

A Conversation with Sujit Kumar Mitra

Nitis Mukhopadhyay

Abstract. Sujit Kumar Mitra was born on January 23, 1932, in Calcutta, India. He earned his B.Sc. degree in statistics from Presidency College, Calcutta, in 1949, an M.Sc. degree in statistics from Calcutta University in 1951 and a Ph.D. degree in statistics from the University of North Carolina at Chapel Hill in 1956, under the guidance of Professor S. N. Roy. He has made pioneering contributions in many areas of statistics and mathematics—including survey sampling, linear models, design of experiments, goodness-of-fit tests and linear algebra. He has been particularly credited for path-breaking contributions in the area of generalized inverses of matrices, culminating in a jointly authored landmark book with Professor C. R. Rao (published in 1971). He was Professor at the Indian Statistical Institute (ISI), both in Calcutta and Delhi, for almost 40 years. He has held visiting positions at Indiana University, Purdue University, University of Texas at Dallas, and Keio University, Japan. He retired from ISI in January 1992 and is currently Professor Emeritus. Unfortunately, he contracted Parkinson's disease in 1978. Despite an uphill battle against constant physical discomfort, he has continued as a leading contributor in many directions of mathematics and statistics. He is well known for his zest in attacking and solving some of the most difficult problems in linear algebra. He has earned many awards, honors and titles, including Fellow of the Indian National Science Academy and of the Indian Academy of Science. He was also elected President of the Statistics Section of the Indian Science Congress in 1988. He has been associated with many journals, including *Sankhyā*, and has edited or coedited several special volumes.

The following conversation took place on August 1, 1995, first at the Department of Statistics, University of Connecticut, Storrs, and then continued at the Mukhopadhyay residence in Manchester, Connecticut.

EARLY UPBRINGING AND FAMILY HISTORY

Mukhopadhyay: Let us start at the very beginning. Would you give a few details about your parents and your childhood?

Mitra: My maternal grandparents lived in Calcutta where I was born. My father and grandfather used to live in Jajpur town, Orissa, and both were lawyers in the subdivisional court in the same town.

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I studied in the local school. However, since the instructional medium was in the Oriya language, which conflicted with my mother-tongue which is Bengali, I skipped most of the elementary grades and sought direct admission to the eighth grade through an entrance examination. I passed the matriculation examination in the year 1944.

Mukhopadhyay: Was that from the same school in Orissa?

Mitra: Yes, it was in Orissa. In fact, that was the first batch of students qualified to appear in matriculation examination under the newly formed Utkal University.

Mukhopadhyay: Were mathematics and mathematical subjects emphasized a lot?

Mitra: Actually I was fortunate to have a good mathematics teacher, Mr. Ballav Panda, who was very inspiring. In fact he possibly taught mathematics to my father also, but I am not one hundred percent sure about that. The other person who in-



FIG. 1. With parents and sisters in home town, December 1979.

spired me was my mother. Even though she did not sit for the matriculation examination, she was very good in arithmetic and geometry and I learned a lot from her about mathematics.

Mukhopadhyay: Do you have brothers and sisters? Can some of them also be called mathematicians?

Mitra: We had a large family. We were five brothers and five sisters. I am the eldest of them all. Among the brothers, the next in parity is an electrical engineer from the B.E. College in Shibpore, West Bengal. The third brother is a professor of electrical engineering at the University of California at Santa Barbara, and the fourth brother is an accountant. My youngest brother is a chemical engineer.

Mukhopadhyay: It seems that some sort of mathematical trend runs among your brothers. What about your sisters?

Mitra: The local school where we studied was then exclusively for boys. Comparable facilities for girls were lacking. The alternative would have been to send the girls to a larger city like Cuttack, the capital of the State of Orissa. But with the limited resources of the family, and firmly committed to upbringing the first three children (all boys), this was ruled out. All my sisters passed the matriculation or equivalent examinations as private candidates and eventually earned B.A. degrees in humanities. My youngest sister married a structural engineer in Los Angeles and, with three children, took care of the family and found the time to earn her B.S. degree in computer science at UCLA. She now holds a part-time job as a programmer.

Mukhopadhyay: While you were at school, did you show some particular aptitude or liking of mathematical and quantitative ideas?

Mitra: I was good in mathematics and I was equally good in the Sanskrit language. When I moved to grade nine, I had a dilemma whether to take mathematics or Sanskrit as the additional subject, and I almost decided in favor of Sanskrit. My father intervened and said that if I did not wish to starve, I had better take up mathematics instead, and of course I chose mathematics.

Mukhopadhyay: It looks as if mathematics and statistics had almost lost a great mind to Sanskrit.

Mitra: (laughs) If I had pursued the Sanskrit route, by now I would probably be a pundit of some sort somewhere. Since there were not enough students who could be admitted to the colleges, those of us who were in the final year of the various schools in the state were given a one time option. If so recommended by the Headmaster of the school, we could sit in the special final exam within the next six months instead of waiting another year. Since I ranked first in the mock examination which the school conducted for this purpose, the school approved my appearing in the special matriculation examination, along with 11 others of my classmates.

COLLEGE LIFE

Mukhopadhyay: After you finished your matriculation, did you go straight to college?

Mitra: When I passed the matriculation examination I was a little over 12 years of age. My father thought I was too young to go to college right away, particularly because there was no college nearby where we lived. So I ended up wasting one year, sort of fooling around. A year later (1945) I joined the St. Paul's College in Calcutta, where I studied for two years. In 1947, I passed the Intermediate Science degree examinations conducted by Calcutta University. Then I went to Presidency College in Calcutta, where I enrolled in their Statistics Honors program.

Mukhopadhyay: So by then a regular statistics department had already been in existence at the Presidency College. Who headed that department?

Mitra: Professor Anil Bhattacharyya was the head at that time. This is the same Bhattacharyya of the Bhattacharyya bounds and Bhattacharyya inequalities.

Mukhopadhyay: Was there someone in particular, or perhaps a book or a paper, that influenced you to change your college course of study from mathematics to statistics?

Mitra: That is an interesting story really. In the Intermediate Science examination, again I did quite



FIG. 2. Married Sheila in May 1958; photo dated August 1958.

well in mathematics as well as chemistry, and I was not aware of a statistics program at the Presidency College. So I applied for the Chemistry Honors program and was selected for admission. Meanwhile, my uncle, who was perhaps two or three years senior to me, wrote to me suggesting that I try for admission in the statistics program. My uncle was a student in the Geology Honors program at the Presidency College at that time, and he mentioned this new program in Statistics Honors which is why I opted to join. I recall that I was in their third or fourth batch of graduating students from the Honors Program. Before I went there, I was absolutely ignorant about what statistics was all about or what I was going to learn there. In spite of that I applied for enrollment as a student in the program. I feel like I should mention a particular episode for the record.

Mukhopadhyay: Please go right ahead.

Mitra: I already told you that I applied for the Chemistry Honors program at the Presidency College and I was selected. When I submitted my application there for the Statistics Honors program, I missed the deadline for the application by a day or two. I felt bad thinking that I had missed the opportunity to study statistics at this college. It turned out that the manager of Nath Bank (located nearby in the College Street market), where the Indian Statistical Institute (ISI) had an account, was a close friend of my grandfather's. One day he saw me walking around, fairly depressed, and asked me if I was truly serious about getting into the statistics program. I mentioned that I had applied there but it looked like I missed the application deadline. "Alright then," he said, "let me talk to some people in the college." I did not know at that time that ISI had drawn a large overdraft from Nath Bank. In other

words, the Nath Bank could exert some pressure on the statistical community in that college. So when the manager of the bank spoke with appropriate people in the college, and since my examination results were good, the requirement to meet the strict application deadline was waived. For a long time this had been pinching me inside because in a way my admission to the Statistics Honors program at Presidency College was achieved through the "back door." I was glad that eventually I did well and secured the top rank in the final M.Sc. examination. In a way that justified the "back door" admission.

Mukhopadhyay: So then you continued in statistics, I assume. You finished your course work at the Presidency College in two years?

Mitra: Yes, I received the B.Sc. (Honors) degree in statistics and then I went for the two-year Masters degree program in statistics at the Calcutta University in 1949.

Mukhopadhyay: At Calcutta University, did you come in contact with any individual who significantly influenced your future research career?

Mitra: Yes. Among my teachers, I was particularly impressed by Professor C. R. Rao, who was a part-time lecturer at the University. Professor H. K. Nandi was also very influential. Incidentally, Professors Nandi and Rao were classmates and they appeared in the M.Sc. examination at the same time.

Mukhopadhyay: Will you please elaborate on the teaching styles of these two striking personalities?

Mitra: Both were excellent teachers, each in his own characteristic way. Professor Nandi came to class fully prepared. His treatment was rigorous, complete and left nothing to the imagination. From time to time he would update his teaching material by including recent contributions to a topic that had not yet found a place in the textbooks. We were taught, for example, Mann and Wald's optimum choice of the number of class-intervals in a χ^2 goodness-of-fit test, and Wald's treatment of fitting a straight line when both variables are subject to error. Professor Rao always had something new and original to say. Constantly innovating and improving his own results, his classes were enjoyable to all those who put in the extra effort that was needed to complete his sketches of proofs.

Mukhopadhyay: Who was the Head of the Statistics Department at the Calcutta University? Who were some of your contemporaries?

Mitra: Professor P. K. Bose, who later became one of the Pro-Vice-Chancellors of the University, was the Head. Among my colleagues, R. G. Laha was in my batch. I. M. Chakravarti, J. Roy and Kamal Chanda were each one year senior to me.

Chakravarti, Roy and I were living at the Eden Hindu Hostel, which is similar to a dormitory in the U.S.A.—situated right between the Presidency College and Calcutta University. Also, at times we were living at the postgraduate hostel near the Raja Bazaar tram depot.

Mukhopadhyay: During that period was there any particular book or work in statistics that really inspired you?

Mitra: At that time we had Cramér's book, *Mathematical Methods of Statistics*, and Kendall's two-volume work, *The Advanced Theory of Statistics*. I must say that I was more impressed with Cramér's book because of its mathematical rigor. It was the first time I had seen such rigorous treatment. Wilks' book, *Mathematical Statistics*, was available to us, but Cramér's book was used more frequently. A few years later, C. R. Rao's book, *Advanced Statistical Methods in Biometric Research*, came out. Another book that was available was by F. Yates, *Sampling Methods for Censuses and Surveys* (published in 1949). That was an impressive piece of work. When I read Yates' book, I could feel that I was learning the material from an expert.

Mukhopadhyay: What was the status of books in the area of probability theory? Did you follow any particular book while you were a student?

Mitra: We only had J. V. Uspensky's book, *Introduction to Mathematical Probability* (published in 1937), which was a fairly good book. We were taught from that book. Feller's book, *An Introduction to Probability Theory and Its Applications*, volume 1, appeared after our Master's examination. Shortly after that, Loeve's book, *Probability Theory*, became available.

EARLY CALCUTTA ISI CONNECTIONS

Mukhopadhyay: After receiving your M.Sc. degree in 1951 from Calcutta University, did you join the group at ISI immediately or did you pursue your doctorate degree in statistics? I am a little confused on that.

Mitra: I finished my last practical paper for the M.Sc. examination in November 1951, and I was told to apply for a job at the Indian Statistical Institute. Actually, both Laha and I were told to apply for a job there.

Mukhopadhyay: Who asked you to apply for a job at ISI, and when was this?

Mitra: Let me backtrack. I have missed one important personality while describing the individuals who inspired me the most at early stages. Apart from Professors Nandi and Rao, my uncle, Debabrata Basu, the author of Basu's theorem,

truly influenced me a lot. Basu is my mother's first cousin and he had a mathematical background. He was learning statistics through self-study.

Mukhopadhyay: Was Dr. Basu then affiliated with the Statistical Laboratory (the birthplace of ISI) at the Presidency College?

Mitra: He joined the Statistical Laboratory one year earlier than me. During that period, practically every evening after the college was over and done with, I used to walk down to Basu's house on Harrison road (later renamed as Mahatma Gandhi Road) and then Basu and I would sit down to have lengthy discussions on statistical matters. Let me recall a story. You may know that many of us in Calcutta used to put coconut oil on our head before taking a shower. This was a usual practice to keep one's head cool in otherwise hot weather, but then most of us would end up having very oily hair. In the living room of Basu's house where we used to sit and discuss things for hours at a time—on the wall there were two very distinct imprints of the back of our heads from leaning against the wall. My grandfather often pointed at those two marks and identified one as D. Basu and the other one as Sujit.

Mukhopadhyay: I know that fingerprints almost uniquely identify an individual. Now it seems that oily imprints of the back of one's head can also be used sometimes for the same purpose!

Mitra: (laughter) D. Basu really influenced me to go and join ISI. Also, he taught me how to learn a new subject well. He used to say that one has to stop taking notes during a class, because those who are busy taking notes would miss out on most of the things the professor has to say. He told me to listen to a professor's explanations, motivations, examples and counterexamples in order to learn from a class. Basu emphasized that these were the real important aspects in a class because these are usually not included in the book or a paper. The basic materials can always be found from a book later. I immediately started following Basu's advice.

Mukhopadhyay: I recall with great pride and joy that both Dr. Basu and you were my teachers at ISI in Calcutta when I was a Master of Statistics degree student during 1970–72. Which year did you join ISI and what happened after that?

Mitra: I joined the Statistical Laboratory in January 1952 and remained there for two years. For about six months or so I worked in the Statistical Laboratory inside the Baker Laboratory Building of Presidency College, and I spent the rest of my time on the main campus of ISI at Baranagar, Calcutta, which was then under construction. In September

1954, I went to the University of North Carolina at Chapel Hill. I took a leave of absence from ISI to travel to Chapel Hill in order to pursue a Ph.D. degree in statistics.

CHAPEL HILL YEARS

Mukhopadhyay: Why did you choose the University of North Carolina at Chapel Hill?

Mitra: The statistics program at Chapel Hill had an array of some of the most famous faculty members, including Professors H. Hotelling, S. N. Roy and R.C. Bose. Professor W. Hoeffding also joined the faculty just around that time. The quality of their program attracted me to Chapel Hill. Actually, I also wrote to Princeton and Berkeley, but the application materials from Berkeley were sent to me via sea-mail and arrived in Calcutta after the application deadline was over. Naturally, I could not apply for admission to Berkeley. In the case of Princeton, it is possible that my application for admission to graduate school with financial support was turned down. My memory seems to have faded.

Mukhopadhyay: In September 1954 you arrived at Chapel Hill. Among your fellow graduate students were there any from India?

Mitra: Among the student body there were several from India. I met R. Gnanadesikan and Shanti Gupta. K. V. Ramachandran and Donald Burkholder were then in the concluding stages of their doctoral work. V. N. Murty joined UNC at the same time I did. However, he later returned to his position at the Central Statistical Organization, New Delhi. There were two graduate students from Pakistan—Moinuddin Siddiqui and M. Iqbal. Iqbal returned to Pakistan, while Moin joined the faculty at Colorado State University, Fort Collins. Other names that I recall are Seymour Geisser, presently at Minnesota, and Dick Carter, who served at Rensselaer Polytechnic in New York for a few years before his untimely death. Among other fellow students, I met Norman Draper, who later joined the University of Wisconsin; John Wilkinson, who wrote his Ph.D. thesis under R. C. Bose on paired comparison designs and later joined the Rensselaer Polytechnic; Donald Richter, who later went to New York University; and Donald Morrison, who is still at the Wharton School at the University of Pennsylvania.

Mukhopadhyay: As a Ph.D. student at Chapel Hill, who did you take the majority of your courses from?

Mitra: From Professor S. N. Roy I took courses on multivariate analysis, and from Professor Bose, I audited a course on the design of experiments. I

also took courses from Professor Hoeffding on inference, parametric as well as nonparametric. Professor Walter Smith taught me a course on estimation. I also remember taking a very interesting course from Professor Hotelling on approximation methods.

Mukhopadhyay: Would you care to add any remarks about your teachers at Chapel Hill? What were their personalities like? Eventually, you started working under Professor Roy, right?

Mitra: I found Hotelling most interesting and his lectures were really enjoyable. When I was taking courses from Hotelling, Hoeffding and others, I felt that I was really learning the materials from maestros who themselves had developed, or were in the process of developing, much of the topics they were teaching me. My financial support came from a U.S. Air Force grant for Professor Roy's projects and I started working with him.

Mukhopadhyay: How did you get started on your thesis work?

Mitra: At that time, Professor Roy was advising Marvin Kastenbaum on analyzing data on the atom bomb casualties from Nagasaki and Hiroshima. That was mostly categorical data analysis. I sat through some of their discussions, and eventually I worked on the problem almost completely by myself. When Professor Roy was going through Marvin's thesis, I had already drawn an outline of my own and obtained some results which included a heuristic justification for the goodness-of-fit χ^2 test based on Roy's union-intersection principle. My paper with Professor Roy appeared in *Biometrika* (Roy and Mitra, 1956).

Mukhopadhyay: Outside the classrooms, what was the general lifestyle in and around the department at Chapel Hill?

Mitra: All the teachers at Chapel Hill were very friendly. I recall that Hotelling organized tea parties on the first Saturday of each month at his home in which all graduate students and faculty were invited. Bose and Roy used to invite all the graduate students to their homes for Indian food, once each semester. I certainly enjoyed these informal get-togethers. During the tea parties, Professor Hotelling would often tell jokes, one of which I particularly remember. At the Hotelling residence, in almost every room, one would find packets of cigarettes and match boxes, but Hotelling did not smoke. Once he remarked, "What I don't smoke, my wife makes up for both of us, thus we are able to maintain the U.S. average."

Mukhopadhyay: (laughter) Since Professor Roy was your thesis advisor, you certainly had close contacts with him. Any recollections about Roy's personal and human side?



FIG. 3. With Professor S. N. Roy and Mrs. Bani Roy at Chapel Hill, November 1954: (standing, from right to left) K. V. Ramachandran, V. N. Murty, M. Iqbal and S. K. Mitra; (front) S. S. Gupta.

Mitra: Professor Roy was a warm-hearted person with a humility that only knowledge and education can impart. I arrived at Chapel Hill one evening in mid-September 1954. The next morning I met Professor Roy in his office in Phillips Hall which he used to share with Professor R. C. Bose. He gave several pertinent pieces of advice—one of which I can hardly forget. “Sujit,” he said, “I think for some time you should get up early so that you can use the bath and toilet before others on the same floor of the dormitory wake up.” He did not elaborate any further, but the implication was clear to me in a couple of days. One had to wait in a long line once everyone in the dorm woke up! During the same Fall semester, with the advent of cold weather, I became ill with the flu. As soon as he learned about my illness, Professor Roy came to my dormitory room with a thermos full of warm (Horlicks) drink and some fruit. In traditional Indian society the Guru–Sishwa (Teacher–Disciple) relationship is not much different from that of the father and his son. To be defeated by his student is a noble wish fondly held by the Guru.

In the early 1960s, Professor Roy and his family came for a visit to Calcutta to see his younger brother unmistakably in death-bed. Even though I was aware of his busy schedule, I picked up

courage and invited him and his family for dinner at my home. Professor Roy accepted the invitation. What stunned me even more was that upon his arrival, when I introduced him to my mother, he bent forward to touch my mother’s feet. “Janani-Janmabhūmischa Swargadapi Garioshi” (the mother and the motherland are more sublime than even the Heaven). No one could imagine that this would turn out to be Professor Roy’s last visit to the country where he was born and raised.

RETURN TO CALCUTTA ISI

Mukhopadhyay: After graduating from Chapel Hill did you immediately return to ISI, Calcutta, or did you spend some time elsewhere? Did you join the Research and Training School at ISI as a faculty member?

Mitra: I mentioned earlier that I went to Chapel Hill under a leave of absence from ISI. So right after graduation from Chapel Hill, I returned to ISI at Calcutta around August or September 1956. At that time there were no division of ranks amongst faculty members. Everyone was called a worker, and so I joined ISI as a worker—there were no professors or lecturers. Incidentally, I would like to share a particularly interesting story.

Mukhopadhyay: Please tell the story.

Mitra: Professor Mahalanobis occasionally used to organize gatherings at the large open deck of his home, Amrapali, which was located in the middle of the ISI, Calcutta, campus. These were very informal meetings where Professor Mahalanobis chatted with us. During some of these discussions, I got the impression that Professor Mahalanobis was not in favor of his students going abroad for doctoral programs. So once I arrived in Chapel Hill, I was worried as to whether I had made the right decision, because if I wished to go back and work for ISI, then I should not displease Professor Mahalanobis. I thought that I might already have displeased him by going to Chapel Hill for my Ph.D. degree. After spending a few days in the United States, I felt very homesick and that, coupled with my misapprehension about Mahalanobis, made me decide to go back to Calcutta as soon as possible. I wrote to Mahalanobis apologizing about my misdeed in deciding to come to the U.S.A. and requested him to arrange some suitable training for myself in statistical quality control in some industrial setting in the United States. Statistical quality control was considered a flourishing area by Mahalanobis for both research and applications. Also, ISI had already established a Quality Control Unit which was sort of a separate department. I thought of switching to obtain practical training in quality control in the States in lieu of getting a Ph.D. in statistics, thinking that Professor Mahalanobis would happily approve such a move on my part. Professor Mahalanobis replied to my letter and I recall his mentioning that he had written directly to Professor Ellis R. Ott, who was the Head of the mathematics department at Rutgers and had previously visited ISI to help nurture a program in statistical quality control, requesting him to arrange such practical training for me. But when Dr. Basu and Dr. Rao became aware of this, they immediately wrote back to me emphasizing that it would be foolish of me to abruptly cut short my pursuit of a Ph.D. in order to return to Calcutta. Both Dr. Basu and Dr. Rao advised me to stay at Chapel Hill for the full two-year term, get my Ph.D. degree and then return to ISI, Calcutta. When I returned to ISI, I did not sense any displeasure on Professor Mahalanobis's part.

Mukhopadhyay: Who were some of the "big" names when you returned to ISI, Calcutta? I suppose that Dr. C. R. Rao was there. Was the sampling legend, Professor D. B. Lahiri, there?

Mitra: Dr. D. Basu was there. Also, the year 1956 happened to be the year when Professor Raghu Raj Bahadur came back to India and joined ISI, Calcutta. He finished his Ph.D. from Chapel Hill in

1950 and was probably teaching at Columbia for a while before returning to India. Professor Bahadur stayed with ISI for a full five-year term from 1956 to 1961. Since 1961 he has been associated with the University of Chicago. Dr. Kallianpur was finishing his stay at ISI, Calcutta, and was in the process of returning to the U.S.A. Yes, D. B. Lahiri was at ISI. Professor Mahalanobis used to refer to Lahiri as his conscience-keeper in sampling theory. There was another outstanding person who was not formally trained in statistics and had only a Bachelor's degree. We used to call him Shankarbabu, formally known as J. M. Sengupta.

Mukhopadhyay: You have met many distinguished visitors at ISI. What are some of the memorable lectures given by the outstanding visitors that got your attention and interest?

Mitra: Actually most visitors to ISI stayed only for a couple of days and, of course, gave a couple of lectures. So, one cannot form any opinion based on such small sampling. Among the visitors who stayed longer and gave a whole series of lectures in some area was Professor R. C. Bose, who gave several lectures on coding theory during May through July 1966. I found Professor Bose's lectures very stimulating. Also, the series of lectures given by Wassily Hoeffding were very stimulating. Professor Hoeffding was visiting ISI on a United Nations assignment during October 1964 through March 1965. He presented a paper which appeared in the *Annals of*



FIG. 4. Delhi ISI campus, December 1992: (standing, from left to right) S. K. Jain and C. R. Rao.

Mathematical Statistics (Hoeffding, 1965). It dealt with asymptotically optimal tests for the multinomial distribution.

APPLICATIONS OF STATISTICS AT CALCUTTA ISI

Mukhopadhyay: In the beginning of your career after receiving the Ph.D. degree, I recall that you got involved with statistical projects having real applications.

Mitra: Yes. I worked on several applied projects in which C. R. Rao was involved. For example, in the Bengal anthropometric survey having data on 18 variables measured on nearly 4,000 individuals, R. G. Laha and I became involved in the scrutiny of the data and analysis. We spent about 8 to 10 hours daily, for nearly six months—including Saturdays and Sundays, to carry out the analysis, with hand-operated calculators (see Majumdar and Rao, 1958). I also became involved in applications of sampling to crop estimation. You may know that for a long time there was a big controversy between ISI and the PUSA Institute (Delhi) about the nature of cuts to be used for actual crop estimation. The controversy was whether one should use a rectangular cut or a circular cut, the latter one being advocated by Mahalanobis. It was generally known that crop estimation usually provided an overestimate of the true yield. This was because of two underlying facts. One of the facts is a direct consequence of customary undersampling of border areas. In the field, the border areas of a plot usually have less yield, and on top of this if one undersamples these boundary areas, then naturally the final estimate would tend to overestimate the yield. In other words, the low-yield areas are often under-represented in comparison with what they should be. The other source of overestimation came from the general tendency observed among the investigators to include the yield near the boundary areas of the cut as part of the cut yield. Professor D. B. Lahiri came up with a very interesting method to circumvent such practical difficulties, and the methodology runs as follows.

Suppose we wish to estimate the yield Y of a plot. First, the plot is unambiguously split, laying a rope for example, to create two half-plots serially numbered 1 and 2. One of the two half-plots is selected at random with equal probability. Then, the selected half-plot is further divided into two quarter plots. Thus, if half-plot #1 is selected initially, it is divided into quarter-plots 11 and 12, whereas if half-plot #2 gets selected initially, it is then divided into quarter-plots 21 and 22. In either case from the available

quarter-plots (two in number), one is selected at random with equal probability. The selection process may be terminated with the quarter-plot selection, in which case the selected quarter-plot is harvested and its yield determined. Its compatriot, the other quarter-plot in the same half-plot, is well defined but its Y -value remains hypothetical. The quarter-plots in the other half-plot disqualified at the first stage may not have been defined at all. Let X be the harvested yield of the selected quarter-plot and, in general, that of the selected k th-order subplot in a k -stage selection. It is easily seen that $2^k X$ is an unbiased estimate of Y .

Mukhopadhyay: Did Professor Lahiri publish this work?

Mitra: He did not, and let me explain why. Now, going back to the earlier discussion, if one succeeds in dividing the plot into two halves as nearly equal as possible, then the variance of this estimator is correspondingly reduced. In 1963, with a group of students from Calcutta, J. Roy and I went to Giridih, a home of Mahalanobis's, where ISI was running a branch with an attached experimental farm. On that farm, we performed the new crop cutting experiment using Lahiri's method, and we found with pleasant surprise that Lahiri's method almost always produced a near-hit in terms of the estimate, in contrast to the circular or rectangular cuts which continued to bias the yield estimate upward. Lahiri's method is obviously impractical when the survey is to be conducted by the field-staff with inadequate supervision because of the enormous freedom it gave to the investigators. The other limitation is the possibility of damage to the standing crop by trampling while defining the various subplots. This may have been the reason why Professor Lahiri never thought of publishing his methodology. Professor Mahalanobis maintained that while teaching students we should not use artificial data or old data collected elsewhere. To create more interest in the subject matter, students should be encouraged to collect their own data and analyze accordingly. In this context, I still remember some of the experiments that we performed.

Mukhopadhyay: Please elaborate on these particular experiments.

Mitra: An experiment was performed to examine whether the blood-pressure readings differ when measured on both hands of an individual. It is known that blood pressure readings depend on the state of mind of the person concerned at the time of measurement. A relaxed person, for example, may provide a relatively lower reading, whereas if the same person becomes tense, the reading becomes slightly higher. Generally, the very process of

measuring the blood pressure creates some tension in the individual's mind, leading to a raised level of the reading. In order to avoid these extraneous difficulties and their effects on the blood pressure itself, we adopted a new design which may be called the double crossover design. The crossover designs were discussed, for example, in Cochran and Cox's book, *Experimental Designs* (second edition published in 1957). But when we were planning our experiment, the double crossover designs were still not considered by others. In that sense the design that we proposed was a new statistical design which grew out of real practical need. The second crossing over was necessary to balance the order effect on investigators if more than one investigator was involved. The second experiment was performed to verify a prevalent belief that the height of an individual varies as the day progresses. The maximum height is attained when the individual wakes up in the morning rested and perfectly relaxed. The height shrinks to its minimum when he returns to bed at night. The experimental data appeared in Majumdar and Rao (1958).

RESEARCH WITH MATRICES

Mukhopadhyay: At what point did you get started on systematic studies of matrices and their properties? How did the transition take place or was it a transition at all?

Mitra: I was attracted to studies of matrices mostly because of the lectures I attended given by Professor C. R. Rao at Calcutta University around 1950–51. In fact, my Ph.D. thesis presented the proofs of the limiting distribution of the goodness-of-fit χ^2 which was later published (Mitra, 1958). This was a refinement of Cramér's original proof only to the extent of improving the treatment of matrices which Cramér gave in his book.

Mukhopadhyay: After returning to ISI, Calcutta, did you immediately start collaborative research on matrices with Professor Rao?

Mitra: In his 1955 *Sankhyā* paper, Dr. Rao introduced the concept of a pseudoinverse of a matrix and showed its usefulness in the analysis of linear models (Rao, 1955). Since then, I have taught topics on pseudoinverses to our students. In 1966–67, Rao wrote a review paper on the calculus of generalized inverses and then gave me the paper to read and comment upon. A pseudoinverse is actually a generalized inverse, but it was given a different name in the beginning. Rao had defined a pseudoinverse arising out of a special type of matrix in linear models, so initially the concept of a pseudoinverse was introduced and its computation provided while aim-

ing at some special types of matrices that are relevant to solving certain normal equations. That is why a pseudoinverse can be viewed as a special type of generalized inverse.

Mukhopadhyay: What happened after you reviewed Dr. Rao's paper?

Mitra: When I read his review paper (Rao, 1967), I could immediately think of making some changes. The first change I thought of was the definition of a generalized inverse itself. The changed definition was published (Mitra, 1968) and later reproduced as the third definition given in the book by C. R. Rao and myself (Rao and Mitra, 1971).

Mukhopadhyay: What was the arrangement between you and Dr. Rao in writing the book?

Mitra: Dr. Rao and I agreed upon our division of labor. Usually the topics relating to applications in statistics were taken care of by Dr. Rao, whereas those dealing with the theory of a generalized inverse were taken care of by me. Then we combined the materials and polished each other's writing to achieve some sort of uniformity.

Mukhopadhyay: How do you generally get started on an original research paper? Can you kindly give some examples?

Mitra: When I read any paper or look at any result, I immediately start asking questions of whatever I am reading. You recall that I had learned to do this from Dr. Basu. For example, when I considered the class of solutions of generalized inverses, the question that immediately came to my mind was this: "We know that an inverse of an invertible matrix determines the original matrix by taking one more inversion, but then similarly does the entire class of generalized inverses of a matrix determine the original matrix?" This was the question I asked myself, and eventually the answer was found in the affirmative.

Mukhopadhyay: The result you just quoted is a fundamental one. When and where did it first appear?

Mitra: It first appeared in 1971. It is known that while inverting a known $m \times n$ matrix A , the rank of a generalized inverse could be any integer between the rank of A and $\min(m, n)$, both endpoints included. The question that I raised was, while solving linear equations, how does one get a generalized inverse with a specified rank s that is within the admissible range. This question was also resolved and led to new findings at the time (Rao and Mitra, 1971).

Mukhopadhyay: In terms of your papers or results in matrices and linear models, is there one particular contribution of yours that you are most fond of?

Mitra: I particularly enjoyed writing the paper that was later published in *Linear Algebra and Its Applications* (Mitra, 1991), on a unified theory of matrix partial orders through generalized inverses. You may recall that the way the minus partial order (see Hartwig, 1980) is generated is as follows. If A and B are two matrices, real or complex, then we say that B is larger than A if, for some generalized inverse A^- of A , $(B - A)$ is killed by multiplication of A^- on either side, that is, $(B - A)A^- = O$, $A^-(B - A) = O$, where O indicates a null matrix. The order of the null matrix is clear from the context. One can restrict the choice of g -inverses that can be used in this context. For example, the star (*) order of Drazin (1978) uses the g -inverse that is known as the Moore–Penrose inverse. The group inverse of A is similarly used to define what I call the sharp order (Mitra, 1987). The sharp order has an interesting limitation.

Mukhopadhyay: And what is that limitation?

Mitra: Because the group inverse can be defined only for a square core matrix, that is, a matrix whose rank is same as the rank of its square, this definition excludes a large class of matrices which are non-core. I raised the question whether the sharp order defined only on core matrices could be extended to all square matrices. Here, “extension” would be in the sense that its restriction on the class of core matrices coincides with the sharp order. After a large number of attempts, including failures and partial successes, I answered the question in the affirmative (Mitra, 1992). This was a paper that pleased me very much.

ADVISING STUDENTS AND VISITS TO PLACES

Mukhopadhyay: You have advised and guided several students’ Ph.D. work. Would you recall some of them and the research areas they worked on?

Mitra: My Ph.D. students are often from different fields. For example, the first student who worked with me was M. T. Subramanya, who is presently working somewhere in Africa. He wrote his thesis on acceptance sampling plans. The next student, Ramanujcharyulu, worked on construction of designs. Next, P. Bhimasankaram wrote his thesis on generalized inverses. Then, Betty Jeanne Moore of Bloomington, Indiana, worked on linear models. C. G. Bhattacharya worked entirely on his own on the common mean estimation problem. N. N. Singh worked on genetics. Actually Singh already had enough results which he obtained entirely by himself. My role was to guide him to consolidate all these results and properly present in a thesis form. Dibyen Majumder worked on linear models.

Mukhopadhyay: You were a Visiting Professor of Mathematics at Indiana University in Bloomington from 1971 to 1974. That is the time when Betty Jeanne Moore worked on her doctoral thesis under you, right? Isn’t it unusual for a Ph.D. student to work under the guidance of a Visiting Professor? How did that arrangement come about?

Mitra: It was unusual, I agree. The University authorities were presented with a *fait accompli*. They were sufficiently broad-minded to approve her entire Ph.D. committee with me as Chairman. If they had found this unacceptable, some other member of the committee might have signed as Chairman.

Mukhopadhyay: You have visited many countries for conferences and workshops. Is there a particular one that you recall?

Mitra: I enjoyed all my trips and meeting colleagues from around the world. In 1974, on my return trip to India from Bloomington, I stayed in Japan for about three weeks. I was most impressed by the efficient design of the total organization of my trip to Japan by my colleagues there at Keio University. The department of statistics at that University functioned in the Faculty of Engineering and was known as the Department of Administration Engineering. (It seems appropriate to recall that Professor Mahalanobis was fond of quoting R. A. Fisher while referring to statistics as a “Key Technology”



FIG. 5. Lake Balatan, Hungary, 1968, at a street named after the Indian poet Tagore.

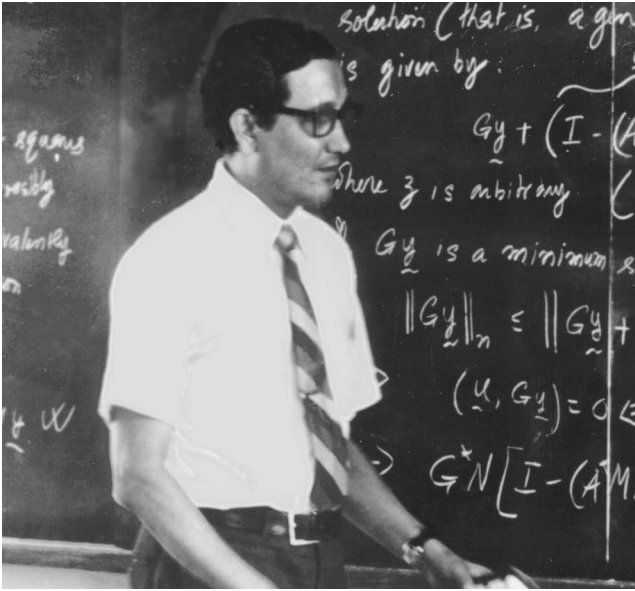


FIG. 6. Lecturing at Keio University, Japan, 1974.

of the twentieth century.) The colleague primarily responsible for the arrangements was Professor Y. Washio, who worked under the advice of Professor H. Sakamoto. That was, I will say, a memorable visit.

MOVE TO DELHI ISI

Mukhopadhyay: I met you at ISI, Calcutta, in the early 1970s. Somewhere down the road you moved to ISI, Delhi. When exactly was that and why did you decide to move to Delhi? If I remember correctly, Dibyen Majumder was probably your first Ph.D. student from ISI, Delhi. Am I right?

Mitra: Your question has two parts. I shall answer them in the order you propose. The route you mention stretches from Calcutta to Delhi, via Bloomington, Indiana. I have often asked myself why I moved to Delhi. Just as I have asked myself why certain other events happened which looked unpalatable at that moment. To cite another example, at some point in my career I had applied for the advertised position of Professor of Statistics at Utkal University in Orissa, India. This would have enabled me to stay near my parents, who were getting old. I was told that even though several members of the selection committee supported my case, the Chairman decided otherwise in the interest of the department. Actually, I had very little role to play in the decision process. It appears that a powerful destiny was making decisions on my behalf, ignoring my personal wishes. I was formally transferred from ISI, Calcutta, to ISI, Delhi, in early 1975. You are right about your surmise re-

garding Dibyen. He moved to Delhi from Calcutta a few months after I made the move.

Mukhopadhyay: You also became the Head of the Delhi Centre of ISI?

Mitra: Strictly speaking I was never the head of the Delhi Centre. Initially, from the second half of 1976, I functioned as the In-Charge of the Delhi Centre as an interim arrangement. When the Headship was formally offered to me, for various reasons I did not accept it, but continued to function as acting Head until a new Head was appointed.

Mukhopadhyay: Would you please elaborate on the major differences between the training programs available in Calcutta and Delhi Centres of ISI? What was your role in establishing the Ph.D. program and other degree programs at Delhi? The readers may not realize that the final degree actually comes from ISI, whether one belongs to Calcutta, Delhi or some other centre of ISI. In other words, ISI is still one organization. Am I right?

Mitra: This query again has several parts, which I will answer in the reverse order. You are right. Unlike what happens in the University of California system, where the different campuses are largely autonomous, there is essentially one Indian Statistical Institute, which decides, for example, the training programs everywhere, conducts the selection tests for trainees, fixes a common calendar and determines the syllabus to be followed. The midterm and final examinations are conducted independently by the different centers, but those who qualify through these examinations are awarded degrees by the ISI at a common convocation ceremony held in Calcutta. Calcutta, which has the largest resources in terms of faculty strength, is the only center which conducts the Bachelor of Statistics program. The Master of Statistics courses are taught at all three centers. The first year of the Master of Statistics program is the same at all three centers. However, for the second year the trainee has an option to choose his or her area of specialty such as Statistical Quality Control and Operations Research (SQC&OR), Applied Statistics and Computing, Planning and Econometrics, Advanced Probability and Mathematical Statistics, and so on. The specializations offered at various centers differ depending on the resources available. You will notice that I did not have any special role to play in developing programs at ISI, Delhi.

Mukhopadhyay: Over the years, you have been involved with the administration at ISI in various capacities. Do you recall any particular joyous moment or a moment of deep regret?

Mitra: Whenever I took part in the administration, I did not feel any joy. I experienced a lot of



FIG. 7. Tree planting at Delhi ISI campus, summer 1976.

unnecessary tension and ill feelings. But, I will say that I was satisfied as an administrator because, in spite of my realization that I do not have the natural talent for such positions, I had accomplished a lot for the Delhi Centre of ISI. I used to solve day-to-day problems in my own way, which involved regular meetings with senior faculty members of the Institute. I became involved in budgetary decisions, particularly when the Delhi Centre of ISI got into large construction projects including the construction of the faculty buildings, residential buildings and the guest house. That took away a lot of my energy and time. But, I can safely say that I have served in almost any administrative capacity that one can think of. For example, I was In-Charge of the “Evening Courses,” “Summer Programs” as well as the International Statistical Education Centre at ISI, Calcutta—organized jointly under the auspices of the Indian Statistical Institute and the International Statistical Institute. I do not have any regrets. I had to take care of some very important responsibilities throughout my career at ISI. I followed my instinct and did the best I could. I took care of these duties as “cheerfully” as possible.

RETIREMENT AND CONTINUED RESEARCH

Mukhopadhyay: Are you now officially retired from ISI and is your designation Professor Emeritus?

Mitra: Yes, I retired from ISI in January 1992. The “Emeritus” status is not automatically given to a retiree. It is awarded and authorized by the Council of the Institute on the basis of the retiree’s long-standing accomplishments. I am a Professor Emeritus at ISI.

Mukhopadhyay: Do you currently have any Ph.D. students at ISI, Delhi, under your supervision?

Mitra: No. This amounts to accepting a responsibility which, because of failing health, I am uncertain of my ability to discharge.

Mukhopadhyay: Are you professionally connected with any other institute or organization in Delhi?

Mitra: Not particularly, but I should add that Emeritus status does not give any monetary benefit. It does, however, allow me to keep my office at ISI, Delhi, use the library, computing and xeroxing facilities and have secretarial assistance. But, after retirement, a fellowship award in March 1992 from the Council of Scientific and Industrial Research (CSIR), Government of India, under the emeritus scientists scheme has been a big help. With CSIR’s prior approval, I can bring in a visitor or a postdoctoral fellow. Also some limited funds are given for attending conferences within India. This fellowship was first awarded for three years and since January 1995 it has been extended for a further period of two years. It cannot be extended beyond five years under any circumstances.

Mukhopadhyay: You are still continuing your research and in fact expanding your research efforts in some areas. What ideas are you pursuing right now?

Mitra: Currently I have written a paper introducing the new concept of separability of vectors and I have found some interesting applications for this concept. But my own feeling is that this concept will see many more interesting applications in the future. It has appeared in a special issue of *Linear Algebra and Its Applications* (Mitra, 1994), dedicated to Professor Chandler Davis of Toronto. The more I work on it, the more impressed I became not only with the beauty of this concept, but also its power.

Mukhopadhyay: You recently coedited a very interesting volume entitled *Glimpses of India's Statistical Heritage* (Ghosh, Mitra and Parthasarathy, 1993), wherein 10 articles were written in an autobiographical style by some of the path-breaking statisticians and probabilists of Indian origin. It is an excellent volume. Whose idea was it, and how did such a project get off the ground?

Mitra: All the three editors of the volume were also editors of *Sankhyā*. J. K. Ghosh was simultaneously the Director of ISI with its headquarter at Calcutta. As the Director, he undertook periodic visits to Bangalore and Delhi—the other two main campuses of the institute—and occasionally even to branches engaged in SQC&OR related work at some of the enlightened local industries. During one such visit to Delhi, he sought our views on *Sankhyā* inviting autobiographical accounts from senior Indian statisticians and probabilists on their research efforts. Apart from their historical value, these essays might also provide a source of inspiration to future generations and attract a few competent Indian students to seek careers in statistics. However, it was felt advisable not to enforce any drastic change in the present composition of *Sankhyā*. At about the same time I was drafting my Presidential Address for the Pune session of the 1988 Indian Science Congress, I decided to speak on the “Method of Overlapping Maps in Survey Sampling,” an area in which I had recently done some work in collaboration with P. K. Pathak and K. Krishnamoorthy. The basic problem was introduced by N. Keyfitz (1950). However, it appears that Dr. D. B. Lahiri (1954) was the first to present a precise formulation of the same, and Des Raj (1957) recognized this to be a special case of the transportation problem. The Presidential Address (Mitra, 1988) was mainly a review paper and it was natural that this should be dedicated to Professor Lahiri on the occasion of his 75th birthday. During the second half of 1987, I

sent one copy of the manuscript to Professor Lahiri for his comments. His response was prompt and unexpected—a 20 page handwritten autobiographical letter. With one unsolicited manuscript already in hand, J. K. Ghosh, K. R. Parthasarathy and I could now look forward to undertaking and completing, within a reasonable time, a volume of invited autobiographies.

HONORS AND AWARDS

Mukhopadhyay: Do you wish to mention any awards or honors you have received over the years? You are a Fellow of the Indian National Science Academy and the Indian Academy of Science. You were also elected President of the Statistics Section of the Indian Science Congress. Any recollections?

Mitra: In the year 1988 I became the President of the Statistics Section of the Indian Science Congress. It was the Platinum Jubilee year for the Congress, and I was really pleased when several important scholars accepted my invitation to come and attend the Science Congress held at Pune. Since that was the Platinum Jubilee year, the Congress decided to invite two outstanding scientists in every area of science and technology, designated as Platinum Jubilee Lecturers. I am very glad to add that the two speakers I could include were Professors C. R. Rao and S. R. S. Varadhan. In 1995, both of them became members of the U.S. National Academy of Science. Among others who attended after accepting my invitation, included Professors D. R. Cox, Raghu Raj Bahadur, J. K. Ghosh, S. K. Chatterjee, S. Dasgupta, K. R. Parthasarathy, B. L. S. Prakasa Rao, C. G. Khatri, B. V. Rao, S. R. Adke, B. R. Bhat and P. Narain. Incidentally, Professor D. R. Cox was a visitor to ISI and was named the Mahalanobis Memorial Lecturer that year.

Mukhopadhyay: You are one of the Editors of *Sankhyā* and an Associate Editor of the *Journal of Indian Society of Agricultural Statistics*. Do these take up a lot of your time?

Mitra: If I could handle papers only in my area, and if the papers were interesting, then I would enjoy putting in the hard work that is needed in the editorships. But most of the papers I handle are not quite connected to my area of active interest, and then handling these papers become something of a chore. Also, most papers submitted for publication are not of very high quality. But, overall, this activity does not take an exorbitant amount of time.

Mukhopadhyay: I remember that a conference was organized in your honor at ISI, Calcutta, upon your retirement. This was organized by some of your former students and other admirers. That must have been a gratifying experience.

Mitra: It was indeed. I was most pleased to receive the affection and respect from so many students and colleagues.

IMMEDIATE FAMILY

Mukhopadhyay: Now I would like to turn to your immediate family. Please tell me a few things about your wife, children and anything else that comes to mind.

Mitra: I have already mentioned that I first became interested in mathematics because of my mother's initiatives. Next, I would like to mention my wife, Sheila. Whatever I have achieved would not have been possible without her. I did not have to worry about running the household, nor my children's education, because it was she who determined which areas would suit each child.

Mukhopadhyay: When did you and Mrs. Mitra get married?

Mitra: We got married in 1958, two years after I received my Ph.D. from Chapel Hill. It may surprise you that this marriage negotiation took place at the suggestion of Dr. D. Basu.

Mukhopadhyay: (laughter) So it looks like Dr. Basu has influenced your life in more than one way. You have three daughters and a son. Where are they?

Mitra: Dr. Basu has influenced my life in many ways. I do not have any direct blood-relation with my wife, Sheila. But, both of us have close blood-relation with Dr. Basu. Sheila is the first cousin of Dr. Basu through his mother's side and my mother is the first cousin of Dr. Basu on his father's side. All three daughters are married. Two daughters live with their husbands in the U.S.A. and the eldest

daughter and her family live in Wales. Our son is now pursuing a Ph.D. degree in economics at the University of Southern California.

Mukhopadhyay: You have a physical handicap and disability that is a natural consequence of Parkinson's disease. Would you feel comfortable talking about this?

Mitra: My physical disability started in 1978, when it was noticed by the neurologists at the All India Institute of Medical Sciences, Delhi. This defect is associated with Parkinsonism. Over the years, the disease has become more and more acute. However, this has not been much of a handicap in my research career. I believe that if God gives one some disability, He also simultaneously enriches one somewhere else to compensate for it. Because I have difficulties in writing, I found, for example, that I could solve many of the research problems mentally. I often go through several steps of computations and algebra without using any pen or paper.

Mukhopadhyay: So, I take it that even if you have difficulties in bodily movements, you are otherwise feeling alright. But, do you ever feel frustrated thinking that if you had not contracted this debilitating disease, you could perhaps have "achieved" much more?

Mitra: I feel fine. I have no negative feelings about my physical disability in terms of my achievements in research. But I believe that this mostly affected my wife. Most certainly my children have also been greatly affected by this disease. We all had to make adjustments.

Mukhopadhyay: When you are not thinking about statistics or mathematics, how do you spend your time? Do you have any hobbies?

Mitra: I do not have any hobbies as such. When I am not doing mathematics or statistics, I watch TV, listen to music and try to read books. I say "I try" because I cannot study continuously for any length of time as I feel pain in my eyes and my vision gets blurred. Then I have no option but to revert to mathematics or statistics or to start thinking about my health, all of which scare me since on several occasions they have led to sleepless nights.

Mukhopadhyay: Have I missed anything in particular?

Mitra: During this conversation, I have mostly mentioned my papers related to matrices and generalized inverses of matrices. But there is one particular paper in statistics, a very short note, which I very much enjoyed writing (Mitra, 1969). This was in connection with a conjecture made by Dr. Rao concerning characterization of the Wishart distribution. I was unable to prove the conjecture, but eventually



FIG. 8. At the Indian National Science Academy, August 1981, signing the Fellowship Register as President M. G. K. Menon looks on.



FIG. 9. Kansas City, Missouri, July 1995.

I was able to produce a very interesting counterexample which showed that Dr. Rao's conjecture was not quite correct. This was certainly a paper that I enjoyed writing. The paper appeared in *Sankhyā, Series A* (1969). In fact Dr. Rao had openly declared that conjecture as a million-dollar problem.

Mukhopadhyay: After your paper came out, did you receive a million dollars from Dr. Rao?

Mitra: He did not give me a million dollars by saying that he would give me a million dollars if I could prove his conjecture, but I had instead disproved his conjecture by producing a counter-example (laughs).

Mukhopadhyay: In other words, your one-time opportunity to become a millionaire was actually killed by yourself!

Mitra: (laughter) That is right.

Mukhopadhyay: It has been my good fortune to sit down with you and have this conversation. I thank you for everything you have done for our profession and for being yourself. But, most of all, I thank you for being my teacher. May you have many more years of productive life in statistics, mathematics and otherwise.

Mitra: Thank you.

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