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Relevance and Disjunctive Syllogism

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Introduction In this paper I present a novel account of the correctness in everyday contexts of informal uses of disjunctive syllogism from the perspective of relevant logics. This account can be regarded as an extension of the Anderson/early-Belnap position which has been much criticized of late. It also draws on intuitions underlying Mortensen's "normal context" approach and uses a similar strategy. Furthermore, it sits well with relevantist methodology because it makes disjunctive syllogism enthymematically valid using a premise which expresses what the relevantist claims is presumed when disjunctive syllogism is taken to be valid. I illustrate this novel approach using examples extant in the literature. This makes it clear that in most everyday, "normal" reasoning situations the formal relevantly valid construal of informal uses of disjunctive syllogism is also sound. It also highlights the shortcomings of classical formal construal of informal uses of disjunctive syllogism. As a further example I prove that the γ result (for R, say) can be used—if A and $\sim A \vee B$ are theorems then B is a theorem—using a relevantly valid argument.

1 An important distinction We must distinguish between informal use of disjunctive syllogism

$$A \text{ and (not-}A\text{ or } B\text{), hence } B,$$

which we will call disjunctive syllogism(or), and contenders for formal reconstruction of the informal argument. An example of the latter is the classical formal reconstruction

$$A, \sim A \vee B \vdash B,$$

which we will call disjunctive syllogism($\vee$). So in speaking of the "correctness of usage . . . of informal uses of disjunctive syllogism" above, I am not suggesting that it is ever correct to use the formal argument disjunctive syllogism($\vee$).

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That would only follow if we were to grant that the classical formal reconstruction of the informal disjunctive syllogism(or) is in fact the correct formal rendering of such informal arguments. Meyer points out the importance of making this distinction in [6], pp. 41–42.

It must not be presumed that in rejecting disjunctive syllogism(∨) the relevantist also rejects disjunctive syllogism(or); rather the relevantist simply rejects the former as an adequate formalization of the latter.

Disjunctive syllogism(∨) is not valid according to the relevant position because it sanctions false inferences in inconsistent or nonprime reasoning contexts and, symptomatically, the validity of paradoxes of relevance such as *ex falso quodlibet* (using other accepted principles). However, disjunctive syllogism(or) is used in many reasoning contexts. So an account must be given of when and why its usage is permitted.

2 The accounts advanced so far In the next section I define $b(A,B)$ and show why it is an appropriate enthymeme. But first a brief description of the two major accounts that have been advanced by relevantist logicians so far.

The Anderson/early-Belnap position (cf. [1], [2]) is that the English "or" can have both an intensional and extensional sense. (Denoting the former by $+$, $A + B \rightarrow . \sim A \rightarrow B$ is valid—standardly, $(A + B) =_{df} \sim A \rightarrow B$—so that disjunctive syllogism for $+$ is modus ponens for $\rightarrow$.) The intensional sense supports disjunctive syllogism, but the extensional sense does not. The empirical claim is that correct everyday usage of disjunctive syllogism occurs only with the intensional "or", i.e., modus ponens is being used, from which the paradoxes of relevance are not forthcoming. So on this account the informal disjunctive syllogism(or) has the following formal reconstruction: $A, \sim A + B \vdash B$. The difficulty with this position is that there appear to be many cases of sound disjunctive syllogism(or) which are rejected as unsound because the truth conditions of the "or" in the informal argument do not ensure that $\sim A + B$ is true, rather than just $\sim A \vee B$.[1]

Mortensen ([7]) presents a different account, according to which disjunctive syllogism(∨) is valid in a proper subclass of reasoning contexts, the "normal" contexts. On the assumption that our metatheory is such a context, consistency and primeness ensure that disjunctive syllogism(∨) is admissible for a theory. So, it is argued, in prime and consistent contexts, such as normal, everyday reasoning situations, one is perfectly entitled to use disjunctive syllogism(∨).

The novel approach involves moving the question of normality from the level of metatheoretic non-deductive persuasion into the object language itself. So rather than moving a level up in order to address the question of normality of a reasoning context, the question of normality is regarded as simply part of the reasoning context.

3 A natural assertion of normality which makes disjunctive syllogism(∨) enthymematically valid A natural assertion of consistency and completeness (normality) is that of any two possibly independent sentences A and B:

(α) Either A is true and B is true, or A is true and B is false, or A is false and B is true, or A is false and B is false[2] where "or" is exclusive; i.e., (α) means that exactly one of the cases obtains. The following is an object language expression of (α):

(β) Either A and B or A and $\sim B$ or $\sim A$ and B or $\sim A$ and $\sim B$.

I will argue that in formalese (α) is best expressed by:

(γ) $(A \,\&\, B)*(A \,\&\, \sim B)*(\sim A \,\&\, B)*(\sim A \,\&\, \sim B)$[3]

using $*$ to denote exclusive intensional disjunction.

Let us verify that this formulation captures the intended meaning of (β). Being an assertion of consistency and completeness whether or not (β) is true cannot be based on the mere fact of truth of one of the disjuncts. "A and B" must be inferentially connected to the other disjuncts in a way in which those in

either A and B or Bach wrote the Coffee Cantata or the Van Allen belt is doughnut shaped

are not. Part of the meaning of (β), as an expression of normality, is that if "A and B" is false, then one of the other disjuncts is true. Clearly (α) means, in part, that at least one of the cases obtains. So part of the meaning of (α) is that if none of $(A \,\&\, B)$, $(A \,\&\, \sim B)$, and $(\sim A \,\&\, B)$ is true then $(\sim A \,\&\, \sim B)$ is true. Now this conditional would not be true if it merely happened that $(\sim A \,\&\, \sim B)$ were true. For the truth of $(\sim A \,\&\, \sim B)$ is insufficient to establish that were it false, then one of the other conjunctions would be true. But if the "or" in (β) were extensional, this observation would be contradicted. Hence, to capture adequately the sense of (α), the "or" of (β) must be the intensional $+$. (α) expresses an intensional relation between the disjuncts which cannot be captured using extensional disjunction.

Clearly, to capture the sense of (α), the truth conditions of "A and B" in (β) must simply be that it is true iff both A and B are true; that is, "and" is extensional. Thus (γ) is the correct formulation of (β), where (β) is intended to capture (α). (That every "or" is intensional, and every "and" extensional, follows from the corresponding symmetry of (α).)

So a local assumption of consistency and completeness can be expressed in the object language by:

$$b(A,B) =_{df} (A \,\&\, B)*(A \,\&\, \sim B)*(\sim A \,\&\, B)*(\sim A \,\&\, \sim B).$$

We will say that a theory is locally Boolean at A and B iff it contains $b(A,B)$.

Using the Tautological Entailments, $\vdash A + B \to A \vee B$, $\vdash A + B \to. \sim A \to B$, rule-prefixing, rule-suffixing, either rule-contraposition or de Morgan equivalents for $+/\circ$, and commutativity of $+$, it follows that

$$\vdash (A \,\&\, B) + (A \,\&\, \sim B) + (\sim A \,\&\, B) + (\sim A \,\&\, \sim B) \to. A \,\&(\sim A \vee B) \to B.$$

And so, using the fact that $\vdash A * B \to A + B$

$$\vdash b(A,B) \to. A \,\&(\sim A \vee B) \to B.$$

So if $b(p,q)$ is in a theory Γ, then $p \,\&\, (\sim p \vee q) \to q$ is also in Γ (on the above minimal assumptions about the logic). That is, the corresponding instance of the conditional form of disjunctive syllogism($\vee$) is in Γ. Assuming a "deductive" notion of theoryhood (a theory contains all the logical theses and is closed under the primitive rules—in particular modus ponens), it follows that the rule form of disjunctive syllogism($\vee$) is locally available in Γ.[4]

Noting that the weaker premise $(A \& B) + (A \& {\sim}B) + ({\sim}A \& B) + ({\sim}A \& {\sim}B)$[5] suffices to obtain disjunctive syllogism($\vee$), hereafter $b(A,B)$ will be used to refer to it and (γ) collectively (even though the former doesn't really express local consistency). That this sentence does suffice suggests there are further issues to be addressed concerning $+$ and negation, though here is not the place to do such.

The above considerations show that $b(A,B)$ renders disjunctive syllogism($\vee$) enthymematically valid. Furthermore, since (α) is a natural expression of normality of a reasoning context, and $b(A,B)$ is its object language expression, $b(A,B)$ is appropriate from the relevantist perspective. $b(A,B)$ expresses what the relevantist claims is being presumed when disjunctive syllogism($\vee$) is regarded as valid. (In Section 7 I provide a formal metatheoretic argument which further justifies the claim that $b(A,B)$ is an object language expression of consistency and completeness.)

Using the Tautological Entailment $A \& {\sim}A \rightarrow A \& ({\sim}A \vee B)$, contraposition, rule-prefixing, and rule-suffixing, it follows that: $\vdash b(A,B) \rightarrow . A \& {\sim}A \rightarrow B$ and $\vdash b(A,B) \rightarrow . B \rightarrow A \vee {\sim}A$, which justifies the label "locally Boolean" for $b(A,B)$.

That a reasoning context is assumed to be normal can be signaled by supposing that all instances of the schema $b(A,B)$ are true. It then follows that all instances of $A \& ({\sim}A \vee B) \rightarrow B$ are true, so that disjunctive syllogism($\vee$) is available by modus ponens.

So correct everyday usage of disjunctive syllogism(or) can be explained by the fact that such correct usage occurs where $b(A,B)$ is true.

Whilst the $b(A,B)$ are object-language expressions of local consistency and completeness, assuming all instances of $b(A,B)$ to be true is of course no guarantee that the reasoning context is in fact consistent and complete. A theory may contain all instances of $b(A,B)$ but not be consistent (for example, the trivial theory containing all sentences); and a theory may be consistent and complete yet not contain $b(p,q)$ (for example, the consistent and complete extension of $\{$theorems of $\mathbf{R}\} \cup \{{\sim}b(p,q)\}$, generated using Henkin's method, say). It follows that a theory may contain all instances of $b(A,B)$ yet not be normal and, conversely, such a normal theory may not contain $b(p,q)$. $b(A,B)$ is an object language expression of "normality", necessarily fallible from the metatheoretic perspective. (This does not constitute a weakness of the account. It simply means that some theories are false in that they affirm $b(A,B)$ when in fact the theory is not consistent and complete, or they fail to affirm $b(A,B)$ (or they affirm its denial) when in fact they are consistent and complete. So: some theories are false.) Whether or not one is in a normal reasoning situation is simply another piece of information to be deliberated by the reasoner and used in the reasoning process.

4 A modification of the Anderson/early-Belnap account which accords with that presented above The local Boolean approach bears some similarity to the Anderson/early-Belnap approach. We can prove: $\vdash b(A,B) \rightarrow . A \vee B \rightarrow A + B$ (alternatively $\vdash b(A,B) \rightarrow . A \supset B \rightarrow . A \rightarrow B$). That is $b(A,B)$ enables the extensional "or" $\vee$ to be strengthened to the intensional "or" $+$. So although it is not claimed that the "or" in correct everyday usage of disjunctive syllogism is intensional, it is claimed that the argument occurs in a context of background truths which elevate the extensional "or" to the logical strength of an intensional "or".

Thus the locally Boolean approach is in complete agreement with the following modification of the Anderson/early-Belnap account:

An informal argument of the form

$$A,\ {\sim}A \text{ or } B;\ \text{hence } B$$

is sound only if the sentence ${\sim}A + B$ is true; so that the formal argument

$$A,\ {\sim}A + B \vdash B$$

is also sound.

Here it is not claimed that the informal ${\sim}A$ or B means, or should be formalized as, ${\sim}A + B$. The claim is only that where the informal argument is sound, ${\sim}A + B$ is also true. As such it is virtually tautologous, for it simply makes the obvious point that if we can infer B from A (plus whatever), then we must have an inferential license $A \rightarrow B$ (from whatever) to do so. ${\sim}A + B$, but not ${\sim}A \vee B$, provides such an inferential license. (This appears to be the position adopted by Read ([9]) where he speaks of the player of Mystery Cards being "supplied . . . with a true intensional premiss".) Since $b(A,B) \rightarrow .\ {\sim}A \vee B \rightarrow {\sim}A + B$ is relevantly valid, if $b(A,B)$ and ${\sim}A \vee B$ are true, then ${\sim}A + B$ is also true. Thus whenever an informal disjunctive syllogism(or) argument is sound on the locally Boolean account, ${\sim}A + B$ is true and $A,\ {\sim}A + B \vdash B$ is also sound.

5 Examples We consider two examples in the literature for illustrative purposes. These were put forward by Burgess to counter the Anderson/early-Belnap position ([4]).

The first example involves a game of Mystery Cards, the details of which need not concern us. A player argues: It isn't both the deuce of hearts and the queen of clubs; but it is the deuce of hearts; so it isn't the queen of clubs. In a real-world card game the corresponding local Boolean assumptions are true, in particular: Either the queen of clubs is on the table and the deuce of hearts is on the table, or the queen of clubs is on the table and the deuce of hearts isn't, or the queen of clubs isn't on the table and the deuce of hearts is, or neither is on the table; where this proposition has the sense of (α). The player's argument follows relevantly from this fact.

Thus to formally reconstruct the player's informally correct argument, we simply need to take account of a suppressed premise. Burgess' subsequent claim that had the player "been a relevantist, unwilling to make a deductive step not licensed by the Anderson-Belnap systems **E** and **R**, he would have been unable to eliminate the queen of clubs from his calculations, and would have lost the game" is clearly false. There is no need for the relevantist to "betray in practice the relevantistic principles he espouses in theory".

The second example is a mathematical one. Zeeman has a proof that for all natural numbers n, $A(n)$ or $B(n)$. Wyberg independently proves that $\sim A(1)$ and uses Zeeman's result to infer that $A(1)$ or $B(1)$ holds, and hence that $B(1)$ holds. Once again formal reconstruction (using the appropriate $b(A,B)$) delivers the corresponding relevantly valid argument (the procedure is completely mechanical). Soundness is a different question. Whilst it is clear in the Mystery Card game that local Boolean assumptions are true, this is not so in the case of mathematical theories. Classical Peano arithmetic is incomplete if consistent. There are grounds for supposing that in the case of "the" Gödel sentence G,[6] $b(G,G)$ ought not be a thesis of the appropriately extended system (cf. the incompleteness result). Thus if Wyberg's claim is about the theses of (say) classical Peano arithmetic, it is not straightforward that the required enthymeme is true. If his claim is about ARITHMETIC then issues concerning the ontological status of this animal, and how it relates to formal theories, need to be addressed to justify the locally Boolean assumptions. Wyberg has made a significant philosophical assumption about the nature of mathematical theories and mathematics itself, but classical reconstruction of his argument is too impoverished to indicate that he has done so.

The examples highlight the following facts:

- In all cases of informal arguments involving use of disjunctive syllogism, there is a corresponding relevantly valid formal reconstruction of the argument. Given any application of disjunctive syllogism(or)

  $$A,\ \sim A \text{ or } B;\ \text{hence B}$$

  we have the following corresponding reconstruction

  $$b(A,B),\ A\ \&\ (\sim A \vee B) \vdash B.$$

- In most such cases (everyday reasoning situations) the formal argument is also sound, because the corresponding local Boolean assumption (that is, (α)) is true.
- The relevant reconstruction displays a substantive assumption which is open to question in some reasoning situations, such as mathematics, yet is not even acknowledged by the corresponding classical reformulation.[7]

6 Comparison with the "normal context" account In this section we compare Mortensen's "normal context" account with the novel "locally Boolean" account.

Mortensen's argument concerns conditions under which disjunctive syllogism(v) holds; that is, what is needed to guarantee the truth of the metatheoretic sentence: $(A \in \Gamma)\ \&\ (\sim A \vee B \in \Gamma) \rightarrow (B \in \Gamma)$ (letting Γ denote an arbitrary theory). He assumes that if a reasoning context is normal then disjunctive syllogism(v) holds of it. Taking for granted that the metatheory is normal, it follows that

$$[\sim(A \in \Gamma) \vee \sim(\sim A \in \Gamma)]\ \&\ [(\sim A \vee B \in \Gamma) \rightarrow (\sim A \in \Gamma) \vee (B \in \Gamma)]$$
$$\rightarrow .\ (A \in \Gamma)\ \&\ (\sim A \vee B \in \Gamma) \rightarrow (B \in \Gamma).$$

Thus Mortensen concludes that consistency and primeness are sufficient to ensure that disjunctive syllogism(v) holds.

But to provide an object language formal reconstruction of informal arguments using disjunctive syllogism(or), the fact of closure under disjunctive syllogism(v) needs to be reflected in the object language itself. The obvious way to do this is to regard

$$(A \text{ is true}) \;\&\; (\sim A \vee B \text{ is true}) \rightarrow (B \text{ is true})$$

as sufficient grounds for the truth of

$$A \;\&\; (\sim A \vee B) \rightarrow B$$

in the case of a normal reasoning context. For this we simply require of such a context

$(C \;\&\; D \text{ is true}) \rightarrow (C \text{ is true}) \;\&\; (D \text{ is true})$
and $[(C \text{ is true}) \rightarrow (D \text{ is true})] \rightarrow [(C \rightarrow D) \text{ is true}]$.

Thus if a theory adequately captures the truths of a normal reasoning context, it is consistent and prime, and by Mortensen's argument is closed under disjunctive syllogism(v); and, by the above considerations should also contain $A \;\&\; (\sim A \vee B) \rightarrow B$.

So we can extend Mortensen's account as follows: The object language deductive argument begins with the implicational form of disjunctive syllogism(v) itself. Considerations about consistent and prime theories are simply part of the nondeductive persuasion that $A \;\&\; (\sim A \vee B) \rightarrow B$ is true in a particular reasoning context. Thus the corresponding formal reconstruction of informal uses of disjunctive syllogism is just

$$A \;\&\; (\sim A \vee B) \rightarrow B,\ A \;\&\; (\sim A \vee B) \vdash B.$$

Thus, $A \;\&\; (\sim A \vee B) \rightarrow B$ functions as an object language assumption of local consistency and primeness.

7 Local consistency and completeness suffice for the truth of b(A,B) In this section we use the type of metatheoretic argument displayed in the previous section and show that if a reasoning context is consistent and complete (locally), then $b(A,B)$ is true in it. Thus it follows that $b(A,B)$ ought be in the corresponding theory (exactly as in the case of $A \;\&\; (\sim A \vee B) \rightarrow B$).

Let τ denote the truths of a reasoning context, *cons* the conjunction of $(X \notin \tau) \vee (\sim X \notin \tau)$, and *comp* the conjunction of $(X \in \tau) \vee (\sim X \in \tau)$, where X ranges over $\{A, B, A \;\&\; B, A \;\&\; \sim B, \sim A \;\&\; B, \sim A \;\&\; \sim B, (\sim(A \;\&\; B) \rightarrow A \;\&\; \sim B)\}$. The argument has the following structure.

1. Assume that the metatheory is normal, so that all instances of $b(C,D)$ are true where C and D are sentences of the metatheory.
2. It follows that

   $$\textit{cons} \;\&\; \textit{comp} \;\&\; b((A \in \tau),\ (B \in \tau)) \rightarrow (A \;\&\; B \in \tau) + (A \;\&\; \sim B \in \tau) + (\sim A \;\&\; B \in \tau) + (\sim A \;\&\; \sim B \in \tau).$$

   And using our assumption 1 we can disjoin $b((A \in \tau),\ (B \in \tau))$ from the above antecedent ($\circ$ has been collapsed by this assumption to &).

3. As in the previous section we assume the following adequacy condition for the truth of $A \to B$:

$$(A \in \tau) \to (B \in \tau) \to. (A \to B) \in \tau.$$

This, together with *comp*, gives:

$$(A \in \tau) + (B \in \tau) \to (A + B \in \tau).$$

Applying this to the above, we get

cons & *comp* $\to (b(A,B) \in \tau)$

as required.

Thus consistency and completeness are sufficient guarantors that $b(A,B)$ is true in a reasoning context. This means that in order to show that the premise $b(A,B)$ of a formal reconstruction of some informal use of disjunctive syllogism(or) (on the locally Boolean plan) is true, we simply need to show that the reasoning context is consistent and complete.

8 Some facts In the next section I will show how use of the fact that a logic such as **E** or **R** is closed under γ can be justified using the locally Boolean assumptions. But first let me list some of the properties of $b(A,B)$.

Note that since $b(A,B) \to. A \,\&\, (\sim A \vee B) \to B$ is valid, a theory which contains all instances of $b(A,B)$ and the Tautological Entailments will contain all first-degree classical entailments (replacing $\supset$ by $\to$). Using the assumptions needed to prove $\vdash b(A,B) \to. A \vee B \to A + B$, together with full permutation and & and $\vee$ introduction for $\to$, we can prove

$$\vdash b(A \to C \to. A \to C, A \to C \to. A \to C) \,\&\, b(B \to B, B \to B)$$
$$\to. (A \to C \to. B \vee A \to B \vee C).$$

It follows that a consistent theory containing all instances of $b(A,B)$ collapses into the classical propositional calculus (with $\supset$ replaced by $\to$); as all elements of the Principia Mathematica formulation of propositional logic are available. So if our base logic is **R**, adding all instances of $b(A,B)$ delivers the classical propositional calculus.

Suppose that $D.S.(p,q)$ denotes the conjunction of all instances of $A \,\&\, (\sim A \vee B) \to B$ using only the set of atoms $\{p,q\}$. Then the following is not valid in **R**: $D.S.(p,q) \to b(p,q)$. So it doesn't follow from an assumption that disjunctive syllogism is true locally that the theory is locally Boolean.

With strong enough fission properties (such as in **R**), if $b(x,y)$ is in a theory Γ for all x,y members of some set of sentences S, where Γ is (as throughout) deductively closed, then $b(X,Y)$ is in Γ for all truth-functional (&, $\vee$, $\sim$) compounds X, Y of elements in S.

9 The relevantist can use γ In this section we show that the relevantist can use the γ result. This has been denied by some relevant logicians and remains a somewhat contentious issue. This provides a further example of the application of the locally Boolean approach to disjunctive syllogism(or).

1. Assume that we have a proof of γ, viz

$$\sim[\vdash\sim A \;\&\; \vdash A \vee B] \vee \vdash B$$

(adopting the consensus view that the closure proposition proved of **E** and **R** has only extensional connectives; i.e., it is not the stronger $\sim[\vdash\sim A \circ \vdash A \vee B] + \vdash B$).

2. We suppose that theoremhood (here denoted by '$\vdash$') is effective (as in the case of **E** and **R**).

We can use arguments along the lines of those put forward by Meyer ([6]) to justify the use of γ (in effect his target is metatheoretic disjunctive syllogism for statements of the form $\vdash A$), in order to justify the truth of $b(\vdash A, \vdash B)$. Since $\vdash$ is effective, it cannot be that for some sentence A both $\vdash A$ and $\sim(\vdash A)$ (A is both deducible and not deducible); nor can there be any sentence A such that neither $\vdash A$ nor $\sim(\vdash A)$ (neither A is deducible nor A is not deducible). That is, the set of sentences of the form $\vdash A$ is both consistent and complete, and clearly for any wffs A and B $b(\vdash A, \vdash B)$ is true. (Note that this reasoning fails where the negation is to the right of the $\vdash$; i.e., in the object language, which is precisely Wyberg's problem.) Using the last-mentioned fact of the above section, it follows that $b(\sim[\vdash\sim A \;\&\; \vdash A \vee B], \vdash B)$ is true for any pair of wffs A and B (this could have been argued for directly). Hence,

$$[\vdash\sim A \;\&\; \vdash A \vee B] \;\&\; (\sim[\text{ditto}] \vee \vdash B) \rightarrow \vdash B$$

is true (on minimal assumptions about the metatheoretic '$\rightarrow$' and assuming that our metatheory is deductively closed). By 1 the right conjunct of the above antecedent is true for all A, B; and if $[\vdash\sim A \;\&\; \vdash A \vee B]$ is true for some A and B then the antecedent above is true, and so the consequent follows. Applying this argument, if $\sim A$ and $A \vee B$ are theorems of (for example) **R** then it follows that B is a theorem of **R**.

10 Conclusion As mentioned in Section 1, it is incumbent upon the relevantist to give a satisfactory account of sound everyday use of informal disjunctive syllogism(or). The "locally Boolean" approach admirably does this for it renders disjunctive syllogism(v) enthymematically valid using a suppressed premise which expresses features of normality of reasoning contexts, and the relevantist claims that such normality is precisely what is presumed when disjunctive syllogism(v) is taken to be valid.

Clearly it is wrong to suppose that the relevantist must object to informal use of disjunctive syllogism(or). The relevantist's objection is rather against a particular formal reconstruction of such informal arguments (viz., disjunctive syllogism(v)). The argument of this paper demonstrates that the onus entailed by such an objection, to provide a satisfactory alternative account, can be amply fulfilled.

NOTES

1. This is not to say that $\sim A + B$ isn't true, for it may be true in virtue of background assumptions other than "not-A or B". See the discussion in Section 4 of a modification of the Anderson/early-Belnap account.

2. Here, as throughout, I allow formal schemata to be self-naming. The context makes it clear whether or not they are being mentioned, or used.

3. Here I take for granted that there is a distinction between extensional "or" (weakening "or") which fails to support an inference license, and intensional "or" which does have inferential force. The argument makes no headway against one who denies this distinction, believes disjunctive syllogism($\vee$) to be valid, and points to the fact that the extensional formulation of (α) is a tautology.

4. Lloyd Humberstone pointed out to me that one can use the weaker $(A \& B) * (A \& \sim B) * \sim A$, expressing local consistency of A and primeness of $\sim A \vee B$.

5. Burgess ([5]) refers to a sentence of this form in his criticism of Read [9]. But he claims it expresses the relevance of A to B, which is false as only the relevance of each of the conjunctions to the fissjunction of the remaining three is implied. (Thus undermining his criticism.)

6. "the" is in scare-quotes because there are an infinite number of Gödel sentences corresponding to the infinite number of codes with which one can arithmetize the metatheory of Peano arithmetic—all materially equivalent.

7. Consistency and completeness assumptions are built in to classical logic, and must be excised in order to avoid the fallacies of relevance and provide a logic capable of applying in all reasoning situations. This paper shows that the relevant reasoner can introduce these assumptions and so reach the world of Boolean wonders of the constrained classical reasoner. Of course the classical reasoner has made these assumptions but cannot express them in his or her impoverished logic. The relevant reasoner denies (contra the classical reasoner) that (β) is a logically necessary truth. The assumption of consistency and completeness is simply another premise, and not a prior condition for reasoning, guaranteed by Logic Itself.

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