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ON A PASSAGE OF ARISTOTLE

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Prior Analytics B 22, 68a16-21, exhibits some points of interest and one difficulty, long known but never seriously faced. We give in I. a translation, in II. some logical and historical comments, and in III. a formalized version.
I. "When (1) A belongs to the whole of $B$ and (2) C, and (3) is predicated of nothing else, and (4) B belongs to all C, (5) A and B must convert; for since (6) A is said only of B and C, and (7) B is predicated both of itself and (8) of C, it is clear that (9) B will be said of everything of which A is said, (10) excepting ( $\pi \lambda \eta \eta^{\prime} \nu$ ) of A itself."
II. (1)-(4) are the data, in modern style: (1) $A b a$, (2) Aca, (3) ( $x$ ) (Axa $\supset$ $(x=b \vee x=c))$, (4) Acb. (2) is syllogistically implied by (1) and (4), and is in any case not used in the subsequent deduction.

The formalization of (3) by means of the universal quantifier and identity might be said to go beyond what the text warrants. But Aristotle is dealing only intuitively with the totality of the unique subjects of $A$, and it seems clear that if his intuitions are to be formalized, this is the way to do it.
(5) is the probandum. The usage of the Analytics shows that it means that given $A b a$, as we are in (1), then we have also $A a b$.
(6) resumes (3).
(7) asserts $A b b$. Since this is taken for granted, and is not among the data (1)-(4), one seems justified in supposing that it is drawn from the underlying logic, being an instance of the syllogistic law of identity, $(x) A x x$. This passage is, I believe, the only evidence that Aristotle accepted this law. A further instantiation of it, Aaa, is implied by (10) whether the text is allowed to stand or emended along the lines suggested below, and is essential to the argument.
(8) resumes (4).
(9) is an intermediate conclusion: $(x)(A x a \supset A x b)$.
(10) is the crux. The probandum, $A a b$ is clearly obtainable by way of (9) and Aaa. One expects (10) to read 'and so of A itself". Percipient translators and commentators have frequently let this expectation rule
their pens, struggling to get an inclusive sense into the final exceptive phrase. Thus Magentinus (In Priores Aristotelis Resolutorias Explanatio, Venice, 1544) has: "'praeterquam quod etiam de A' hoc est et de ipso A enunciabitar. Illud autem 'praeter' hoc in loco 'quoque' significat." That is a desperate expedient. However a scholion of John Baptist Monlorius (Perfectissima in Aristotelis Anal. Prior., seu de Ratione libros duos, latinitate a se donatos, Paraphrasis et Scholia, Frankfurt, 1593) attempts to make that line of thought more respectable, summarizes the state of affairs, and leaves the matter about where it remains still. He translates: "quin etiam de ipso $a$ ", remarks correctly that the textual tradition appears constant, and sees that (10) is at odds with (5). He then quotes his recent predecessor Burana to the effect that Alexander had judged the text corrupt, and had proposed "and" in place of "except".

It seems a hopeless task to give $\pi \lambda \eta \dot{\eta} \nu$ an inclusive sense as early as Aristotle, though much later it could take the sense of 'besides', and that $\kappa \alpha i ́$ could have been accidentally altered to $\pi \lambda \dot{\eta} \nu$ is implausible. What might Aristotle have written, that would both make logical sense and have been altered to $\pi \lambda \dot{\eta} \nu$ by some prehistoric copyist? The best I can suggest is $\pi \lambda \epsilon \in \rho$, "more". The passage would then finish: "B will be said of everything of which A is said, all the more of A itself".
III. We can now give formal expression to the argument, keeping as closely as possible to the text. Points $1-4$ resume (1)-(4) above. 5 is justified in the comment on (7). 6 is an indispensable law of extensionality, the Aristotelian character of which is guaranteed by Topics H 1, 152a31 ff.

1. $A b a$
2. $A c a$
3. $A c b$
4. $(x)(A x a \supset(x=b \vee x=c))$
5. $(x) A x x$
6. $(x, y, z)(A y x \supset(x=y \supset A x z))$
7. $A b b$ by 5
8. $(x)(x=b \supset A x b) \quad$ by 6,7
9. $(x)(x=c \supset A x b) \quad$ by 6,3
10. $(x)(A x a \supset A x b) \quad$ by $4,8,9$. (Cf. (9) above)
11. Aaa by 5
12. $A a b$ by 10,11 .

It may be noted that 1-4 could be equivalently expressed in the single proposition, $(x)(A x a \equiv(x=b \vee x=c))$.

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