

TOWARDS AN AXIOMATIZATION OF VALUE-THEORY

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1 Introduction We begin the formalization of Clarence Irving Lewis' theory of value* by quoting various of the central concepts of his theory from his *An Analysis of Knowledge and Valuation* [5]. These quotations are analyzed informally and possible symbolizations of some of the central concepts are given. The function of the semantic definitions of the formal vocabulary is to relate this vocabulary to its English analogue. The choice of primitive constants was determined by the suitability of the formal analogue to the original English version and the economy of definitions. We chose our primitive predicates by attempting to define the largest number of derivative predicates by means of the smallest number of primitives and choosing these primitives so that they would fit Lewis' expressions.

One difference between a primitive and a defined constant is the fact that a defined constant can be translated into an expression containing only the primitive constants and the devices of simplified type theory. Hence, all defined or derivative constants are merely shorthand for the corresponding expression containing only primitive constants. Another difference between a derivative constant and a primitive constant is the fact that the derivative constant is here introduced by means of two types of definition whereas the primitive constant is introduced by means of only one type of definition. A primitive constant is introduced into the formal analogue by a semantic definition which consists of a formal expression on the left-hand side as *definiendum* and an informal reading for this expression on the right-hand side as *definiens*. A semantic definition is a means of interpreting a formal expression in terms of ordinary language. The formal language constructed in Section 2 is interpreted throughout by means of such semantic definitions.

The choice of primitives was of course partly arbitrary. There are undoubtedly many different formal analogues fitting the textual evidence equally well. The reduction of the number of primitive predicates to a

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small set required checking whether Lewis indicated which predicates he considered more fundamental. Some of the quotations and informal explanations in 2 constitute evidence of the textual suitability of our choice of primitives. We follow Lewis' order of exposition as closely as possible and begin by symbolizing most of the concepts which he considers fundamental to his theory. Once we have identified the notions which are to be primitive in our formal analogue all the other constants, obtained by translation of further material from Lewis into our formal analogue, are defined formally in terms of these primitives.

The defined constants were chosen by selecting from Lewis' book those statements which seemed to be definitions of the derivative concepts of his theory. We looked for all statements in Lewis which seemed to be definitions of his value-vocabulary, and then we gave possible translations of them by means of semantic and formal definitions. The quoted material in Section 2.2 can be regarded as further evidence for the suitability of our formalization, both of the primitives and of the defined vocabulary. Section 2.2 completes the presentation of the formal vocabulary of that part of Lewis' value-theory which we have selected.

Since we use certain non-value predicates in constructing our formal analogue of Lewis' value-vocabulary it seems advisable to make the distinction between value and non-value predicates somewhat clearer. In our formal analogue a value-predicate is either one of the two primitive value-predicates or a predicate defined in terms of these. A non-value-predicate is a predicate which is neither a primitive value-predicate nor a predicate defined in terms of these. These non-value-predicates are required in our formal analogue in order to enable us to mirror Lewis' statements containing such predicates. The set of non-value-predicates contains one pragmatic predicate, two psychological predicates, a time-order predicate, and a phenomenal predicate. The predicate "Test" is called a pragmatic predicate because it enables us to define value-predicates in the modality of actuality from corresponding value-predicates in the mode of potentiality. The two predicates "Wants" and "Avoids" are called psychological since they are used to describe psychological states of affairs. These two predicates occur in our formal theory only because their analogues are contained in one of Lewis' definitions. They enter into the definition of intrinsic value in the antecedent of a hypothetical clause. Thus our formal analogue is capable of countenancing beings which do not have these psychological relations. The predicate "Part" is called phenomenal because it denotes a relationship which holds only between phenomenal entities. The last of the non-value-predicates is a physical or time-order predicate. It merely serves to indicate that certain entities stand in relation to each other as earlier, later, or contemporaneous. Hence, it allows us to characterize entities as temporally ordered.

One reason for not packing these non-value-predicates into our primitive predicates is that Lewis did not do so. Also, this would only have made the formal language less perspicuous. Since these predicates are well defined in the theories of which they are an essential part we merely indicate

the points of contact which our value-theory has with these other systems of well-defined scientific concepts [1, 3]. Note that we do not investigate the question of the consistency of the system which results from adding the analogues of Lewis' value-vocabulary to these scientific theories. Such a task is beyond the scope of this paper.

As Carnap [2] and Martin [6] pointed out, the axiomatic method consists of two phases, formalization and interpretation. To complete the interpretation of our formal analogue we must proceed to a systematic assignment of denotations to some of the primitive vocabulary. Another way of characterizing an interpretation of an axiom system is to say that we must give rules which determine the meaning of the axiomatic terms. These rules are of a semantical nature and the system obtained from a formal system by means of the addition of such rules is a semantical system.

To arrive at our choice of axioms we first selected from Lewis' text those statements which he considered both true and important. We symbolized these statements, then attempted to prove them from a simple axiomatic basis suggested by the definitions. After some attempts at proof it became obvious that certain statements would be needed as additional premisses in these proofs. Having decided which premisses would be most fruitful in producing further theorems, we again sought textual evidence from Lewis. The quoted material from Lewis corroborates our choice of axioms. All the axioms we chose for the characterization of our analogue have corresponding statements, usually italicized in Lewis' work. Those important statements of Lewis which did not become axioms in our analogue were then shown to be theorems derivable from the definitions and axioms by means of simplified type theory. Again our selection was guided by central statements in Lewis' theory.

Section 2.4 is an ordinary language translation of the formal axioms and theorems of our formal analogue, presented in Section 2.3. We try to have the formulation of our theory as close as possible to Lewis' language. Section 2.4 does for the axioms and theorems what the ordinary English statements do for the formal statements they immediately succeed.

In the conclusion, Section 3, we use the formal analogue to characterize contrasting alternatives to Lewis' value-theory by suppressing or altering some of the axioms and theorems, or by adding further axioms and theorems to the formal analogue. Finally, we evaluate Lewis' theory from the viewpoint of our present formalization and place it among the contrasting alternatives considered.

2 Formalization of Lewis' theory of valuation

2.1 The primitive vocabulary We characterized our method in Section 1. To begin the task of explicating the primitive vocabulary, we quote Lewis' own characterization of value-theory:

- (1) in value-theory . . . there must be initial statements explicative of those properties which are to be dealt with; and such statements can only be definitive and *a priori*. Thus in the field of values, such basic statements will not represent valuations of anything but rather are required to be analytic of the nature of value

itself, or of some species of value, and indicate the criteria of the valuable. [5], pp. ix-x

Our intended task is to represent and analyze Lewis' theory of value by means of a formal analogue. The above statement is a characterization of our own endeavour. Our first task is the formal expression of Lewis' definitions describing the "properties which are to be dealt with" in value-theory.

However, we need to point out that the present order of exposition and the choice of the primitive predicates is the result of a choice among various alternative formalizations and repeated reductions to a minimum number of primitives. Hence, a careful textual justification of both our choice of primitive predicates and our choice of the forms of their semantic definitions is here called for. This is the main task of this subsection.

In our theory it becomes possible to define the formal analogue of Lewis' entire value-vocabulary in terms of three primitives, one of which is not a value-predicate at all, but what we call a 'pragmatic-predicate'. The formal analogue of Lewis' theory can be made to rest upon two primitive predicates of value-theory proper, and is developed with the help of a pragmatic-predicate, a time-order-predicate, the predicate "is a part of", and the usual devices of sentential connectives, quantification theory, and simplified type theory. In simplified type theory one has variables and entities over which these variables range which are subdivided into types or levels. The variables of the first or lowest type range over some totality of objects. The variables of the second type range over classes of these objects. The variables of the third type range over classes of classes of these objects, etc. In simplified type theory the only added logical primitive constant is ϵ , to be read "is a member of", and the atomic formulae are of the form $x \epsilon y$, where y is a variable of type one higher than that of x .

The two-place descriptive predicate "Part" has its places significantly filled by values of variables of the first type. The appearance of the predicate "Part" is a signpost, in our notation, that we are talking about entities of the lowest type. We leave the type designation implicit in order to make our notation more perspicuous. For example, the predicate "Circum" is of the second level when its variables are all of the first level. The same notation also signifies a predicate of the third level when one or more of its variables are of the second level, and so on for the predicates of a higher level. The reader may reintroduce the type level designations systematically by retracing the steps to the appearance of the predicate "Part". It must be remembered that in our official language no such ambiguity is introduced.

It appears that we are forced to use type theory to reproduce in part the ordinary distinction between concrete and abstract entities. Lewis' distinction between entities which are and those which are not themselves properties of other entities introduces into the formal analogue at least all of the machinery of simplified type theory.

We anchor the primitive descriptive constants in Lewis' text by means of material quoted from Lewis. The quotations are followed by semantic

definitions telling us how to read the symbols so introduced. Following the definitions we show why these predicates are suitable formal analogues of Lewis' vocabulary and also why they are central to Lewis' theory.

One of the weightiest reasons for choosing our particular formulation of the first primitive predicate in our formal analogue is the fact that Lewis uses almost the same words as we do. That is, Lewis' informal definition of what it means to attribute value to an objective existent is almost the same as our semantic definition of the first primitive predicate introduced:

- (2) attributing value to an existent, *O*, means that under circumstances *C*, *O* will or would, or probably will or would, lead to satisfaction in the experience of somebody, *S*; or it intends the joint assertion of many such affirmations. [5], p. 512

In the above definition Lewis indicates that he considers "leads to satisfaction" to be more primitive than "is valuable", since he is defining the latter in terms of the former. Consider a number of quotations in which "yields satisfaction", "yields dissatisfactions", or analogues of these appear:

- (3) But the term 'valuable' is to be applied to objects and other existents solely with the meaning 'capable of conducing to satisfaction in some possible experience'. [5], p. 414
- (4) gratification of some desire, some enjoyment or satisfaction, the realization in direct experience of a positive value-quality, is the peculiar or the decisive kind of confirmation of objective value in a thing. [5], p. 380.
- (5) moral goodness would not be enough: virtue is the supreme good, but the highest and complete good requires also satisfaction of the human capacity for happiness. [5], p. vii

There are quite a number of additional quotations which give evidence of the suitability and central nature of the chosen predicates but economy of space prevents us from listing them all here (e.g., [5], pp. 414, 448, 513, 523, 525).

We introduce the two main predicates by semantic definitions before assessing the evidence for the suitability and centrality of our primitive predicates. [All semantic definitions are denoted as "Def. 1", "Def. 2", etc., and all *formal definitions* are denoted as "D 1", "D 2", etc.]

Def. 1 "Sat *abcdef*" read "*a* is capable of yielding satisfaction *b* to person *c* in experience *d* at time *e* under circumstances *f*".

Def. 2 "Dis Sat *abcdef*" read "*a* is capable of yielding dissatisfaction *b* to person *c* in experience *d* at time *e* under circumstance *f*".

Both predicates seem to be required and at least it is not obvious how "dissatisfaction" could be defined in terms of "satisfaction". Hence, both these predicates will be retained as primitive.

It is immediately apparent that the reading of "Sat" is similar to

quotations (2) and (3) above. The reading of "*Dis Sat*" differs from the reading of "*Sat*" only by having "dissatisfaction" in the place of "satisfaction". In his definitions Lewis used a predicate which lacks but one place in order to be identical with ours. Where Lewis merely referred to circumstance *C*, we have added another place to the predicate in order to be able to insert values for a particular kind of circumstance, namely the circumstance of temporal order. The predicates chosen seem suitable formal analogues for the material quoted from Lewis.

As to the centrality of the above predicates to Lewis' theory, the quotations also give some evidence: Firstly, (2) and (3) show that Lewis defines the term "valuable" in terms of "satisfaction".

Secondly, (4) shows that the notion of satisfaction is central to Lewis' theory since it is by reference to this notion that he defined the notion of decisive confirmation of value. Since Lewis defined "objective value" in terms of "satisfaction", the belief that an objective existent has objective value is decisively confirmed by a realization of some satisfaction in experience in the presence of, or through the instrumentality of, the object.

Thirdly, (5) shows that the notion of satisfaction is essential to the characterization of the notion of the highest and complete good, the *sum-mum bonum*.

The above evidence shows that "*Sat*" and "*Dis Sat*" are suitable to express the central predicates in Lewis' theory. The formalization which is to follow, and its interpretation, must of course be corroborating before we can determine how suitable our formalization is and how central the predicates chosen for formalization are.

All the value predicates may be had in the two modalities of actuality and potentiality, the main use of the pragmatic-predicate is found in defining the predicates in the modality of actuality from the predicates in the modality of potentiality. Thus, "*Actual Sat*" can be defined from "*Sat*" by means of the pragmatic-predicate "Test".

- (6) the only kind of test of objective value which would be direct and ruling, is the test of finding such immediate value or disvalue in the presence of, or through the instrumentality of, the object which should be in question. [5], p. 413

Def. 3 "*Test abcdef*" read "upon test *a* yields *b* to person *c* in experience *d* at time *e* under circumstances *f*".

With the help of this pragmatic-predicate we can indicate what is required to enable us to move from a predication of potential value to a predication of actual value.

This concludes the introduction and analysis of the primitive value and pragmatic vocabulary. Next, the rest of Lewis' value-vocabulary is introduced by definition on the basis of these primitives.

2.2 The principal definitions We introduce Lewis' value-vocabulary by quotations from Lewis and semantic definitions as we did in the first section, except that following the informal introduction we give formal definitions of each predicate in terms of the primitive value-predicates introduced in the previous subsection.

The following is a list of the usual symbols which occur in the literature together with the typographical form that they shall have in this paper. All lower case Latin letters with or without primes are used as variables.

- “ $N. . .$ ” for “not. . .”.
- “ $(. . . \equiv . . .)$ ” for “ $. . .$ if and only if. . .”.
- “ $(. . . \supset . . .)$ ” for “if. . . then. . .”.
- “ $(\exists a)$ ” for the existential quantifier.
- “ (a) ” for the universal quantifier.
- “ $(\exists a, b, c)$ ” for “ $(\exists a)(\exists b)(\exists c)$ ”.
- “ (a, b, c) ” for “ $(a)(b)(c)$ ”.
- “ $(a \in b)$ ” for “ a is a member of b ”.
- “ $(. . . = . . .)$ ” for “ $. . .$ equals. . .”.
- “ $(. . . \neq . . .)$ ” for “ $. . .$ not equal. . .”.
- “ $(. . . \vee . . .)$ ” for “ $. . .$ or. . .”.
- “ $(. . . \& . . .)$ ” for “ $. . .$ and. . .”.

The next predicate is defined in terms of “*Sat*” and “*Dis Sat*”. We are defining what it means in the language of our formal analogue to say that something is a *possible value-object*. The notion which is now to be defined is exemplified by the quotations given in Section 2.1. There we extracted the entire predicate characterized by *Def. 1* and *Def. 2*, and hence all notions packed into these predicates are exemplified in Lewis.

Def. 4 “*Val Obj a*” read “ a is capable of being a value-object”.

D 1 “*Val Obj a*” for “ $(\exists b, c, d, e, f) (Sat abcdef \vee Dis Sat abcdef)$ ”.

According to *D 1*, a possible value-object is that which is capable of yielding some satisfaction or dissatisfaction to some person in some experience at some time under some circumstances.

To express certain distinctions among values Lewis used the concept of *value-quality*:

- (7) the ultimate aim of every sensible action is some realization of positive value-qualities in experience, . . . value in an existent consists in its potentialities for the realization of directly findable value-qualities in experience. [5], pp. 392, 394

Def. 5 “*Valqu bcdef*” read “ b is capable of being a value-quality characterizing experience d for person c at time e under circumstances f ”.

Quotation (7) contains references to value-qualities. Our symbol for a value-quality is “*Valqu bcdef*” which is a five-place predicate. Hence, we express *is capable of being a value-quality* by a five-place predicate in which the first-place variable ranges over part of the domain of directly experienced qualities, i.e., the domain of directly experienced value-qualities. The second place variable ranges over the domain of persons, the third place variable ranges over the domain of experiences, the fourth place variable ranges over the domain of time-values, and the fifth place variable ranges over the domain of circumstances.

D 2 "*Valqu bcdef*" for " $(\exists a)(\text{Sat } abcdef \vee \text{Dis Sat } abcdef)$ ".

The concepts of *immediate value* and *immediate disvalue* are easily introduced on the basis of our primitives:

- (8) the only kind of test of objective value which would be direct and ruling, is the test of finding such immediate value or disvalue in the presence of, or through the instrumentality of, the object which should be in question. [5], p. 413

Def. 6 "*Imed Val acd*" read "*a is capable of being an immediate value for person c in experience d*".

Def. 7 "*Imed Dis Val acd*" read "*a is capable of being an immediate disvalue for person c in experience d*".

In terms of our primitive predicates these two predicates are defined as follows:

D 3 "*Imed Val bcd*" for " $(\exists a, e, f) \text{Sat } abcdef$ ".

D 4 "*Imed Dis Val bcd*" for " $(\exists a, e, f) \text{Dis Sat } abcdef$ ".

Additional references can be found in [5], pp. 397, 404-5, 425, 479, 483, 485, 488.

Before we can introduce the next predicate we need to introduce two auxiliary primitive predicates which belong to psychology. Since we are not investigating Lewis' psychological theory we omit analysis of these predicates at this time. However, before we introduce these primitive predicates by definition, let us present some quotations from Lewis exemplifying the need for both these psychological predicates and the value-predicates defined with their help:

- (9) Value-disvalue is that mode or aspect of the given or the contemplated to which desire and aversion are addressed; and it is that by apprehension of which the inclination to action is normally elicited. [5], p. 403
- (10) *values ascribable to objects are always extrinsic values*; intrinsic value attaching exclusively to realizations of some possible value-quality in experience itself. [5], p. 389
- (11) In this sense of 'intrinsic value' as the value of that which is valued for its own sake, *no objective existent has strictly intrinsic value; all values in objects are extrinsic only*. [5], p. 387
- (12) the dichotomy 'intrinsic or extrinsic' is here restricted to signifying the distinction between what is valuable for its own sake and that which is valuable for the sake of something else. [5], p. 392

We need to refer to values in ways which indicate whether the valuer *wants* or *avoids* the entity valued for the sake of itself or for the sake of something else.

Def. 8 "*Want abc*" read "*a wants b for the sake of c*".

Def. 9 "*Avoid abc*" read "*a avoids b for the sake of c*".

Since these are auxiliary primitive predicates of psychology, we will

not be able, of course, to give formal definitions of these notions in our formal analogue of Lewis' theory of valuation. However, the formal definition of intrinsic value and intrinsic disvalue can be given as follows:

Def. 10 "*Intrins Val bc*" read "*b* is capable of being an intrinsic value for *c*".

Def. 11 "*Intrins Dis Val bc*" read "*b* is capable of being an intrinsic disvalue for *c*".

D 5 "*Intrins Val bc*" for " $(\text{Ea}, d, e, f)(\text{Sat } abcdef \ \& \ (g)(\text{Want } cbg \supset g = b))$ ".

D 6 "*Intrins Dis Val bc*" for " $(\text{Ea}, d, e, f)(\text{Dis Sat } abcdef \ \& \ (g)(\text{Avoid } cbg \supset g = b))$ ".

According to *D 5*, *b* is capable of being an intrinsic value for *c* if and only if there exists a possible value-object *a* such that *a* is capable of yielding satisfaction *b* to person *c* in some experience *d*, etc., and if person *c* wants satisfaction *b* then person *c* must want *b* for the sake of itself.

Lewis used the predicate "*ultimate value*" in making important statements regarding the values of experiences and lives.

(13) nothing has really intrinsic and ultimate value except such goodness as might characterize a life found good in the living of it, . . . [5], p. 407

Def. 12 "*Ultimate Val ac*" read "*a* is capable of being an ultimate value for person *c*".

Def. 13 "*Ultimate Dis Val ac*" read "*a* is capable of being an ultimate disvalue for person *c*".

D 7 "*Ultimate Val ac*" for "*Intrins Val ac* & $(d)(\text{Part } da \supset \text{Intrins Val } da)$ ".

D 8 "*Ultimate Dis Val ac*" for "*Intrins Dis Val ac* & $(d)(\text{Part } da \supset \text{Intrins Dis Val } dc)$ ".

D 7-8 involve a phenomenal primitive predicate which indicates a phenomenal situation in which one thing is said to be a part of another, or two things are said to be parts of one and the same thing. Since "*is a part of*" is a primitive predicate of a non-valuational kind we give only a semantic definition of this predicate.

Def. 14 "*Part ab*" read "*a* is a part of *b*".

The predicates "*extrinsic value*" and "*extrinsic disvalue*" have already appeared in quotations (10)-(12). But before we proceed to the formal definition of these two predicates we have to define the pragmatic predicate "*Instrumental*" in terms of "*Test*", which was introduced in 2.1.

Def. 15 "*Instrumental ab*" read "*a* is instrumental to *b*".

D 9 "*Instrumental ab*" for " $(\text{E } c, d, e, f) \text{ Test } abcdef$ ".

(14) we shall say that *A* has *extrinsic value*, or *instrumental value*, only if *B* (or some eventual *Z* to which it may lead) has intrinsic value. [5], p. 385

The above quotation readily indicates how to define extrinsic value and disvalue:

Def. 16 "*Ext Val abc*" read "*a* is capable of being an extrinsic value for person *c* with respect to *b*".

Def. 17 "*Ext Dis Val abc*" read "*a* is capable of being an extrinsic disvalue for person *c* with respect to *b*".

These are defined formally in terms of the predicates "*Instrumental*", "*Intrins Val*" and "*Intrins Dis Val*":

D 10 "*Ext Val abc*" for "*Instrumental*^{>0} *ac* & *Intrins Val cb*".

D 11 "*Ext Dis Val abc*" for "*Instrumental*^{>0} *ac* & *Intrins Dis Val cb*".

According to D 10, *a* is capable of being an extrinsic value for person *b* if and only if *a* is eventually instrumental to some *c* and *c* is capable of being an intrinsic value for person *b*. ">0" denotes the ancestral of a relation, see Carnap [1], p. 147.

The following quotation introduces the concept of *contributory value*:

- (15) One experience is better than another; characterized by a positive value-quality which is higher in degree; and we are concerned to make such comparisons of different experiences. [5], p. 462

Def. 18 "*Comp Val ab*" read "*a* is capable of having comparative value relative to *b*".

Def. 19 "*Comp Dis Val ab*" read "*a* is capable of having comparative disvalue relative to *b*".

The formal definition of the analogue of "comparative value" requires the ability to express that one satisfaction is greater or equal to another for some person:

Def. 20 "*GrEqu bb' for c*" read "value *b* is greater or equal to value *b'* for person *c*".

D 12 "*GrEqu bb' for c*" for " $(a, d, e, f)(\text{Sat } abcdef \supset (\exists a', d', e', f') \text{ Dis Sat } a'b'cd'e'f')$ ".

Def. 21 "*Equ ab for c*" read "value *a* is equal to value *b* for person *c*".

D 13 "*Equ bb' for c*" for "*GrEqu bb' for c* & *GrEqu b'b for c*".

D 14 "*Comp Val ab*" for " $(\exists c, d, e, f, g, h)(\text{Sat } agcdef \& \text{Sat } bhcdef \& \text{GrEqu } gh \text{ for } c \& \text{NEqu } gh \text{ for } c)$ ".

D 15 "*Comp Dis Val ab*" for " $(\exists c, d, e, f, g, h)(\text{Dis Sat } agcdef \& \text{Dis Sat } bhcdef \& \text{GrEqu } gh \text{ for } c \& \text{NEqu } gh \text{ for } c)$ ".

The following material quoted from Lewis indicates the need for the predicate "*contributory value*":

- (16) consider only the relation of an earlier to a later experience, whose value-quality is affected directly and not merely indirectly and causally; or we may speak of the

relation of these ingredients to a whole passage of experience, which includes both, and in which this qualification of the value-quality of the one by the other may be mutual. We shall use the word 'contributory' for either of these two relationships: . . . [5], p. 487

- (17) the relation of an experience to a whole in which it is included, and in which the distinguishable constituent experiences intimately and mutually qualify one another with respect to their value-quality. [5], p. 487
- (18) Here the one experience qualifies the other not only causally and instrumentally but also directly and in the manner of ingredients in a temporal *Gestalt*. [5], p. 486

The above quotations indicate that we require a predicate which will allow us to characterize experiences as having contributory value for other experiences and for wholes made up of experiences. In order to be able to define a predicate such as "contributory value" we have to be able to say what it means for the value-quality of one experience to directly influence or affect the value-quality of another experience. We also need a time-order predicate to express temporal relations among experiences.

Def. 22 "*Dir Val abc*" read "value *a* directly affects value *b* for person *c*".

Def. 23 "*Dir Dis Val abc*" read "disvalue *a* directly affects disvalue *b* for person *c*".

The formal definitions of these two predicates are in terms of "*Sat*", "*Actual Sat*", "*Dis Sat*", "*Actual Dis Sat*" and "*GrEqu bb'* for *c*".

The definitions of "*Actual Sat*" and "*Actual Dis Sat*" are similar to our semantic definitions *Def. 1* and *Def. 2*. The only difference between them is that the earlier definitions are of predicates in the mode of "potentiality" whereas the definitions to be introduced allow us to make assertions in the mode of "actuality".

Def. 24 "*Actual Sat abcdef*" read "*a* yields satisfaction *b* to person *c* in experience *d* at time *e* under circumstances *f*".

Def. 25 "*Actual Dis Sat abcdef*" read "*a* yields dissatisfaction *b* to person *c* in experience *d* at time *e* under circumstances *f*".

D 16 "*Actual Sat abcdef*" for "*Sat abcdef* & *Test abcdef*".

D 17 "*Actual Dis Sat abcdef*" for "*Dis Sat abcdef* & *Test abcdef*".

D 18 "*Dir Val bac*" for " $((a')(Ee, f)(((E d, g)(Actual\ Sat\ g b c d e f) \supset (E d', g') (Sat\ g' a c d' e f)) \& ((E d, g)(NActual\ Sat\ g b c d e f) \supset (E d', g') (Sat\ g' a' c d' e f)))) \supset (GrEqu\ aa' \text{ for } c \& NEqu\ aa' \text{ for } c))$ ".

D 19 "*Dir Dis Val bac*" for " $((a')(Ee, f)(((E d, g)(Actual\ Dis\ Sat\ g b c d e f) \supset (E d' g') (Dis\ Sat\ g' a c d' e f)) \& ((E d, g)(NActual\ Dis\ Sat\ g b c d e f) \supset (E d', g') (Dis\ Sat\ g' a' c d' e f)))) \supset (GrEqu\ aa' \text{ for } c \& NEqu\ aa' \text{ for } c))$ ".

D 18 reads: value b directly affects value a for c is equivalent to if, if b is a satisfaction for c entails that a is a satisfaction for c and if b is not a satisfaction entails that a' is a possible satisfaction, then a' will be smaller than a for c .

The last predicate required for our definition of "contributory value" is the time-order predicate. Since this is not a value-predicate to be defined in terms of our primitive value-predicates we introduce it here by means of a semantic definition.

Def. 26 "*EarSame* ab " read " a is a time earlier or the same as b ".

For convenience sake let us introduce additional time-order predicates in terms of "*EarSame*".

Def. 27 "*Same* ab " read " a is a time the same as b ".

D 20 "*Same* ab " for "*EarSame* ab & *EarSame* ba ".

Def. 28 "*NSame* ab " read " a is a time different from b ".

D 21 "*NSame* ab " for "*N(Same* ab)".

Def. 29 "*Ear* ab " read " a is a time earlier than b ".

D 22 "*Ear* ab " for "*EarSame* ab & *NSame* ab ".

Now we are in a position to define our next value-predicate which is read respectively for value and disvalue:

Def. 30 "*Cont Val* ab " read "value a is capable of contributing to value b ".

Def. 31 "*Cont Dis Val* ab " read "disvalue a is capable of contributing to disvalue b ".

However, as indicated by quotations (16)-(18), Lewis used one predicate to cover two slightly different notions ambiguously. We are now in a position to define the first of these notions but not the second which is somewhat more complex. Let us denote the first notion by "*Cont Val*₁" and "*Cont Dis Val*₁" and proceed to their formal definitions.

D 23 "*Cont Val*₁ ab " for " $(\exists c, d, d', e, e', f, f', g, h)(\text{Sat } agcdef \ \& \ \text{Sat } bhcd'e'f' \ \& \ \text{Dir } Val \ ghc \ \& \ \text{Ear } ee')$ ".

D 24 "*Cont Dis Val*₁ ab " for " $(\exists c, d, d', e, e', f, f', g, h)(\text{Dis } Sat \ agcdef \ \& \ \text{Dis } Sat \ bhcd'e'f' \ \& \ \text{Dir } Dis \ Val \ ghc \ \& \ \text{Ear } ee')$ ".

D 23 reads: value a is capable of contributing to value b , in the first way, if and only if there exists some c, g, h, e and e' , such that a is capable of yielding satisfaction g to person c , etc., and b is capable of yielding satisfaction h to person c , etc., and value g is capable of directly affecting value h for c and e is a time earlier than time e' .

The second of the predicates now being introduced is referred to in the quoted material above as "qualifying each other. . . in the manner of a temporal *Gestalt*". This same notion of a *Gestalt* and a temporal *Gestalt* is also used by Lewis in order to define his notion of a life. Since we need the

notion of temporal *Gestalt* for the definition of both the notion of contributory value of the second kind and the notion of a life, we define the notions of a *Gestalt*, temporal *Gestalt*, life, and part of a life. The latter two notions are particularly important here since the second kind of contributory value is the kind of contributory value that constituent experiences of either a life or a part of a life have for each other and for the whole of experience in question.

Def. 32 “*Qualif ba*” read “*b* qualifies *a*”.

D 25 “*Qualif ba*” for “ $(\exists c, d, e, f)(\text{Actual Sat } abcdef \vee \text{Actual Dis Sat } abcdef)$ ”.

Before we proceed to a formal definition of *Gestalt*, however, we need to anchor this notion in Lewis’ text by means of the following quotations:

- (19) a phenomenal *Gestalt*; in relationships of constituent elements which make it some kind of configurational whole. [5], p. 478
- (20) Constituent experiences compose a temporal *Gestalt* of experience in the sense of being its included and mutually qualifying parts. [5], p. 503
- (21) the relations of good or bad experiences in constituting a good or bad life is not that of a series of temporally juxtaposed and externally related moments but is that of ingredients which affect or qualify one another; the relation of components in a temporal *Gestalt*. [5], p. 486

We define firstly a *Gestalt* and secondly a temporal *Gestalt*.

Def. 33 “*Gest a*” read “*a* is a *Gestalt*”.

Def. 34 “*Temp Gest a*” read “*a* is a temporal *Gestalt*”.

These two predicates are defined in terms of “*Part*” and “*Qualif*” for D 26, and “*Part*”, “*Ear*” and “*Gest*” for D 27.

D 26 “*Gest a*” for “ $(b, c)((\text{Part } ba \ \& \ \text{Part } ca) \supset \text{Qualif } bc \ \& \ \text{Qualif } cb \ \& \ \text{Qualif } ab \ \& \ \text{Qualif } ba \ \& \ \text{Qualif } ac \ \& \ \text{Qualif } ca)$ ”.

D 27 “*Temp Gest a*” for “ $\text{Gest } a \ \& \ (b, c)(\text{Part } ba \ \& \ \text{Part } ca \ \& \ N(\exists d)(\text{Part } db \ \& \ \text{Part } dc) \supset \text{Ear } bc \vee \text{Ear } cb)$ ”.

D 26 indicates that *a* is a *Gestalt* if and only if for any *b* and for any *c* if *b* and *c* are parts of *a* then *a*, *b*, and *c* must all mutually qualify each other.

According to D 27, *a* is a temporal *Gestalt* if and only if *a* is a *Gestalt* and for any *b* and for any *c*, if *b* and *c* are ultimate parts of *a* then *b* is earlier than *c* or *c* is earlier than *b*.

Now we are ready to define what it means for something to be a life and a part of a life. Lewis referred to a life in the following manner:

- (22) The final evaluation of any particular experience is evaluation of it as a contributing to a whole of experience which it enters a constituent. And the overarching temporal *Gestalt* which is final is the purview of a whole life. [5], p. 503

Quotation (22) also refers to the relationship between constituent experiences and the life of which they are constituents. Hence, let us try to catch Lewis' meaning in our definition of life.

Def. 35 "Life abn " read " a is the life of person b up to and including time n ".

This notion can be defined in terms of the previously defined predicates "Temp Gest", "Part", and "EarSame", plus "Exper" which is defined as follows:

Def. 36 "Exper dce " read " d is an experience of person c at time e ".

D 28 "Exper dce " for " $(\exists a, b, f)(\text{Sat } abcdef \vee \text{Dis Sat } abcdef)$ ".

D 29 "Life abn " for " $\text{Temp Gest } a \ \& \ (a')(\text{Part } a'a \equiv (\exists c)(\text{Exper } a'bc \ \& \ \text{EarSame } cn))$ ".

According to *D 29*, a is the life of person b up to and including time n if and only if a is a temporal *Gestalt* and for any a' , a' is a part of a if and only if there exists a c such that a' is an experience of person b at time c and c is a time earlier or the same as time n .

Now that we have defined our notion of a life, we are in a position to define the second kind of contributory value. From quotations (16), (17), and (22), we see that experiences which enter as ingredients into a whole of experience have this kind of contributory value for each other and for the whole of experience in question whether it be a part of a life or a whole life. Since we will need to refer to a part of a life let us now define, for convenience sake, what it means for something to be a part of a life.

Def. 37 "Part Life a " read " a is a part of a life".

This predicate is formally defined in terms of our predicates "Part" and "Life".

D 30 "Part Life a " for " $(\exists b, c, e)(\text{Life } bce \ \& \ \text{Part } ab)$ ".

D 30 indicates that a is a part of a life if and only if there exists some b , some c , and some e such that b is the life of person c up to and including time e and a is a part of b .

Now we are finally ready to give a definition of the second type of contributory value. Let us denote this by "Cont₁Val₂" and "Cont Dis Val₂".

D 31 "Cont Val₂ ab " for " $(\exists n, c)((\text{Life } bcn \vee \text{Part Life } b) \ \& \ (a, a')(\text{Part } ab \ \& \ \text{Part } a'b)) \supset \text{Cont Val}_1 aa' \ \& \ \text{Cont Val}_1 a'a \ \& \ \text{Cont Val}_1 ab \ \& \ \text{Cont Val}_1 ba \ \& \ \text{Cont Val}_1 a'b \ \& \ \text{Cont Val}_1 ba')$ ".

D 32 "Cont Dis Val₂ ab " for " $(\exists n, c)((\text{Life } bcn \vee \text{Part Life } b) \ \& \ (a, a')(\text{Part } ab \ \& \ \text{Part } a'b)) \supset \text{Cont Dis Val}_1 aa' \ \& \ \text{Cont Dis Val}_1 a'a \ \& \ \text{Cont Dis Val}_1 ab \ \& \ \text{Cont Dis Val}_1 ba \ \& \ \text{Cont Dis Val}_1 a'b \ \& \ \text{Cont Dis Val}_1 ba')$ ".

According to *D 31*, value a is capable of contributing to value b in the second way if and only if if b is a life or b is a part of a life and a and a' are any parts of b then the values of the parts a and a' and the value of the whole b are all capable of mutually contributing to each other in the first way.

This completes the introduction of value predicates required for the subsequent sections of this paper. Lauer [4] contains a larger number of value predicates and the reader is referred to it for further detail.

2.3 Axioms and theorems We need six axioms to connect all value-predicates in our formal analogue. The first and most general axiom we call the *Axiom of Value-theory* or the *Axiom of Objective Value*. This reads formally:

Axiom 1: $(a)(Val\ Obj\ a).$

It states that everything is a value-object. This means that the range of the variable of each type is the class of possible value-objects of that type.

“Test” is used to define “actual satisfactions” from the corresponding predicate in the mode of possibility. But we take “Instrumental” to signify a relation among both possible and actual entities. Thus we have to add an axiom of instrumentality which relates the signification of the predicates in the mode of possibility to the relation signified by “Instrumental”. Of course our analogue of this axiom expressed in the mode of actuality would be directly derivable from the definition of “*Actual Sat*” and “*Actual Dis Sat*”. Axiom 2 is the formal version of the *Axiom of Instrumentality*.

Axiom 2: $(a, b)((\exists c, d, e, f)(Sat\ abcdef \vee Dis\ Sat\ abcdef) \supset Instrumental\ ab).$

The following quotation illustrates the next axiom which we call the *Axiom of Intrinsic Value or Intrinsic Disvalue*.

(23) *The goodness of good objects consists in the possibility of their leading to some realization of directly experienced goodness. What could by no possibility ever be an instrument for bringing any satisfaction to anybody, is absolutely without value, or the value of it is negative.* [5], p. 387

This quotation shows that if an object is capable of yielding satisfaction then this satisfaction must be directly experientiable by some person, and since Lewis characterized direct experiences of satisfactions as the only intrinsic good, if an object is capable of yielding satisfaction then the satisfaction yielded must have intrinsic value for some person. Axiom 3 is a formal analogue of quotation (23).

Axiom 3: $(a, b, c)((\exists d, e, f)(Sat\ abcdef \vee Dis\ Sat\ abcdef) \equiv (Intrins\ Val\ bc \vee Intrins\ Dis\ Val\ bc)).$

A reading of this is: for any possible value-object, this value-object is capable of yielding either a satisfaction or dissatisfaction to some person if and only if that satisfaction or dissatisfaction is an intrinsic value or disvalue, respectively, for this person.

The next two quotations illustrate the *Axiom of Progressiveness*:

- (24) And since experiences in general have both such intrinsic value and such instrumental value, the *final* assessment of the value of any experience must include reference to both. [5], p. 485
- (25) there is no moment whose quality and value fails to have further significance as contributory or subversive of further and more inclusive aims. [5], p. 495

The content of these two quotations is expressed formally by Axiom 4, which we have called the *Axiom of Progressiveness*.

Axiom 4: $(a, b, c, d, e, f)((\text{Sat } abcdef \vee \text{Dis Sat } abcdef) \supset (Ea', b', d'e', f')((\text{Sat } a'b'cd'e'f' \vee \text{Dis Sat } a'b'cd'e'f') \& \text{NEqu } bb' \text{ for } c \& (\text{Cont Val}_2 bb' \vee \text{Cont Dis Val}_2 bb'))))$.

This axiom reads: for any a , b and c , such that if a is capable of yielding satisfaction or dissatisfaction b to person c , etc., then there exists an a' , b' and d' , such that a' is capable of yielding satisfaction or dissatisfaction b' to person c , etc., and b is different from b' and value b contributes in the second way to value b' or disvalue b contributes in the second way to disvalue b' .

The next axiom needed in our formal analogue is illustrated by the following quotation from Lewis:

- (26) every simple value-modality can be determined in degree. Whatever thing it is which is valued and whatever respect it is in which a value of it is to be determined, there will be other things which can be compared with it as better or worse, and a place of it in some linear order of values can be thus assigned. [5], p. 543

This axiom, which we call the *Axiom of Dimensionality of Values*, can be rendered formally thus:

Axiom 5: $(a)(\text{Val Obj } a \supset (E b)(\text{Val Obj } b \& a \neq b \& (\text{Comp Val}_1 ab \vee \text{Comp Dis Val}_1 ab)))$.

A reading of this axiom would be: for anything, if it is a possible value-object then there exists some other possible value-object and the first value-object has possible comparative value or disvalue, of the second kind, relative to the second value-object. A more perspicuous reading would be: for anything, if it is a possible value-object then there exists a different possible value-object and the immediate value or disvalue of the first is greater than the immediate value or disvalue of the second or the immediate disvalue of the second is greater than the immediate disvalue of the first.

The last axiom to be introduced might be called the *Axiom of Immanence of Values*. The material for this axiom is introduced by the following quotation from Lewis:

- (27) nothing has really intrinsic and ultimate value except such goodness as might characterize a life found good in the living of it, . . . [5], p. 407

A formal rendering of this axiom would be:

Axiom 6: $(b, c)((Ultimate\ Val\ bc \vee Ultimate\ Dis\ Val\ bc) \equiv (Ea, e)(Life\ ace \ \& \ (Ed, f)(Sat\ abcdef \vee Dis\ Sat\ abcdef)))$.

Now we come to perhaps the most interesting and crucial part of this study. We quote certain central statements of Lewis, statements which he considered both important and true, and render them into their formal analogues. In [4] we proved the formal statements thus obtained to be theorems in our formal analogue which indicates that our formal explicate has passed a crucial test of adequacy.

We introduce the first such statement by a quotation from Lewis:

(28) In this sense of 'intrinsic value' as the value of that which is valued for its own sake, *no objective existent has strictly intrinsic value; all values in objects are extrinsic only*. [5], p. 387

A formal translation of the first part of the italicized statement would be:

Theorem 1 $(a)(Obj\ a \supset (c)(Nintrinsic\ Val\ ac \ \& \ Nintrinsic\ Dis\ Val\ ac))$.

This would read: for anything, if it is an objective existent then for any c such that c is a person it is true that a is not capable of being an intrinsic value or an intrinsic disvalue for person c .

A symbolization of the second italicized sentence in (28) is:

Theorem 2 $(a)(Obj\ a \supset (E\ b, c)(Ext\ Val\ acb \vee Ext\ Dis\ Val\ acb))$.

This would read: for anything if it is an objective existent then there exists a b and c such that a is capable of being an extrinsic value for person c with respect to b or a is capable of being an extrinsic disvalue for person c with respect to b .

We introduce the next theorem by means of a quotation from Lewis:

(29) *values ascribable to objects are always extrinsic values; intrinsic value attaching exclusively to realisations of some possible value-quality in experience itself*. [5], p. 389

The second half of the above quotation is the statement we want to render formally.

Theorem 3 $(a, c)((Intrinsic\ Val\ ac \vee Intrinsic\ Dis\ Val\ ac) \equiv (E\ d, e, f)Valqu\ acdef)$.

A reading of this would be: for any a and c a is capable of being an intrinsic value or an intrinsic disvalue for person c if and only if a is capable of being a value-quality characterizing experience d for person c at time e , etc.

From the next two quotations from Lewis we extract Theorems 4 and 5, which are easily provable:

(30) Immediate or directly findable value is not so much one quality as a dimension-like mode which is pervasive of all experience. [5], p. 401

The content of this quotation is rendered formal by the following two theorems:

Theorem 4a $(a, c, d)(Imed\ Val\ acd \supset (\exists a', d')(Imed\ Val\ a'cd' \ \&\ (GrEqu\ aa' \text{ for } c \vee GrEqu\ a'a \text{ for } c)))$.

Theorem 4b $(a, c, d)(Imed\ Dis\ Val\ acd \supset (\exists a', d')(Imed\ Dis\ Val\ a'cd' \ \&\ (GrEqu\ aa' \text{ for } c \vee GrEqu\ a'a \text{ for } c)))$.

The reading of Theorem 4a is: for anything, if it is an immediate value then there exists another immediate value such that the first is greater or equal to the second or the second is greater or equal to the first for any person c .

Theorems 4a and 4b are direct consequences of Axiom 5. There it is stated that, for anything, if it is a possible value-object then there exists a different possible value object and the immediate value of the first is greater than the immediate value of the second or the immediate disvalue of the second is greater than the immediate disvalue of the first. Hence, Theorems 4a and 4b are part of the *Axiom of Dimensionality of Values*.

The following material from Lewis presents us with an illustration for the last theorem proven:

- (31) if in one sense the determinations of values must be eventually in terms of the value-qualities of direct experiences, still in another sense no immediately experienced good or bad is final, but rather is further to be evaluated by its relation to the temporal whole of a good life. [5], p. 483

The content of this quotation from Lewis is rendered formal by means of the following theorem:

Theorem 5 $(a, c, d)((Imed\ Val\ acd \vee Imed\ Dis\ Val\ acd) \supset (\exists b, d')(Imed\ Val\ bcd' \vee Imed\ Dis\ Val\ bcd') \ \&\ (Cont\ Val_2\ ab \vee Cont\ Dis\ Val_2\ ab))$.

This reads: for anything, if it is a possible immediate value for person c or a possible immediate disvalue for person c , then there exists some other immediate value or disvalue for person c and the first value is contributing to the second value in the second way or the first disvalue is contributing to the second disvalue in the second way.

2.4 The evaluation of the axioms and theorems Here we evaluate the axioms and theorems dealt with in the preceding section. This interpretation shows how these features of our formal analogue of Lewis' theory of value characterize this theory.

Axiom 1 is the most general axiom and it states that everything is a value-object. That means that the range of the variables of each type is the class of possible value-objects of that type and characterizes the theory as a value-theory. This axiom was called the *Axiom of Value-theory* or the *Axiom of Objective Value*.

The second axiom is the *Axiom of Instrumentality* and in it the assertion is made that if any a is capable of yielding satisfaction or dissatisfaction b to some person c then a is instrumental to b . This axiom is a reflection in our formal analogue of the pragmatist's general *dictum* that whatever has any being whatsoever is instrumental to something else. Or,

if anything has being then there must exist something to which it is instrumental. The axiom is a particular case of this general *dictum*. The range of the variable *a* in this axiom is the domain of possible value-objects and the range of the variable *b* is the domain of satisfactions capable of being yielded by these possible value-objects.

The dispositional property signified by "is instrumental to" is in the special case of value-theory the dispositional property signified by "is capable of yielding satisfaction or dissatisfaction".

Thus, this axiom characterizes Lewis' theory as a pragmatist theory of value. The pragmatic element in Lewis' theory could be expressed in the *dictum*: The total value or disvalue of a possible value-object is equal to the *Gestalt* of all the possible values or disvalues that could be yielded by it to any person in any experience, at any time and under any circumstances. This may be seen to be a special case of the pragmatist *dictum* that the entire meaning of an intellectual conception is the totality of the practical consequences which might conceivably result from that conception.

The third axiom is the *Axiom of Intrinsic Value or Intrinsic Disvalue*. It states that: any possible value-object, if this value-object is capable of yielding either a satisfaction or dissatisfaction to some person, then the satisfaction or dissatisfaction must be an intrinsic value or disvalue, respectively, for this person. Or, that all satisfactions or dissatisfactions had by some person in some experience are intrinsic values or disvalues for that person. Or, that actual satisfactions or dissatisfactions had in personal experiences are the touchstone of intrinsic values and disvalues.

Hence, Axiom 3 can be thought of as a reflection in our formal analogue of the fact that Lewis is a naturalist and a humanist in value-theory. Let us consider what Lewis himself said in this respect about his theory:

- (32) a naturalistic or humanistic conception of values; . . . holds that the natural bent of the natural man stands in no need of correction in order validly to be the touchstone of *intrinsic* value. It repudiates the conception that with respect to intrinsic values we are natively incompetent, or born in sin, . . . (it repudiates) . . . transcendental norms which would impose themselves as imperatives which must overrule our natural desires, . . . [5], p. 398

This quotation shows that Lewis maintained that man is not by nature perverse and that man can come to a knowledge of the good life through a study of man himself. His view is that there are elements in man's experience which are clues as to how man ought to live. One does not have to be redeemed in order to live as one ought to live. But besides repudiating redemptionist norms, Lewis also repudiated transcendentalist norms. Axiom 3 is intended to maintain similar distinctions in our formal analogue and thus the axiom characterizes our formal analogue a naturalistic or humanistic value-theory.

The fourth axiom is the *Axiom of Progressiveness*. It states that no value-quality characterizing a momentary experience for some person fails to have beneficial or adverse effects on further momentary experiences or on wholes made up of experiences. Hence, this axiom would characterize

Lewis' theory as repudiating hedonism and relativism in value-theory. It would of course repudiate the crass hedonism of the Cyrenaics but it also repudiates the more sophisticated theory of a hedonistic calculus such as that of Bentham and Mill, because such a calculus depends upon the possibility of considering satisfactions or dissatisfactions characterizing momentary experiences as atomic and final.

The axiom characterizes Lewis' theory in another way in that it is a reflection in our formal analogue of the principle of prudential ethics. Lewis' theory repudiates a form of atomism in value-theory by maintaining that no value or disvalue characterizing a momentary experience is capable of being a final value. The final evaluation of all experiences must take into account their contributory value as contributory to some further whole of experience.

The fifth axiom is the *Axiom of Dimensionality of Values*. This axiom states that for anything whatever, if it is a possible value-object then there exists a different value-object and they have comparative value or disvalue of the second kind with respect to each other. This axiom again reflects Lewis' repudiation of atomism in value-theory, in that it states that nothing is valued in only one mode, but everything is valued in a variety of modes. And all values are capable of comparison with other values in a definite order of rank of values. As Aristotle might have said, all values fall within the domain of objects to which the distinctions greater or lesser apply. Hence, we could call Lewis' theory a special kind of relativistic value-theory. There are no absolute values of objects but all possible value-objects can be valued in a variety of ways and therefore one and the same value-object can possess values of various kinds in various degrees.

The sixth and last axiom is the *Axiom of Immanence of Values* or the *Axiom of Ultimate Value*. It states that for any person and any value or disvalue, the value or disvalue is ultimate if and only if there exists a life such that that life is the life of that person up to and including some time and the life is capable of yielding satisfaction or dissatisfaction to that person at that time.

This axiom is another way of asserting that this value theory is humanistic. Lewis has a twofold humanism in his theory if we consider Axiom 3 and Axiom 6 together. His value-theory is humanistic and naturalistic because unredeemed human nature is the touchstone of intrinsic value, and because it is the value-quality characterizing a personal life found good in the living of it which is the only kind of value which is ultimate.

This brings us to a consideration of the theorems. The first two theorems jointly state that: all possible objective existents have only extrinsic value or disvalue; no value or disvalue in an objective existent is an intrinsic value. A theory which holds such a view would be one which is humanistic in the sense of maintaining that the only intrinsic values are actual satisfactions had by some person in some experience. No possible objective existents are ever actual satisfactions or dissatisfactions, and therefore, no possible objective existents are intrinsic values or disvalues. One could say that there are no intrinsic values unless there are persons having

satisfactions in experiences. The value or disvalue of all objective existents is determined by what they contribute by way of satisfactions or dissatisfactions to persons.

The third theorem again asserts some consequences of the *Axiom of Intrinsic Value or Disvalue*. This theorem states that for any value or disvalue it is intrinsic value or disvalue if and only if it is a value-quality characterizing some experience for some person.

In Theorem 4a and 4b is to be found one of the consequences of the *Axiom of Dimensionality*. It states that if anything is an immediate value or disvalue for some person in some experience then there exists a different immediate value or disvalue for the same person in some other experience and the first immediate value or disvalue is related to the second immediate value or disvalue as greater or equal. In other words, all immediate values or disvalues are relative in the same sense that they can all be compared with other immediate values. Again this is an assertion of relativism in value-theory.

The final theorem is one of the immediate consequences of the *Axiom of Progressiveness* and characterizes Lewis' theory in the same way as that axiom.

To summarize, Lewis' theory of value is a naturalistic or humanistic value-theory. Lewis' recognition of the fact that the natural man is the touchstone of intrinsic value makes his theory a naturalistic one. Man does not need divine grace in order to know the intrinsically and ultimately good. Lewis' theory is absolutistic since he maintains that all possible values are comparable by means of one scale of values. Lewis' theory is relativistic since he maintains that no objective existent has value or disvalue attributed in only one mode. No value in an objective existent is *the* value of that objective existent. It is also anthropocentric in the sense discussed in the gloss on Theorem 2. That is, the value or disvalue of all objective existents is determined by what they contribute by way of satisfactions or dissatisfactions to persons.

3 Conclusion and assessment of Lewis' theory of valuation

3.1 The accomplishments of the present formalization First and foremost we have constructed a formal analogue of Lewis' theory of valuation. We have carried out this formalization on the basis of two primitive descriptive predicates of value-theory proper together with the aid of a pragmatic predicate, two psychological predicates, a time-order predicate, the predicate "Part", and the usual devices of propositional logic, quantification logic and simplified type theory.

Another way of stating this accomplishment would be to say that we have succeeded in constructing a formalized naturalistic value-theory solely by means of adding two descriptive constants of value-theory proper to the devices of propositional and quantification logic and simplified type theory.

Section 2.4 gives some evidence of the success of our formalization in representing Lewis' theory of valuation. We hope that we have been able to

avoid the danger of constructing a formal analogue which is too artificial to mirror many of the meanings of ordinary language statements.

The mirroring in our formal analogue of many of the concepts of value-theory could be regarded as a contribution to a complete axiomatization of value-theory. Such an axiomatization of value-theory is of intrinsic philosophic interest.

3.2 *Some of the principal choices in theories of valuation* In this section we will take cognizance of some of the principal choices in theories of valuation, which result from altering some of the axioms and theorems characterizing our formal analogue of Lewis' naturalistic and humanistic value-theory.

The denial of Axiom 1 would produce a radical change in our formal analogue. Any theory of value which is subjectivist would contain a statement equivalent to the denial of Axiom 1. Such a theory of value would contain an axiom to the effect that: for anything at all, it is not a possible objective value. Or, there are no objective values or disvalues. In such a theory the objectivity of values would be denied.

An emotive theory of values might be based, and is usually based, on the assertion that statements such as "anything is a value-object" are meaningless. Such a theory would contain an axiom or theorem which would deny that the quoted statement and its analogues are meaningful.

Cynics and nihilists in value-theory would be examples of philosophers who would also deny truth or falsity to valuations. These philosophers would be driven to such a view because of some fundamental despair.

A skeptical position in philosophy with regard to values could take various forms. A redemptionist theory, one which maintains that one has to be redeemed in order to live as one ought to live, could accept Axiom 1 but deny human knowledge of intrinsic value altogether or in part. Or again, a redemptionist ethics could accept Axiom 1 but contain an added statement to the effect that human knowledge of intrinsic values is only possible if the intrinsic values are divinely revealed. Such a theory might be characterized a revelationist-authoritarian theory.

A transcendentalist position with regard to values, such as that maintained by Kant, would contain an analogue of the denial of Axiom 1 since the only value countenanced is the good will and the good will is not an object. A transcendentalist view of the nature of ideas might contain an axiom to the effect that it is never the case that all of the conceivable effects of an idea exhaust the full meaning of that idea. Analogously, a transcendentalist value-theory might contain an axiom, the denial of Axiom 2, which might state that: it is never the case that the totality of the possible value-qualities yielded to possible human experience by a possible value-object is equal to that value-object's total objective value, because some or all values are only transcendentially realized. Any philosophical position which would repudiate pragmatism in value-theory would contain a statement analogous to the value-axiom stated above. Hence, a value-theory which would be part of an Absolute Idealism which postulates a realm of ideas or values which is never fully actualized might contain an analogue of the same axiom.

By changing Axiom 3 in the appropriate way we can obtain a transcendentalist view of norms. A transcendentalist in value-theory holds that norms have extra-natural sanctions. Transcendental norms would impose themselves as imperatives which must overrule our natural desires. The intrinsic values are, for Kant, not satisfactions realized or realizable in human experience but the good will which alone has intrinsic worth. The good will is determined not with regard to experience but by the self-determination of a transcendental source, the transcendental Ego. The transcendental Ego transcends the empirical ego or experienced ego at all times. Hence, a study of the empirical ego could not tell us anything about the intrinsically good. This is contrary to Axiom 3 because there it is stated that the natural or unredeemed man is the only touchstone of intrinsic value or intrinsic disvalue.

Another form of transcendentalism, a certain type of redemptionist theory of value holds that the intrinsically valuable is inaccessible to the natural man and is only made accessible by divine or super-human revelation or divine grace. Such theories might be called revelationist and salvationist theories of value respectively. In such theories the corresponding axiom to Axiom 3 would read: for anything, it has intrinsic value if and only if it is divinely revealed or taught by some authoritarian religious body.

A transvaluationist value-theory, such as that of Nietzsche, would contain a statement to the effect that no natural prizings or disprizings have intrinsic value or disvalue, but rather that something is an intrinsic value if and only if it is a transvaluated natural prizing. Hence, for Nietzsche and the early Heidegger, human nature must be overcome or transformed in order to have genuine values.

A deontologist might also deny Axiom 3 in that he would maintain that the right alone is good. Hence, if satisfactions and dissatisfactions are those aspects of experience to which desire and aversion are addressed and if right and wrong are determined independently of desire and aversion, then satisfactions and dissatisfactions are not determinant of the good and the bad.

A Thomistic value-theory would not imply the denial of Axiom 3 but the axiom would not be as strong as in our analogue. Thus, a Thomist might maintain that some intrinsic values are human satisfactions, e.g., the natural virtues in the case of St. Thomas' theory of values. But the supernatural virtues would also be intrinsic values. To maintain such a position one would however have to deny Axiom 6 if it is not the case that only the natural virtues can be ultimate.

If we change Axiom 4 appropriately we obtain various forms of hedonism. In Lewis' theory of value this axiom reads: no value-quality characterizing a momentary experience for some person is ever a final value because, as explained earlier, all final values are contributory values. A philosophically crude hedonism would deny this axiom and it would maintain that all value-qualities characterizing momentary experiences for some person are final values.

A more sophisticated hedonism, such as ⁱⁿ exemplified by Bentham's

utilitarianism with its conception of a hedonistic calculus, would still have an axiom to the effect that all compound values are equal to the aggregate of their constituent values.

In a prudential ethics Axiom 5 would be retained in the form in which it characterizes Lewis' theory of values. This axiom would also be retained by any position which repudiates the atomic and final nature of immediate values had in momentary experiences. However, a hedonistic theory of values containing the concept of a hedonistic calculus would also wish to retain Axiom 5 in the form in which it is found in our formal analogue. This is so because the success of the calculus of pleasures at least in Bentham depends upon the classification of pleasures with respect to degree.

A philosophical position which would maintain a doctrine of absolute and atomic values would contain a statement to the effect that no possible value-object is comparable with respect to values with any other possible value-object.

A transcendentalist or redemptionist theory of values would also imply an appropriate change in Axiom 6. This axiom would now read: for anything, it is an ultimate value if and only if it is not a satisfaction yielded to some person by some life in some experience apart from the divine intervention. Thus the *summum bonum* would not be natural but could only be attained after death or after the cessation of natural human life. A transvaluationist would maintain that only super-human values are capable of being ultimate values.

Axiom 6 of our formal analogue states that nothing is an ultimate value or disvalue except the value-quality characterizing the life of some person in some experience. Such an axiom would characterize all naturalistic and humanistic value-theories.

A redemptionist or salvationist value-theory and a transvaluationist theory would all deny Axiom 6 in that they would contain statements to the effect that no value-quality characterizing a life for some person is ever an ultimate value or an ultimate disvalue.

Various value-theories might retain Axiom 6 but the logical relation between ultimate values and value-qualities characterizing lives of persons would not be as strong. A self-realization ethics might maintain that some ultimate values are satisfactions yielded to persons by their lives but might further maintain that there are some ultimate values which are not satisfactions so yielded. In our formal analogue there is derivable the following theorem: for anything, if it is a person or a life or an experience then it is not a satisfaction or dissatisfaction. This theorem follows from the definition of a possible objective existent. Persons, experiences, and lives are all objective existents and all objective existents can only *yield* satisfactions and dissatisfactions but never themselves *be* satisfactions or dissatisfactions. Hence, if we wanted to have a self-realization ethics, we would have to add a philosophical psychology of self to the theory expressed by our formal analogue. In such a theory we would have to have an axiom in value-theory to the effect that, for anything, it is a good self if and only if

it is a satisfaction. Such an axiom would be required because a good self is an ultimate value in a self-realization ethics.

The following is a list of the value-theories which we have considered in this section as contrasting alternatives to Lewis' theory. This list is divided into cognitive and non-cognitive theories and is intended to serve the reader as a summary of the alternative positions considered in this section. The reader might find it useful to refer to it from time to time while reading the critical evaluation of Lewis' theory which follows.

I. Non-cognitive theories of value:

1. Emotive theories of value.
2. Cynicism in value-theory.
3. Nihilism in value theory.
4. Skepticism in value theory.

II. Cognitive theories of value:

5. Hedonism.
6. Prudential ethics.
7. Utilitarianism.
8. Humanism and naturalism in value-theory.
9. Self-realization ethics.
10. The deontologist.
11. Redemptionism, revelationism and salvationism.
12. Thomistic value-theory.
13. Transvaluationism.
14. Transcendentalism in value-theory.
15. Absolute idealistic value-theory.
16. Contemplative conceptions of the realization of values.

3.3 *A critical evaluation of the formal analogue of Lewis' theory* This final section of the paper serves the purpose of locating our formal analogue of Lewis' value-theory within the list of contrasting alternative theories of value considered. This is done by indicating how the theories contained in the list are grouped with regard to the six axioms of our formal analogue.

The list of sixteen possible alternative value-theories is divided into two groups, containing objectivist and subjectivist theories, by the fact that they either accept or reject Axiom 1, respectively. This is further divided into cognitive and non-cognitive theories according to whether valuations are considered to be meaningful or meaningless, respectively. All theories which would contain statements denying the *Axiom of Objective Value* are subjectivist theories. Because the first three theories of the list would contain statements denying that Axiom 1 is meaningful, these three theories are characterized as non-cognitive. Hence, by means of Axiom 1, our analogue is differentiated from all subjectivist and all non-cognitive theories of value and characterized as an objectivist theory of value. That our analogue is a cognitive theory and hence that values are knowable was presupposed by Lewis.

Theories 5-16 of our list are all cognitive theories. By means of Axiom 2, the *Axiom of Instrumentality*, these theories are further subdivided. Thus, all the cognitive theories are subdivided into at least two groups depending on whether they are based upon the value-theory analogue of the pragmatist theory of cognition or the value-theory analogue of some other theory of cognition. Our analogue of Lewis' theory, hedonism, prudential ethics, and utilitarianism and some theories of self-realization ethics would rest on such a value-theory analogue of the pragmatist theory of empirical cognition. The remaining cognitive theories in the list would be based upon the value-theory analogue of some other theory of cognition and hence these theories are differentiated from the pragmatist-based theories of value.

The *Axiom of Intrinsic Value or Intrinsic Disvalue* serves to distinguish our analogue from those theories which would deny that the natural and normal man is the *only* touchstone of intrinsic value or disvalue. Hence, the deontologist, redemptionist, revelationists, salvationists, Thomists, transvaluationists, transcendentalists and absolute idealists in value-theory would partly or wholly deny Axiom 3 in their value-theories.

Axiom 3 thus serves to characterize our analogue as a humanistic and naturalistic value-theory and differentiates it from all non-humanistic and non-naturalistic or supernaturalistic value-theories.

Some self-realization theories of value and some deontologists would be differentiated from our analogue by the fact that they only accept Axiom 3 in a weaker form than the form in which it is here stated.

Axioms 2 and 3 served to group together hedonism, prudential ethics, utilitarianism, humanism, and naturalism in value-theory, and some forms of self-realization ethics. Axiom 4, the *Axiom of Progressiveness*, enables us to divide this group further. Hence, because hedonism and Benthamite utilitarianism both contain a denial of Axiom 4, they are differentiated from our formal analogue.

Axiom 4 could also be called the *Axiom of Prudential Ethics* and hence prudential ethics seems to be an integral part of Lewis' theory. An ideal utilitarianism could also be accommodated in Lewis' theory as long as the possibility of a summation of value-qualities characterizing momentary experiences is denied. [7]

Axiom 5, the *Axiom of Dimensionality of Values*, serves to repudiate all views which would maintain the absolutistic and atomic nature of values. Thus, this axiom serves to differentiate our analogue once again from any crude form of hedonism, or any atomistic value-theory.

The *Axiom of Ultimate Value*, Axiom 6, serves to differentiate further our analogue from Theories 11-15 in our list. Hence, Axioms 3 and 6 serve to differentiate Lewis' theory from any form of transcendentalist value-theory. By making Axiom 6 weaker we could extend our analogue and accommodate certain forms of self-realization ethics.

To summarize: Axiom 1 differentiates theories of value into subjectivist and objectivist and cognitive and non-cognitive theories; Axiom 2 differentiates cognitive theories of value into theories based on the

value-theory analogue of the pragmatist-empiricist theory of cognition and theories based upon the value-theory analogue of some other theory of cognition; Axioms 3 and 6 differentiate theories of value into humanistic-naturalistic theories, hedonistic-utilitarian (in the narrow sense of containing a calculus of pleasures) theory, and prudential-naturalistic-utilitarian (in the broad sense of utilitarian) theories; Axioms 4 and 5 assert the continuity of the realm of values and thus differentiate atomistic from non-atomistic value-theories.

Ethical transcendentalism and hedonism can be thought of as two extreme poles of a spectrum. Both positions depend on an absolutistic conception of values. Ethical transcendentalism would absolutize values by making them independent of human satisfactions and dissatisfactions. Hedonism would absolutize values by declaring that value-qualities characterizing momentary experiences are in some sense atomic and final. Hence, Axioms 3, 6, and 4 serve to characterize our analogue as a value-theory which would fall somewhere between the two extreme poles of the spectrum. Our analogue relativizes values had in momentary experiences with regard to the values had in whole lives. But, objective-values are also relativized with respect to satisfactions yielded in human experiences.

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