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USE OF SINGULAR PERTURBATION METHOD TO FORMULATE ELECTRICAL NETWORK EQUATIONS

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ABSTRACT. Formulation of dynamic equations of a linear network using singular perturbation method is considered. It is well known that the differential order of a given network may be less than the number of energy storing elements it contains. This happens whenever a network contains (a) loops of capacitances and voltage sources only, (b) cutsets of inductances and current sources only and/or (c) coupling elements such as ideal transformers and gyrators in certain topologically degenerate manner. It is also known that these degeneracies can be corrected by adding small resistances in series or small conductances in parallel with certain branches of the given network. This facilitates the formulation of dynamic equations. Since small resistances and conductances are invariably present in any physical network, it is essential to know the dependence of the network's behavior on these small parameters.

It is pointed out here that singular perturbation theory can be used to analyze the limiting behavior of network state variables as the small parameters go to zero. For this purpose, dynamic equations need to be in the form

$$\frac{dx}{dt} = A_1x + A_2z + B_1u$$

$$\epsilon \frac{dz}{dt} = A_3x + A_4z + B_2u,$$

where x, z are state variable vectors, u is an input vector, A_1, A_2, A_3, A_4, B_1 and B_2 are matrices of appropriate dimension and ϵ denotes small parameters. If inductive currents and capacitive voltages are chosen as state variables, the resulting equations may not be in this form. However, it can be shown that from the knowledge of the topological degeneracy of a given network, a nonsingular transformation of variables can always be performed to put the equations in the required form.

Reduced order equations can be obtained by setting ϵ equal to zero. Tikhonov's standard theorem can then be used to obtain network theoretic conditions for which the solution of the reduced equations will approximate the behavior of the given network. This pro-

cedure gives explicitly not only the reduced order dynamic equations, but also the initial values of the state variables in a straightforward way. The initial values thus obtained are consistent with what can be obtained by using principles of conservation of charge and conservation of flux linkages.

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