field, Hellman, and others. I shall not here discuss the effectiveness of these attacks. Instead I want to consider the extent to which Chihara’s own use of modality is reasonable and defensible.

Chihara addresses the issue of the modality in his theory toward the beginning of Chapter 7, noting that “[i]n the phrase ‘it is possible to construct’, the term ‘possible’ needs some explanation” (p. 171). His answer, in brief, is that he is making use of a “conceptual” or “broadly logical” possibility, “a kind of metaphysical possibility,” and one which is correctly formalized by the modal logic **S5**. In particular, he is concerned to make clear that something may be possible to construct in this sense even if we have no idea of how to go about constructing it (p. 172). Does Chihara give a sufficiently precise characterization of the sense of possibility that he needs for his project? And are the resources he uses in this characterization legitimately open to him as a nominalist?

To see how even apparently quite simple examples can reveal the often complex nature of modal notions in mathematics, it is worth focusing on one of Chihara’s own favorite parts of mathematics, namely, Euclidean geometry. As mentioned, the first of Chihara’s five puzzles concerns the relation between modality and existence in different formulations of geometry, and the whole of Chapter 2 is devoted to this particular issue. Later in the book he returns to geometry in the course of attacking Michael Resnik’s views on modality. Resnik is a noncognitivist about modal claims; in other words, he thinks that categorical statements of necessity and possibility lack

truth value. Chihara thinks that this fails to do justice to actual mathematical practice and gives as an example the geometrical assertion: “it is impossible to draw with a compass and straightedge a square that is equal in area to a given circle” (p. 215). This was shown to follow from the proof (given in the nineteenth century) of the transcendence of π .

It is certainly a problem for Resnik’s noncognitivism that modal statements of this sort are treated as the targets of proof (or disproof) by mathematicians and that their putative truth or falsity is routinely discussed. However, geometric examples of this sort seem to also pose problems for Chihara’s analysis of modality. What, exactly, does the above theorem mean? On the one hand, it seems that someone *might*—armed only with compass and straightedge—accidentally construct a square with precisely the right area. On the other hand, no method of construction in Euclidean geometry (even something simple like quadrupling the area of a given square) will precisely achieve the desired result, since there will always be some human error involved. Perhaps we can thread our way between these two objections by parsing the original claim as one about the availability of a method which will consistently yield arbitrarily accurate instances of the proposed construction. Yet even here it seems *possible* for someone to repeatedly achieve highly accurate square-circlings through the fortuitous application of some haphazard procedure.

Given the above sorts of complication concerning modality in mathematics, the issue of how to go about determining the truth of the modal mathematical facts needed for Chihara’s Constructibility Theory becomes more pressing. Chihara’s response to Resnik, who raises similar worries concerning how we can come to know the facts underlying the Constructibility Theory, is, firstly, to give up the word “know” in this context, arguing that all he needs is that we have “plausible grounds” for believing the axioms (p. 217) and, secondly, to argue for the power of what he calls “theoretical reasoning” as a means of determining what is and is not possible. Chihara asserts that this approach is more fruitful than either modal logic or logical intuition, two of the means enumerated by Resnik as possible avenues to modal knowledge. It should be noted—in support of Chihara—that modal logic alone, even a specific system such as **S5**, does not tell us which atomic claims are possible. So something else in addition is needed to motivate the basic claims of constructibility. But Chihara is decidedly vague about what “theoretical reasoning” amounts to in this context. He talks of constructing “theories of possibility” and of testing them against what is deemed to be possible by our logical, linguistic, and scientific theories, and also against the results of modal logic and of logical intuition. The effectiveness of such a method is highly doubtful. The former kinds of theories deal with various notions of possibility—logical, epistemic, physical—which makes it unlikely that they can themselves function as templates against which to measure our “theories of possibility.” And the latter approaches, modal logic and intuition, have already been conceded to be insufficiently powerful on their own to generate the required justification of modal facts. How then can they underpin the allegedly more fruitful “theoretical reasoning” as required by Chihara’s approach?

Elsewhere in the book, Chihara returns to the issue of justifying his constructibility claims, this time in the context of applying mathematical reasoning to a simple scenario involving the number of coins on a table. The premise

[1] There are five dimes on table A at time t

is rendered in the Constructibility Theory as

[c-1] ‘ x is a dime on table A at t ’ satisfies a 5 attribute.

As already mentioned, Chihara is not a hermeneutic nominalist so he does not believe that [c-1] gives the *meaning* of [1]. Nonetheless, he thinks that our confidence in the truth of [1] carries over, in some fashion, to give us confidence in the truth of [c-1]. Indeed he suggests that the link between [1] and [c-1] is akin to a “Moorean fact”: I can know that this connection holds, even in the absence of a precise semantic analysis, in the same way that I can know that *this* (pointing to my right hand) is a hand if and only if *that* (pointing to my left hand) is a hand. The analogy here looks weak for a number of reasons. Firstly, Moore’s whole point was to focus on claims that have not been through the mill of philosophical analysis; [1] may have this character, but [c-1] certainly does not! Secondly, an important feature of the coins example is that there is no empirical evidence for [1] which is not equally evidence for [c-1]. Yet this symmetry of evidence is not present in the hands case. A one-handed person may have empirical evidence for *this* being his right hand, yet no corresponding evidence for the presence of a left hand. Yet [1] and [c-1] are not supposed to be able to come apart in this manner.

One other resource which Chihara uses to map out the use of modality in his Constructibility Theory is the semantics of possible worlds. Chihara is well aware that the literal construal of possible worlds talk is liable to violate the nominalistic strictures of his project, so he is careful to stress that he takes such talk to be “an elaborate myth, useful for clarifying and explaining the modal notions, but a myth just the same” (p. 196). This leads into a debate between him and Stewart Shapiro concerning whether he can legitimately use the apparatus of possible worlds for explanatory purposes while simultaneously maintaining that it is just a “myth.” I will return to this issue in the next section.

4 Indispensability

Indispensability is another implicitly modal concept that features prominently in contemporary philosophy of mathematics. Chihara devotes the bulk of Chapter 5 to discussing the Quine-Putnam indispensability argument for platonism in mathematics. There is much to commend in Chihara’s critique of indispensability arguments. He is right, for example, to point out the reliance of many contemporary versions of these arguments on problematically strong versions of holism. Quine, a great promoter of indispensability-style arguments for mathematics, was of course himself an arch-holist. A more interesting question is the extent to which indispensabilist arguments for mathematical platonism *need* an assumption about holism. Recent defenders of indispensability such as Mark Colyvan [4] tend to invoke weaker versions of holism in support of their views. But it is unclear whether this is necessary: even if the various posits of our theories do not necessarily face the “tribunal of experience” collectively, the nominalist still needs a convincing argument for why we should treat the postulation of numbers in our best scientific theories any differently from, say, the postulation of electrons. Chihara is also critical of the sweeping generality of many indispensability arguments (“it is philosophy of science done with a five-foot brush” (p. 126)), a feature which is perhaps also inherited from Quine’s relative lack of interest in the details of actual scientific practice.

Less convincing is Chihara's unpacking of what is meant by the term "indispensable." It is typically taken to be the case in the literature that "Fs are indispensable for science" is true if there are no adequate alternatives to our current scientific theories which avoid positing Fs. The question then is what we mean by "adequate" in this context. Chihara rejects Quine's notion of evidence and argues for a firm distinction between epistemic and pragmatic considerations of theory choice. Only epistemic considerations, he argues, ought to count in evaluating the truth of alternative theories and hence in determining the plausibility of indispensability claims. Even if he is right about this (and there is some doubt about how clearly such a line can be drawn), he goes on to claim that theoretical features such as simplicity and elegance are to be counted as (merely) pragmatic. This is contentious and indeed seems to fly in the face of scientific practice. The fact is that scientists do seem to take many forms of simplicity to be indications of truth, not just matters of convenience. Moreover, there are facets of simplicity (for example, ontological economy) which may in fact make a theory less practical and easy to use.

I shall close with an issue which has bearing on all three of the themes discussed above. At one point, Chihara summarizes his nominalistic position as being that "theorems of mathematics . . . do not have to be true to be justifiably used by scientists to draw the inferences they do in their scientific work" (p. 252). Interestingly, this characterization is not in fact strong enough for Chihara's purposes because some of his antinomialist opponents could happily agree with this claim. Take an indispensabilist platonist such as Mark Colyvan. Colyvan's view is that we ought rationally to believe in the existence of abstract mathematical objects in the same way that we ought rationally to believe in the existence of electrons. And as long as mathematical theorems remain an essential part of our best scientific theories, then we continue to be justified in using them even if such posits are in fact false. A better way for Chihara to distinguish his position is to focus not just on use but on belief: his view is that it is quite rational to simultaneously use mathematics in science, believe in the truth of the concrete theoretical entities postulated by science, and yet not believe in the literal truth of the mathematical theorems being used.

Note that if one of the uses to which mathematical theorems are put in science is the explanation of physical phenomena then Chihara's nominalism commits him to the view that there can be acknowledged falsehoods which play an essential explanatory role in explaining facts about the world. This same issue in another context was mentioned at the end of Section 3, where Shapiro objected that Chihara could not explain his modal concepts using the apparatus of possible worlds while simultaneously viewing such apparatus as a "myth." Chihara's defense against Shapiro is to distinguish two sorts of explanation:

(A) Scientific explanations of natural phenomena,
and

(B) Explanations of ideas or the meaning and use of expressions.

Chihara argues that while type (A) explanations may have to be taken to be true, his use of possible worlds talk is in the context of giving a type (B) explanation of the meaning of his modal notions. And he argues that type (B) explanations can legitimately bring in reference to false or mythical or imaginary theories and situations.¹ Even if we grant Chihara's claims about type (B) explanations (and perhaps we should not), this solves the possible worlds problem but not the larger issue of

the explanatory role of mathematics in science. For this role is precisely to give type (A) explanations. There seem to be three options left at this point. Chihara can either follow van Fraassen and give up any link between type (A) explanations and truth, an option that many philosophers have found unappealing. (Chihara writes that he “can think of cases in which one makes use of imaginary states of affairs in order to explain some complicated natural process” (p. 197). Unfortunately he does not describe any case of this sort.) Or he can argue that in no cases where mathematics is applied in science does the mathematical component itself play a genuinely explanatory role. There has been a flurry of recent work defending and attacking putative cases of mathematical explanation in science, with the tide perhaps turning in favor of the “pro-explanatory” camp. (See Melia [6], Colyvan [3], and Baker [1].) Or, finally, perhaps Chihara can return to his structuralist theme and argue that it is only the structural content of the mathematics which is playing a role in the scientific explanation.

Note

1. Chihara’s [2] is largely addressed to defending this claim for the case of possible worlds semantics.

References

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