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QUANTIFICATION, DOMAINS OF DISCOURSE, AND EXISTENCE

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This article* is a presentation of our analysis of the problem of existential import. We will first briefly analyze some basic logical concepts as they relate to the problem and which have for the most part been ignored by the participants in the dispute over existential import. A theory of existential import is presented within the context of a logical system, which is either explicitly indicated or tacitly assumed. The logical system employed is usually either the traditional syllogistic logic or the standard predicate logic. An important exception is Timothy Smiley's view on existential import using many-sorted logic, a system which can be extended to contain formulations of both syllogistic and predicate logic. We will refer to Smiley's adaptation of many-sorted logic to the Aristotelian syllogistic [1].

The first concepts that we wish to consider are those of logical syntax and logical semantics. Logical syntax refers to the strictly formal or symbolic system with no interpretation of any kind. The symbols are regarded precisely as symbols without meaning. The names, 'individual variables', 'predicate letters', etc., enable us to talk about the different symbols we have in our syntactical structure. What is meaningful are the relationships among the symbols which are defined by the formation rules, axioms, and rules of inference. If any interpretation is given to any of the symbols of the logical system, either individually or collectively, then we say that the system has a semantic aspect. Corresponding to the syntactical structure of the system we can construct what is called a model, which gives the rules for assigning interpretations to the symbols of the system and also provides rules for assigning truth-values to the interpreted formulas. It is sometimes convenient to construct an interpretation which is only truth-functional. Properties of the formal system such as

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consistency and completeness may then be proved relative to the model (provided that this is the case). Consistency means that a system does not contain contradictory propositions. In an informal sense, completeness means that a system contains all the formulas that we would like it to have. In the strict sense, a system is complete when every theorem (including axioms) is logically true (i.e., true in every case) for all interpretations. The converse is also usually proved for a formal system, viz.: Every logically true formula is a theorem. This latter property is called semantic completeness and is referred to by Smiley in his development of many-sorted logic. Syntax is sometimes characterized as being essentially deductive, while semantics is considered to be primarily truth-functional. In this article we take syntax to refer to an uninterpreted formal system and semantics to refer to a system having any interpretation.

Theoretically we begin with the syntactical structure and then proceed to the semantics of the logical system. But, in most practical situations we take the reverse route, starting with the interpretation and then analyzing it for its syntactical structure. For example, given the sentence 'All men are rational', and analyzing it with respect to the modern theory of existential import using the standard predicate logic, we have as the syntactical structure: $(x)(Mx \supset Rx)$. If our analysis is undertaken with respect to the many-sorted logic, we can write: $(m)Rm$. We see that we have some flexibility in writing the symbolic form for the original English sentence. The logical system we choose should be determined by its efficiency relative to the circumstances and purposes surrounding our analysis.

The quantifiers, (x) and $(\exists x)$, when considered syntactically, have no significance or meaning other than the relationship of the symbols to one another within the symbolic expression, i.e., the order in which the symbols are written. Theoretically, the quantifiers may be given any interpretation, provided that the overall interpretation in which they appear remains logically consistent. In terms of practical application the interpretation of the quantifiers is greatly limited. The universally accepted interpretation for the universal quantifier, (x), is 'For all x', or 'For every x'. The existential quantifier, $(\exists x)$, is given the meaning 'For some x', but in most contemporary logical contexts this is equivalent to 'There exists at least one x'. However, 'For some x' may be taken to be equivalent to 'For at least one x', where there is no explicit existential denotation, i.e., $(\exists x)$ can be interpreted as being neutral with respect to existence. Essential in understanding the interpretation of the quantifiers is the domain of discourse. In many discussions of existential import, an adequate consideration of the domain of discourse is ignored, although an implicit and often confused dependence upon a domain of discourse can be detected.

The domain of discourse belongs to the model (and may be referred to as the domain of the model) and contains individual elements or values. Each individual constant symbol in the formal system is assigned (i.e., has for its interpretation) one of the elements in the domain. The assignment or interpretation is made in accordance with the appropriate rule (also called a mapping function) in the model. Each individual variable in the formal system ranges over the elements or values of the domain. A universally quantified variable ranges over the entire domain; in a practical sense, this means we are speaking of every individual in the domain. An existentially quantified variable is one that is interpreted as at least one of the elements in the domain, but that element (or elements) is not specifically designated. A rule (or function) in the model assigns to each predicate letter in the formal system a predicate or property in the interpretation.

We now turn to the modern symbolization of the square of opposition and give these forms the following interpretation:

A	$(x)(Sx \supset Px)$	All men are animals.
E	$(x)(Sx \supset \sim Px)$	No men are animals.
I	$(\exists x)(Sx \wedge Px)$	Some men are animals.
O	$(\exists x)(Sx \wedge \sim Px)$	Some men are not animals.

In the modern theory of existential import, the domain of discourse is the real world. Unfortunately, in most discussions of this theory, no mention of the domain of discourse is made but the domain is tacitly assumed to be the real world. The *A* and *I* sentences, viz., 'All men are animals' and 'Some men are animals' are true in the above interpretation, while the *E* and *O* sentences are false.

The variable x ranges over all real things. Thus, the quantifier (x) can be interpreted as 'For all real things', and the quantifier $(\exists x)$ as 'There exists at least one real thing'. S has the interpretation 'is a man', and P has the interpretation 'is an animal'. Sx is 'x is a man', and Px is 'x is an animal'. Then, the literal interpretation for the *A* formula is 'For every real thing, if that real thing is a man then it is an animal', which is taken to be equivalent to 'Every man is an animal' (or 'All men are animals'). The syntactical form indicates that we should be quantifying over 'all real things' in the interpretation, but the English sentence, 'All men are animals', has 'men' universally quantified. The formal system being used here is the (one-sorted) first-order predicate calculus in which we quantify variables, but not predicate letters. The variable should correspond to the subject term in the interpretation, i.e., in the English sentence. We also observe that the symbolic form is hypothetical, whereas the English sentence is declarative. Thus, interpreting $(x)(Sx \supset Px)$ as 'All men are animals' leaves something to be desired from the viewpoint of logical exactness. We find a similar problem with the *I* form. The symbolic form, $(\exists x)(Sx \wedge Px)$, indicates that we should be quantifying over 'at least one real thing' and not over 'men' as in 'Some men are animals'. Also, the symbolic form is a conjunction while the English interpretation is a simple declarative. The problem extends to the other two forms, the *E* and *O*. There is a more adequate symbolism for presenting the square of opposition. Let us consider the following.

Let the domain of discourse be restricted to all men. We now rewrite the square as follows, using the same interpretation as above.

A	$(y)Ay$	All men are animals.
E	$(y) \sim Ay$	No men are animals.
I	$(\exists y)Ay$	Some men are animals.
O	$(\exists y) \sim Ay$	Some men are not animals.

The variable y ranges over the domain of men and the predicate letter A is interpreted as 'is an animal'. The variable now corresponds to the subject term and the logical form parallels the grammatical form of the English sentence. The present formalism preserves all of the traditional relations of the square, while in the previous modern symbolism most of these relations are not valid. The domain in our example is limited to men and this restricts the subject terms in the interpreted sentences to men. In limited cases of this kind our formalism functions very well. We may wish to state, 'All horses are animals', using the same formalism. We can no longer use the domain of discourse containing men, but need to use the domain of 'horses'. In other words, we are now in an entirely different context semantically. We cannot consider both men and horses (as subject terms) within the same context as long as we employ the one-sorted predicate calculus as our logical system. The solution to our problem is to use the many-sorted logic.

Let the domain of discourse contain both men and horses; let the variable x be a universal variable ranging over the entire domain; let the sortal variable m range over the subdomain of men; let the sortal variable h range over the subdomain of horses; and interpret A as 'is an animal'. We can now write 'All men are animals and all horses are animals', which is the interpretation for the logical form $(m)Am \wedge (h)Ah$. Consider the form $(x)Ax$. This is interpreted as 'Everything is an animal', where everything means everything in the domain of discourse, viz., all men and horses. Thus, $(x)Ax$, in this case, can also be interpreted as 'All men and horses are animals'. $(\exists x)Ax$ has the interpretation 'Something is an animal', i.e., at least one individual in the domain is an animal. $(\exists x)Ax$ can also be interpreted 'There exists at least one thing that is an animal', where 'exists' means precisely 'exists in our domain of discourse'. In other words, $(\exists x)$ is being interpreted as 'there is at least one individual in our domain of discourse such that'. It should be evident that the 'existential' status of both quantifiers, (x) and $(\exists x)$, is the same, with (x) quantifying over everything in the domain of discourse and $(\exists x)$ quantifying at least one indeterminate individual in the domain. Our interpretation of the quantifiers and their relation to the domain of discourse is generally in agreement with the position of R. M. Martin [2].

In these examples, all the domains have contained only real things, either the entire real world or part of it. Domains containing non-real members may also be used. The case in which the domain of discourse is the real world is only a special case. We are not restricted to applying only the real domain to natural language situations as the modern theory of existential import attempts to promote. Consider the following. Let the domain contain only unicorns, u be a universal variable, and G be interpreted as 'is green'. Then, $(u)Gu$ is interpreted as 'All unicorns are green'. $(\exists u)Gu$ has the interpretation 'At least one unicorn is green'. This means that at least one unicorn in our domain of discourse is green, not that there is a unicorn in the real world that is green. An intelligent person reading such a statement in mythology or fiction encounters no difficulty in understanding that real existence is not intended. The important point we are making is that 'existence' in the domain of discourse is not identical to existence in the real world. In many cases, these two 'existences' coincide, thus leaving some persons who employ logical systems completely unaware of the distinction. How then would we symbolize a sentence such as 'At least one unicorn is green, but unicorns do not exist'? Well, we would not, since we have a confusion of two different contexts. 'At least one unicorn is green' belongs to a fictional context, while 'Unicorns do not exist' obviously belongs outside of that fictional context. We can symbolize 'Unicorns do not exist', but relative to a domain of discourse different from the domain of 'At least one unicorn is green'.

Let our domain contain both real things and unicorns, where x is a universal variable, r a sortal variable ranging over real things, and u a sortal variable ranging over unicorns. Let G be interpreted as 'is green' (meaning the color green as a property in the real world) and E be interpreted as 'exists' (meaning real existence). 'Unicorns do not exist', i.e., 'No unicorns exist' is symbolized as $(u) \sim Eu$. Within this context we cannot state that unicorns are green. But we can say 'At least one thing is green', which is symbolized as $(\exists x)Gx$, because all real things are included in the domain. The formalism, $(x)Ex$, which is interpreted as 'Everything exists' does not hold, since the domain includes unicorns which are non-existent. However, $(r)Er$, interpreted as 'All real things exist' is true for our domain. We have $(\exists x) \sim Ex$ which is interpreted as 'At least one thing does not exist', and this is also true with respect to the domain. When a domain with more than one kind of existent (real, mathematical, imaginary, etc.) is employed, care need be taken to avoid equivocating upon any predicate terms; we have the example of 'is green' above. We have used existence as a logical predicate and this accords with the use of existence as a grammatical predicate in the interpretation. The only time real existence can be expressed in terms of a quantifier is the special case when the domain of discourse contains only real things. Existence is neutral with respect to logical structure. Or to state it in another way, existence is a semantic concept.

We often run across statements about vacuous subject terms. Take for example, 'Pegasus is a horse', where Pegasus is the mythological flying horse. Some logicians (e.g., Russell) hold that this statement is false because 'Pegasus' is an empty subject term—it does not refer to anything real. Is 'Pegasus' really an empty subject term? Once we accept a term as the subject of a sentence we, at least tacitly, accept that term as a member (or value) of the domain of discourse. Using 'Pegasus' as the subject term in 'Pegasus is a horse' automatically places 'Pegasus' in the domain of discourse. And 'Pegasus' is a perfectly good value to have in the domain of discourse. 'Pegasus' is now one of the values in the domain which makes the propositional function, 'x is a horse', to be either true or false. The concept of empty subject terms is important in the modern theory. When the subject term is 'empty' a universal proposition is true but a particular proposition is false. Let us investigate the following example.

Let the domain of discourse be the real world, and let x be a universal variable; interpret U as 'is a unicorn' and G as 'is green'. The modern version of the square is then written as:

A	$(x)(Ux \supset Gx)$	All unicorns are green.
E	$(x)(Ux \supset \sim Gx)$	No unicorns are green.
I	$(\exists x)(Ux \wedge Gx)$	Some unicorns are green.
O	$(\exists x)(Ux \wedge \sim Gx)$	Some unicorns are not green.

For the modern theory in this case, having an 'empty' subject term the A and E propositions are true and the I and O are false. Let us analyze the A form. Ux, which is interpreted as 'x is a unicorn' where x takes only real values, is always false—since there are no real things which are unicorns. Since Ux is the antecedent of the conditional $Ux \supset Gx$, by definition $Ux \supset Gx$ is always true, thus, making $(x)(Ux \supset Gx)$ logically true (true for all values of x) for the real domain. The E form, $(x)(Ux \supset \sim Gx)$, is also logically true for the same reason. Ux being false in all cases, makes the conjunction, $Ux \wedge Gx$, logically false. Hence, $(\exists x)(Ux \wedge Gx)$ and $(\exists x)(Ux \wedge \sim Gx)$ are both logically false. The modern theory's truth-functional interpretation of these forms is correct. However, nowhere in our analysis do we find a vacuous subject term with respect to the logical form. We may consider U, i.e., 'is a unicorn' to be an empty predicate term relative to the domain of real things. In the modern viewpoint we have what is a predicate term in the formalism becoming a subject term in the interpretation. Because of the logical forms used, viz., conditionals and conjunctions, the truth valuations for Gx and $\sim Gx$ are inconsequential in this example. We can point out that 'is green' as used in the modern theory is the kind of predicate which is meant to apply to real things, and not to such individuals as unicorns.

If we symbolize the same four statements in the many-sorted logic, we have

A	$(u)Gu$	All unicorns are green.
E	$(u) \sim Gu$	No unicorns are green.
I	$(\exists u)Gu$	Some unicorns are green.
O	$(\exists u) \sim Gu$	Some unicorns are not green.

Choose any domain of fiction containing unicorns in which every unicorn is green. Let u be a sortal variable ranging over unicorns, and let G be interpreted as 'is green'. It is now obvious that the affirmative propositions, A and I, are true and the negative propositions, E and O, are false. The predicate 'is green' in this case is the type of property that applies to the fictional individuals in the chosen domain. Nowhere is there an empty subject term or empty predicate term in sight, either in the formalism or in the interpretation, relative to our domain. The logical and grammatical forms are parallel. We also find that all the traditional rules of the square of opposition are expressed as instances of theorems of the many-sorted logic, viz.,

Contradiction: $(u)Gu \equiv \sim(\exists u)\sim Gu$
$(u)\sim Gu \equiv \sim(\exists u)Gu$
Contrariety: $\sim[(u)Gu \wedge (u)\sim Gu]$
Subcontrariety: $(\exists u)Gu \vee (\exists u)\sim Gu$
Subalternation: $(u)Gu \supset (\exists u)Gu$
$(u)\sim Gu \supset (\exists u)\sim Gu$

Within the many-sorted logic we find that the following equivalences are also instances of theorems when x is a universal variable:

$$(u)Gu \equiv (x)(Ux \supset Gx)$$
$$(u)\sim Gu \equiv (x)(Ux \supset \sim Gx)$$
$$(\exists u)Gu \equiv (\exists x)(Ux \wedge Gx)$$
$$(\exists u)\sim Gu \equiv (\exists x)(Ux \wedge \sim Gx)$$

Does this mean that our two previous examples are equivalent? No, since in one case we have a real domain and in the other a fictional domain. An attempt to reconcile the two examples into one would result in an equivocation upon the predicate 'is green'. Besides, the truth values for the E and O propositions in the two examples are in conflict. We do know that the standard one-sorted predicate logic is a subsystem of the many-sorted logic. When considered in the same context, i.e., with respect to the same domain, the above equivalences are logically true. These equivalences cannot be introduced into the first example in which we are using the standard predicate logic and have as our domain the real world. The equivalences do hold for the second example where we are using the many-sorted logic and a fictional domain, when we add the universal variable x which ranges over all the fictional individuals in the domain. Also, within this extended system we now have $(\exists x)Ux$ which has the meaning 'There is at least one unicorn', i.e., in our domain of fictional characters and not in the real world. A logically true statement of the form, $(\exists x)Ax$, does not impart actual existence as is often claimed by modern logicians, including Smiley for his many-sorted system with universal variables where $(\exists x)Ax$ is a theorem in that system. To say that the many-sorted logic with universal variables is necessarily an existential system, i.e., that we must use a domain of real things in order to apply the system, is incorrect. We have just given an example of successfully applying the system to a fictional situation.

We now know that the above four equivalences do hold for any interpretation in which x is interpreted as a universal variable. Let us consider one more example. Let the domain contain centaurs, unicorns and chimeras. Let x be a universal variable, u be a sortal variable ranging over unicorns, y be a sortal variable ranging over both centaurs and unicorns, and z be a sortal variable ranging over both centaurs and chimeras. Let U be interpreted as 'is a unicorn' and G be interpreted as 'is green'. Then the equivalence, $(u)Gu \equiv (x)(Ux \supset Gx)$, is true when interpreted for our domain since x is a universal variable. However, $(u)Gu \equiv (z)(Uz \supset Gz)$ is not true relative to the domain. The specific reason is because the sub-domain of unicorns (over which u ranges) is not contained within the sub-domain of centaurs and chimeras (over which z ranges). But, $(u)Gu \equiv (y)(Uy \supset Gy)$ does hold, precisely because the sub-domain of unicorns is contained within the sub-domain of centaurs and unicorns (over which y ranges). That this is so can be shown more clearly when we reduce our original domain (of centaurs, unicorns, and chimeras) to a new domain containing only centaurs and unicorns—the variable y now becomes a universal variable ranging over the entire new domain of centaurs and unicorns, and $(u)Gu \equiv (y)(Uy \supset Gy)$ is interpreted as being true. Thus, we have two cases in which $(u)Gu \equiv (x)(Ux \supset Gx)$ is interpreted to be true:

1. When x is a universal variable.
2. When x is a sortal variable and the sub-domain over which u ranges is entirely contained within the sub-domain over which x ranges.

The same holds for the E, I, and O forms as for the A form. Smiley fails to point out in [1] the observation which we make in case number 2 above. It is obvious that these two cases can be reduced to one case, viz., $(u)Gu \equiv (x)(Ux \supset Gx)$ is interpreted to be true when the domain (or sub-domain) over which u ranges is entirely contained within the domain (or sub-domain) over which x ranges.

For his many-sorted logic without universal variables, Smiley claims that the presupposition of a non-empty domain endows the traditional propositions of the square with existential import. For Smiley and other modern logicians a non-empty domain is a domain which contains at least one real individual. Subject terms like 'Pegasus' and 'unicorns', which are not real, are considered to be empty subject terms in the modern theory and are not admissible as members of a non-empty domain. We have already shown that domains containing only non-real things can be useful and valid in the performance of logical analyses. Strictly speaking, an empty domain is a domain which contains nothing—neither real things nor non-real things. Thus, positing an empty domain means that in the interpretation the variables are uninterpreted. However, in almost all actual applications the presupposition of a non-empty domain is intended to mean the rejection of any domain containing non-real members, and this we reject as being unnecessary and impractical. The traditional propositions can be either existential or non-existential, depending upon what is in the domain of discourse. The logical structure, with or without universal variables, is indifferent to existence. Hence, we also reject Smiley's method of translation for universal propositions from the many-sorted logic without universal variables to the standard predicate calculus by adding the rider $(\exists x)Ax$, i.e., the translation from $(a)Ba$ to $(x)(Ax \supset Bx) \wedge (\exists x)Ax$ is not accepted.

Conclusion The problem of existential import developed along with the development of modern symbolic logic during the nineteenth century. The problem is peculiar to the standard predicate calculus. There never was a real problem of existential import within the traditional syllogistic logic—it was placed there in retrospect by the modern logicians. The traditional square of opposition was accepted as valid for nearly two thousand years without serious question. Our conclusion is that it is still valid. However, the inferential power of the syllogistic logic is limited and the development of the predicate calculus greatly enhanced the scope of applicability of logic. The predicate calculus added new rules of inference (along with those of the propositional logic) and a very convenient generalized symbolic form. The development of the predicate calculus relied heavily upon mathematical insight, especially from set theory, and this has resulted in some shortcomings relative to natural language application. The syllogistic, developed within the context of natural language and not mathematics, adapts readily to natural language situations—this is now evident with the presence of an adequate formalism (viz., as a subsystem of the many-sorted logic). The subsumption of both the syllogistic and the predicate calculus into the many-sorted logic provides a much more adequate logical system for analyzing the logical structure of natural language. The many-sorted logic is actually a many-sorted predicate logic, while the standard predicate calculus is a one-sorted predicate logic.

The square of opposition may be formalized within the many-sorted logic as

A	$(a)Ba$	E	$(a) \sim Ba$
I	$(\exists a)Ba$	O	$(\exists a) \sim Ba$

We saw that within the many-sorted logic these four forms are equivalent to the forms used in the modern square ($(a)Ba \equiv (x)(Ax \supset Bx)$, etc., where the domain (sub-domain) over which a ranges is contained within the domain (sub-domain) over which x ranges). Two symbolic expressions being equivalent in the formalism means that in any interpretation their respective interpretations are truth functionally equivalent but not that any other aspect of the interpretation is exactly equivalent. Whatever interpretation is given to $(a)Ba$ and $(x)(Ax \supset Bx)$, they will either both be true or both be false; they cannot have opposite truth values. We have the English interpretation: 'All men are animals' and 'For every real thing, if that real thing is a man then it is an animal' for the forms $(a)Ba$ and $(x)(Ax \supset Bx)$, respectively. The two English sentences do not have exactly the same meaning nor the same grammar. Logical equivalence does not guarantee linguistic exactness, but it does guarantee truth functional consistency.

Thus, although the modern square of opposition is logically equivalent to the square given above, which is a formalization of the traditional square, the modern square has some linguistic inadequacies when interpreted which makes it less desirable than the traditional square for natural language interpretation. Therefore, we designate the traditional square as formalized within the many-sorted logic as 'the square of opposition'. We might note at this point that the many-sorted logic is a 'modern' system of logic and is an advance over both the syllogistic and predicate logics.

The problem of existential import evolved primarily because of the following two factors:

1. The unavailability of an adequate logical system for representing the logical syntax of natural language.
2. The failure to clearly understand the role played by the universe of discourse.

With respect to the first factor, an adequate logical system, the many-sorted logic, is now available for analyzing the logical structure of natural language.

The second factor is undoubtedly the more significant. Even with the standard predicate logic the domain of discourse has been and still is badly neglected. More emphasis upon the domain of discourse would have led to a better understanding of how to deal with existence logically. The persistence in attempting to impose existence upon logical structure has led to frustration. As we have seen, existence belongs to logical semantics, not logical syntax; neither the denotations nor the conotations of natural language appear in the logical structure. Existence is a grammatical predicate in natural language. To force an existential interpretation upon $(\exists x)$ is not justified. $(\exists x)$ can be interpreted to quantify subjects which do not belong to the real world. We advocate that $(\exists x)$ be interpreted precisely as a quantifier and nothing more, viz., as 'For some x' without existential denotation. $(\exists x)$ might be better referred to as a particular quantifier rather than as an existential quantifier.

The individuals in the domain of discourse are nothing more than the things which we are talking about in a context at hand. The members of the domain may be specifically identified, or the domain may be given in general terms, e.g., 'the real world', 'all men', etc., the criterion being that we understand clearly what we are talking about in our interpretation. Different types of existents may be included in the same domain, but care should be taken to avoid introducing equivocations or logical inconsistencies into the interpretation.

Summing up: Problems with existential import may be avoided by

1. Using the many-sorted (predicate) logic.
2. Distinguishing between logical syntax and logical semantics.
3. Proper and careful choice of the domain of discourse for the interpretation which is being analyzed or proposed.

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