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ASSOCIATED PRIMES OF LOCAL COHOMOLOGY AFTER ADJOINING INDETERMINATES. PART 2: THE GENERAL CASE

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ABSTRACT. Let A be a domain finitely generated as an algebra over a field, k of characteristic zero, $R = A[t_1, \ldots, t_\ell]$ or $A[[t_1, \ldots, t_\ell]]$, and I an ideal of R. If A has a resolution of singularities, Y_0, which is the blowup of A along an ideal of depth at least 2 and is covered by a finite number of open affines with $H^j(Y_0, \mathcal{O}_{Y_0})$ of finite length over A for $j > 0$, we prove that $\mathrm{Ass}_R H_I^i(R)$ is finite for every i. In particular, this holds when A is a finite-dimensional normal domain with an isolated singularity which is a finitely generated algebra over a field of characteristic 0.

1. Introduction. The problem of when the assassinators of local cohomology modules are finite has been widely studied for some time. Many mathematicians have looked at the general case where one considers the local cohomology of any module, but others have restricted their focus to the case where we only attempt to control the local cohomology of the ring itself. In this setting, the case of regular rings is particularly nice. If S is a regular ring of equal characteristic $p > 0$, Huneke and Sharp showed in **[3]** that $\mathrm{Ass}_S H_I^i(S)$ is always finite. In characteristic 0, Lyubeznik used the theory of D-modules in **[4]** to show that $\mathrm{Ass}_S H_I^i(S)$ is finite for regular domains, S, which are finite algebras over a field of characteristic 0.

My last paper **[7]** began exploring a way to extend these results on the finiteness of $\mathrm{Ass}_S H_I^i(S)$ to polynomial or power series extensions of a ring, whose resolution of singularities is covered by regular rings of the type studied by Lyubeznik. This allowed the use of techniques from algebraic geometry to link the good behavior of the regular rings covering the resolution of singularities to the polynomial or power series extensions of the base ring. In that paper, I considered base

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rings whose resolutions of singularities can be covered by two or three open affines, but here we extend those ideas to the case of a base ring whose resolution of singularities is covered by any finite number of open affines.

The main result of this paper is Theorem 3.1. In it, we take a Noetherian domain which is a finite algebra over a field of characteristic zero as our base ring, A. We form R by adjoining finitely many variables to A, either as polynomials or power series. If A has a blowup, Y_0, along an ideal of depth at least 2 which is covered by finitely many affine patches so that all higher cohomology of its structure sheaf $\mathcal{O}_{Y_0}$ has finite length over A, then Theorem 3.1 shows that $\mathrm{Ass}_R H^i_I(R)$ is always finite.

2. New material. We start by establishing some basic notation. Let k be a field of characteristic 0. Let A be a Noetherian domain finitely generated as a k-algebra, which has a resolution of singularities, Y_0, that is the blowup of A along an ideal of depth at least 2 and has an open cover by affine patches $U_1, \ldots, U_m$ so that $H^j(Y_0, \mathcal{O}_{Y_0})$ has finite length over A for all $j > 0$. We will let S_p be the regular ring corresponding to U_p.

Let $R = A[t_1, \ldots, t_\ell]$ or $A[[t_1, \ldots, t_\ell]]$, and let I be any ideal of R. Note that our proofs will be done in the polynomial case, but the same proofs work for power series.

Our first step in controlling the assassinators of the local cohomology modules of R with respect to I is to create two collections of double complexes.

Take generators of I so that $I = (f_1, \ldots, f_n) \subset R$. Let $A_0^{\bullet\bullet}$ be the double complex formed by tensoring the complex used to compute the sheaf cohomology of $\mathcal{O}_Y$,

$$0 \longrightarrow \oplus S_p \longrightarrow \oplus S_{pq} \longrightarrow \cdots \longrightarrow S_{1\cdots m} \longrightarrow 0,$$

with the complex used to compute local cohomology of R with respect to I,

$$0 \longrightarrow R \longrightarrow \oplus R_{f_i} \longrightarrow \oplus R_{f_i f_j} \longrightarrow \cdots \longrightarrow R_{f_1 \cdots f_n} \longrightarrow 0.$$

Thus, $A_0^{\bullet\bullet}$ is the complex given below, with the 0th column on the left and the $(m-1)$st column on the right.

$$
\begin{array}{ccccccccc}
 & & 0 & & 0 & & & 0 & \\
 & & \uparrow & & \uparrow & & & \uparrow & \\
0 & \longrightarrow & \left(\bigoplus_p S_p\right)_{f_1\cdots f_n} & \longrightarrow & \left(\bigoplus_{p<q} S_{pq}\right)_{f_1\cdots f_n} & \longrightarrow \cdots \longrightarrow & & (S_{1\cdots m})_{f_1\cdots f_n} & \longrightarrow 0 \\
 & & \uparrow & & \uparrow & & & \uparrow & \\
 & & \vdots & & \vdots & & & \vdots & \\
 & & \uparrow & & \uparrow & & & \uparrow & \\
0 & \longrightarrow & \bigoplus_k \left(\bigoplus_p S_p\right)_{f_k} & \longrightarrow & \bigoplus_k \left(\bigoplus_{p<q} S_{pq}\right)_{f_k} & \longrightarrow \cdots \longrightarrow & & \bigoplus_k (S_{1\cdots m})_{f_k} & \longrightarrow 0 \\
 & & \uparrow & & \uparrow & & & \uparrow & \\
0 & \longrightarrow & \bigoplus_p S_p & \longrightarrow & \bigoplus_{p<q} S_{pq} & \longrightarrow \cdots \longrightarrow & & S_{1\cdots m} & \longrightarrow 0 \\
 & & \uparrow & & \uparrow & & & \uparrow & \\
 & & 0 & & 0 & & & 0 &
\end{array}
$$

We filter this complex by subcomplexes $A_0^{\bullet\bullet}\langle k\rangle$, which are simply $A_0^{\bullet\bullet}$ with the first k rows replaced by zeros. Let E_0 be the associated graded complex with respect to this filtration, so that E_1 is the total complex of $A_1^{\bullet\bullet}$ where $A_1^{ij} = \bigoplus H^j(Y, \mathcal{O}_Y)_{f_{k_1}\cdots f_{k_i}}$, the horizontal maps are 0, and the vertical maps are induced by the vertical maps of $A_0^{\bullet\bullet}$.

Here, $d^1 : A_1^{ij} \to A_1^{i+1,j}$, so we are simply taking cohomology along each column. This means E_2 is the total complex of $A_2^{\bullet\bullet}$, where $A_2^{ij} = H_I^i(H^j(Y, \mathcal{O}_Y))$.

We can continue this process to create a set of double complexes,

$$A_1^{\bullet\bullet}, A_2^{\bullet\bullet}, \ldots, A_k^{\bullet\bullet}, \ldots, A_m^{\bullet\bullet},$$

where the differential at stage k is $d^k : A_k^{ij} \to A_k^{i+k,j-(k-1)}$.

Lemma 2.1. *A_k^{ij} is holonomic for all $k \geq 2$ and all $j > 0$.*

Proof. (By induction on k.) Our base case is $k = 2$, so we are looking at $A_2^{ij} = H_I^i(H^j(Y, \mathcal{O}_Y))$, where $j > 0$.

To show this module is holonomic, we use D-module methods. Let $D = k[t_1, \ldots, t_\ell, \partial_1, \ldots, \partial_\ell]$, where ∂_i is differentiation with respect to t_i. We can extend the action of $k[t_1, \ldots, t_\ell]$ on R to an action of D by setting $\partial_i(a) = 0$ for all $a \in A$.

As each S_p is generated over R by finitely many fractions of elements from R, we can define an action of D on them via the quotient rule. This D action extends to all localizations and is compatible with localization maps, making every module in $A_0^{\bullet\bullet}$ a D-module and all its maps D-module maps. In fact, all the modules and maps in every $A_k^{\bullet\bullet}$ will have a D-module structure.

Because $H^j(Y_0, \mathcal{O}_{Y_0})$ has finite length over A for $j > 0$, as a module over k it is a finite dimensional k-vector space. This means $H^j(Y, \mathcal{O}_Y) \cong H^j(Y_0, \mathcal{O}_{Y_0}) \otimes_k k[t_1, \ldots, t_m]$ is just a direct sum of copies of $k[t_1, \ldots, t_m]$, which is holonomic by [**4,** Property 2.2 (a) and Remark 2.9]. Thus, whenever $j > 0$, $A_2^{ij} = H_I^i(H^j(Y, \mathcal{O}_Y))$ is the local cohomology of a holonomic D-module and therefore is also holonomic by [**4,** Property 2.2 (d)].

Now assume that A_k^{ij} is holonomic for every $j > 0$.

We know A_{k+1}^{ij} is the cohomology of the map d^k at the ijth spot. Since d^k maps up k rows and left $k-1$ columns, we have

$$A_{k+1}^{ij} = \frac{\ker(d^k : A_k^{ij} \to A_k^{i+k, j-(k-1)})}{d^k(A_k^{i-k, j+(k-1)})}.$$

By our induction hypothesis, A_k^{ij} is holonomic. Since the kernel of d^k in the numerator is a D-submodule of A_k^{ij}, this means it is also holonomic by [**4,** Property 2.2 (c)].

Similarly, since $j + (k-1) > 0$, we have $A_k^{i-k, j+(k-1)}$ holonomic by our induction hypothesis. Since d^k is a D-module map, the image in the denominator is holonomic by [**4,** Property 2.2 (c)].

Therefore, A_{k+1}^{ij} is a quotient of holonomic of D-modules and hence is holonomic by [**4,** Property 2.2 (c)]. □

Lemma 2.2. *If $k \geq 3$ and $\mathrm{Ass}_R A_k^{i0}$ is finite, then $\mathrm{Ass}_R A_{k-1}^{i0}$ is also finite.*

Proof. We know $A_k^{i0} = \dfrac{A_{k-1}^{i0}}{d^{k-1}(A_{k-1}^{i-(k-1), k-2})}$. Since $k \geq 3$, we have $A_{k-1}^{i-(k-1), k-2}$ holonomic by Lemma 2.1. This means that its image under d^{k-1} is also holonomic, so $\mathrm{Ass}_R d^{k-1}(A_{k-1}^{i-(k-1), k-2})$ is finite.

Since $\mathrm{Ass}_R A_k^{i0}$ is finite, we see A_{k-1}^{i0} has a finite assassinator after a quotient by a module which also has a finite assassinator. Thus, $\mathrm{Ass}_R A_{k-1}^{i0}$ is finite as claimed. □

We can also filter our original double complex $A_0^{\bullet\bullet}$ by columns instead of rows. We will call the double complexes from this new filtration $B_k^{\bullet\bullet}$s to avoid confusion. Here, $B_0^{\bullet\bullet} = A_0^{\bullet\bullet}$, and $B_0^{\bullet\bullet}\langle k\rangle$ is just $B_0^{\bullet\bullet}$ with the first k columns replaced by zeros. Similar to our filtration by rows, let E_0 be the associated graded complex with respect to this new filtration. This makes E_1 the total complex of $B_1^{\bullet\bullet}$ where $B_1^{i0} = H_I^i(\oplus S_k)$, $B_1^{i1} = H_I^i(\oplus S_{k\ell})$ and so on up to $B_1^{i,m-1} = H_I^i(S_{1\cdots m})$. All vertical maps are 0, and the horizontal maps are induced by the corresponding maps in $B_0^{\bullet\bullet}$.

Here, $d^1 : E_1^{pq} \to E_1^{p,q+1}$, so, to get E_2, we are taking cohomology along each row. The ith row is the Čech complex which computes cohomology of the sheaf $\mathcal{H}_I^i(\mathcal{O}_Y)$ with respect to the cover of Y by $U_1, \dots, U_m$. Thus, E_2 is the total complex of $B_2^{\bullet\bullet}$, where $B_2^{ij} = H^j(Y, \mathcal{H}_I^i(\mathcal{O}_Y))$ and the differential is $d^2 : B_2^{ij} \to B_2^{i-1,j+2}$.

As with filtration by rows, we can continue this to create a set of double complexes

$$B_1^{\bullet\bullet}, B_2^{\bullet\bullet}, \dots, B_k^{\bullet\bullet}, \dots, B_{m-1}^{\bullet\bullet},$$

where our differential is $d^k : B_k^{ij} \to B_k^{i-(k-1),j+k}$.

Lemma 2.3. $\mathrm{Ass}_R B_k^{i0}$ *is finite for all* $k \geq 2$.

Proof. (By induction on k.) Our base case is $k = 2$, where $B_2^{i0} = H^0(Y, \mathcal{H}_I^i(\mathcal{O}_Y))$.

We know $H^0(Y, \mathcal{H}_I^i(\mathcal{O}_Y)) \subseteq H_I^i(S_1) \oplus \cdots \oplus H_I^i(S_m)$, where the S_p are all regular. This means $\mathrm{Ass}_{S_p} H_I^i(S_p)$ is finite for $1 \leq p \leq m$. Since the associated primes of $H_I^i(S_p)$ over R will be restrictions of the associated primes over S_p, we conclude that $\mathrm{Ass}_R B_2^{i0}$ is finite.

Now assume that $\mathrm{Ass}_R B_k^{i0}$ is finite.

We know that B_{k+1}^{i0} is the cohomology of the map d^k, which maps down $k-1$ rows and right k columns, at the $i0$th spot. The image of d^k is 0 at this spot, so we have

$$B_{k+1}^{i0} = \ker(d^k : B_k^{i0} \longrightarrow B_k^{i-(k-1),k}) \subseteq B_k^{i0}.$$

Since $\mathrm{Ass}_R B_k^{i0}$ is finite by our induction hypothesis, this means $\mathrm{Ass}_R B_{k+1}^{i0}$ is finite as well, and we are done. □

3. Main result. Our main theorem shows that the local cohomology of our polynomial or power series ring R with respect to any ideal $I \subseteq R$ has only finitely many associated primes. For ease of reference, all our assumptions will be restated in the statement of Theorem 3.1.

We will act in the proof as if the number of affine patches covering the resolution of singularities Y_0 is three or more. For the case of two affine patches, see [**7,** Theorem 2].

Theorem 3.1. *Let A be a domain, finitely generated as an algebra over a field, k of characteristic 0, and let $R = A[t_1, \ldots, t_\ell]$ or $A[[t_1, \ldots, t_\ell]]$. If A has a resolution of singularities, Y_0, which is the blowup of A along an ideal of depth at least 2 and has an open affine cover by $U_1, \ldots, U_m$, where $H^j(Y_0, \mathcal{O}_{Y_0})$ has finite length over A for all $j > 0$, then $\mathrm{Ass}_R H_I^i(R)$ is finite for any i and any ideal $I \subset R$.*

Proof. Consider our double complexes $A_k^{\bullet\bullet}$. From [**7,** Lemma 1], we get $H^0(Y, \mathcal{O}_Y) \cong R$. This means $A_2^{i0} = H_I^i(H^0(Y, \mathcal{O}_Y)) \cong H_I^i(R)$, so we are really just interested in controlling the associated primes of A_2^{i0} for every i.

To do this, let us consider the last double complex in this sequence: $A_m^{\bullet\bullet}$. Our differential here is $d^m : A_m^{ij} \to A_m^{i+m,j-(m-1)}$. There are only m nonzero columns, so this map is only nontrivial if $j = m-1$, i.e., $d^m : A_m^{i,m-1} \to A_m^{i+m,0}$. This means that our associated graded complex $E_{m+1}^{\bullet\bullet}$ has

$$E_{m+1}^{i0} = \frac{A_m^{i0}}{d^m(A_m^{i-m,m-1})},$$
$$E_{m+1}^{ij} = A_m^{ij} \quad \text{for all } 1 \le j \le m-2,$$

and

$$E_{m+1}^{i,m-1} = \ker\left(d^m : A_m^{i,m-1} \longrightarrow A_m^{i+m,0}\right).$$

Since we have $d^r \equiv 0$ for all $r \ge m+1$, meaning that $E_{m+1} = E_\infty$, we get an exact sequence for every i,

$$0 \longrightarrow E_\infty^{i-m,m-1} \longrightarrow A_m^{i-m,m-1} \longrightarrow A_m^{i0} \longrightarrow E_\infty^{i0} \longrightarrow 0,$$

where the middle map is d^m.

Letting $\mathcal{T}^\bullet$ denote the total complex of E_∞, then $\mathcal{T}^i = E_\infty^{i0} \oplus E_\infty^{i-1,1} \oplus \cdots \oplus E_\infty^{i-(m-1),m-1}$, gives us an exact sequence

$$\mathcal{T}^{i-1} \longrightarrow A_m^{i-m,m-1} \longrightarrow A_m^{i0} \longrightarrow \mathcal{T}^i,$$

where the first map is a projection onto $E_\infty^{i-m,m-1}$, the middle map is d^m and the last map comes from the projection of A_m^{i0} onto $E_\infty^{i0} \subseteq \mathcal{T}^i$.

We can therefore think of A_m^{i0} as the center of the short exact sequence

$$0 \longrightarrow d^m \left(A_m^{i-m,m-1}\right) \longrightarrow A_m^{i0} \longrightarrow \operatorname{im}\left(A_m^{i0}\right) \longrightarrow 0,$$

where $\operatorname{im}(A_m^{i0})$ is the image of A_m^{i0} inside $\mathcal{T}^i$.

Since $m > 3$, and hence $m-1 > 0$, Lemma 2.1 implies that $A_m^{i-m,m-1}$ is holonomic. This means that its image under the D-module map d^m is also holonomic, and hence, $d^m(A_m^{i-m,m-1})$ has finitely many associated primes. Thus, to show $\operatorname{Ass}_R A_m^{i0}$ is finite, we need only control $\operatorname{Ass}_R \operatorname{im}(A_m^{i0})$.

To do this, we look at the $B_k^{\bullet\bullet}$s. The last nonzero such complex is $B_{m-1}^{\bullet\bullet}$, where our differential is $d^{m-1} : B_{m-1}^{ij} \to B_{m-1}^{i-(m-2),j+(m-1)}$. Since there are only m nonzero columns, the differential is the zero map except for $d^{m-1} : B_{m-1}^{i0} \to B_{m-1}^{i-(m-2),m-1}$. This means that E_m is the total complex of $B_m^{\bullet\bullet}$, where

$$B_m^{i0} = \ker(d^{m-1} : B_{m-1}^{i0} \to B_{m-1}^{i-(m-2),m-1}),$$
$$B_m^{ij} = B_{m-1}^{ij} \quad \text{for all } 1 \le j \le m-2,$$

and

$$B_m^{i,m-1} = \frac{B_{m-1}^{i,m-1}}{d^{m-1}(B_{m-1}^{i+(m-2),0})}.$$

Because $d^m \equiv 0$, we have $E_\infty = E_m$. For each i, this gives us the short exact sequence

$$0 \longrightarrow E_\infty^{i0} \longrightarrow B_{m-1}^{i0} \longrightarrow B_{m-1}^{i-(m-2),m-1} \longrightarrow E_\infty^{i-(m-2),m-1} \longrightarrow 0,$$

where the middle map is d^{m-1}.

Again, letting $\mathcal{T}^\bullet$ be the total complex of E_∞, we have

$$\mathcal{T}^i = E_\infty^{i0} \oplus E_\infty^{i-1,1} \oplus \cdots \oplus E_\infty^{i-(m-1),m-1}.$$

This means that $\mathcal{T}^i$ maps onto $E_\infty^{i0} = B_m^{i0}$ via the short exact sequence

$$0 \longrightarrow B_m^{i-1,1} \oplus \cdots \oplus B_m^{i-(m-1),m-1} \longrightarrow \mathcal{T}^i \longrightarrow B_m^{i0} \longrightarrow 0.$$

Since A_m^{i0} maps to $\mathcal{T}^i$, its image in $\mathcal{T}^i$ also maps to B_m^{i0}, and the kernel of this mapping is the intersection of the images of $\operatorname{im}(A_m^{i0})$ and $B_m^{i-1,1} \oplus \cdots \oplus B_m^{i-(m-1),m-1}$ inside $\mathcal{T}^i$.

Lemma 3.2. *The images of A_m^{i0} and $B_m^{i-1,1} \oplus \cdots \oplus B_m^{i-(m-1),m-1}$ inside $\mathcal{T}^i$ have a trivial intersection.*

Proof. Elements of $\mathcal{T}^i$ are equivalence classes of m-tuples $(z_0, \ldots, z_{m-1})$ in $A_0^{i0} \oplus \cdots \oplus A_0^{i-(m-1),m-1}$, where $z_0 \mapsto 0 \in A_0^{i+1,0}$ and $\operatorname{im}(z_k) = \operatorname{im}(z_{k+1}) \in A_0^{i-k,k+1}$ for all $0 \leq k \leq m-2$.

The module A_m^{i0} comes from our filtration by rows, so its elements can be thought of as equivalence classes of elements $z \in A_0^{i0}$ for which $z \mapsto 0 \in A_0^{i1}$ and $z \mapsto 0 \in A_0^{i+1,0}$. Therefore, elements of A_m^{i0} map to $\mathcal{T}^i$ by $[z] \mapsto [(z, 0, \ldots, 0)]$.

Turning our attention to $B_m^{\bullet\bullet}$, the piece $B_m^{i-k,k}$ for each $1 \leq k \leq m-2$, comes from the filtration by columns. This means it consists of equivalence classes of elements $w_k \in B_0^{i-k,k}$, where

$$w_k \longmapsto 0 \in B_0^{i-k+1,k}$$

and

$$w_k \longmapsto \operatorname{im}(B_0^{i-k-1,k+1}) \subseteq B_0^{i-k,k+1},$$

so we have an element $w_{k+1} \in B_0^{i-k-1,k+1}$ with $\operatorname{im}(w_k) = \operatorname{im}(w_{k+1}) \in B_0^{i-k,k+1}$. Thus, the map $B_0^{i-k,k} \to \mathcal{T}^i$ is just

$$[w_k] \longmapsto [(0,\dots,0,w_k,w_{k+1},0,\dots,0)],$$

where w_k and w_{k+1} appear in the kth and $k+1$th spots, respectively.

Our last module, $B_m^{i-(m-1),m-1}$ also comes from the filtration by columns, so its elements are equivalence classes of elements $w_{m-1} \in B_0^{i-(m-1),m-1}$, where

$$w_{m-1} \longmapsto 0 \in B_0^{i-(m-1)+1,m-1}.$$

This means that our map $B_m^{i-(m-1),m-1} \to \mathcal{T}^i$ sends

$$[w_{m-1}] \longmapsto [(0,\dots,0,w_{m-1})].$$

Since $\operatorname{im}(A_m^{i0})$ has entries only in the 0th component, while the images of the $B_m^{i-k,k}$s have entries only in the first through $(m-1)$st components, it is clear that the images of A_m^{i0} and $B_m^{i-1,1} \oplus \cdots \oplus B_m^{i-(m-1),m-1}$ have a trivial intersection. □

Lemma 3.2 tells us that $\operatorname{im}(A_m^{i0}) \subseteq \mathcal{T}^i$ is a submodule of $\operatorname{im}(\mathcal{T}^i) \subseteq B_m^{i0}$. Since, by Lemma 2.3, we know $\operatorname{Ass}_R B_{m-1}^{i0}$ is finite, this means that $\operatorname{Ass}_R \operatorname{im}(A_m^{i0})$ is also finite. Therefore, $\operatorname{Ass}_R A_m^{i0}$ is finite as well.

Now that we know $\operatorname{Ass}_R A_m^{i0}$ is finite, we can repeatedly apply Lemma 2.2 to show that $\operatorname{Ass}_R A_2^{i0}$ is finite. Since $A_2^{i0} \cong H_I^i(R)$, this means $\operatorname{Ass}_R H_I^i(R)$ is finite for all i as claimed. □

The following corollary is a special case of the previous theorem whose assumptions are perhaps more familiar.

Corollary 3.3. *Let A be a finite-dimensional normal domain with an isolated singularity, where A is finitely generated as an algebra over a field of characteristic* 0. *If $R = A[t_1,\dots,t_\ell]$ or $A[[t_1,\dots,t_\ell]]$, then* $\operatorname{Ass}_R H_I^i(R)$ *is finite for any i and any ideal I of R.*

Proof. Let $\mathfrak{m} \subset A$ be the maximal ideal which defines the non-singular locus of A. Since $\dim(A) = m$ is finite, we know $\mathfrak{m}$ is generated, up to radical, by at most m elements. Let Y_0 be the blow-up of A along $\mathfrak{m}$. It is clear that Y_0 is covered by at most m affine patches corresponding to the generators of $\mathfrak{m}$, and also that $\operatorname{depth}_{\mathfrak{m}} R \geq 2$

since $\mathrm{ht}(\mathfrak{m}) \geq 2$ and R is normal. Finally, we know that the higher cohomology of the structure sheaf of A's desingularization will consist of finitely generated A-modules supported only on the singular locus of A. Since $\mathrm{Sing}\,(A) = \{\mathfrak{m}\}$, this means all higher cohomology of Y_0 is killed by some power of $\mathfrak{m}$, and hence is of finite length over A. Theorem 3.1 now implies that $\mathrm{Ass}_R H^i_I(R)$ is finite. □

The only part of the proof of Theorem 3.1 which uses the fact that we are in characteristic 0 is the D-module theory needed for Lemma 2.2. (We also use D-module theory in the proof of Lemma 2.1, but this lemma is used only in the proof of Lemma 2.2, not in the proof of Theorem 3.1.) In equal characteristic $p > 0$, Lyubeznik and others have successfully used the theory of F-modules to control the local cohomology of regular local rings in a way analogous to the use of D-modules in characteristic 0, see **[5]**. Since many results about local cohomology which are proved in characteristic 0 using D-modules can be proved in equal characteristic $p > 0$ using F-modules, it would be interesting to see if Lemma 2.2 can be proven using F-module theory. If it were possible, this would extend Theorem 3.1 to the equal characteristic p case.

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