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SYMMETRIZATION AND CONCENTRATION INEQUALITIES FOR MULTILINEAR FORMS WITH APPLICATIONS TO ZERO–ONE LAWS FOR LÉVY CHAOS

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We consider stochastic processes $\mathbf{X} = \{X(t),\ t \in T\}$ represented as a Lévy chaos of finite order, that is, as a finite sum of multiple stochastic integrals with respect to a symmetric infinitely divisible random measure. For a measurable subspace V of $\mathbf{R}^T$ we prove a very general zero–one law $P(\mathbf{X} \in V) = 0$ or 1, providing a complete analog to the corresponding situation in the case of symmetric infinitely divisible processes (single integrals with respect to an infinitely divisible random measure). Our argument requires developing a new symmetrization technique for multilinear Rademacher forms, as well as generalizing Kanter's concentration inequality to multiple sums.

1. Introduction. Zero–one laws for *subspaces* are known phenomena in probability theory which have been observed in a number of circumstances. A typical zero–one law of this type says that, for a certain family $\mathscr{P}$ of stochastic processes $\mathbf{X} = \{X(t),\ t \in T\}$ and a certain family $\mathscr{V}$ of measurable subspaces V of $\mathbf{R}^T$,

$$P(\mathbf{X} \in V + x) = 0 \text{ or } 1 \tag{1.1}$$

for every $\mathbf{X} \in \mathscr{P}$, $V \in \mathscr{V}$ and $x \in \mathbf{R}^T$. The family $\mathscr{V}$ often (but not always) contains all measurable subspaces of $\mathbf{R}^T$. Zero–one laws may also hold for measurable subgroups of $\mathbf{R}^T$, but in this paper we will only be concerned with subspaces.

Historically, the first case considered was that of $T = [0, 1]$ and $\mathscr{P}$ consisting of the Wiener processes. Here the zero–one law for any measurable subspace of $C[0, 1]$ has been proved by Cameron and Graves (1951). Within the next 20 years zero–one laws were proved for all Gaussian processes and all measurable subspaces of $\mathbf{R}^T$. We only mention in this context the papers of

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Kallianpur (1970) and Jain (1971), and we refer the reader to the survey article of Janssen (1984) for more details and an extensive survey containing many important results in this area.

Gaussian zero–one laws were commonly derived using the theory of reproducing kernel Hilbert spaces. However, no such theory is available for, say, stable processes. It turns out nonetheless that there is another reason why zero–one law should hold valid for both Gaussian and stable processes, and it relies on the defining scaling property of these processes. This was the argument used by Dudley and Kanter (1974) and Fernique (1974) and extended to semistable processes by Louie, Rajput and Tortrat (1980) and Tortrat (1980). For this family of stochastic processes zero–one laws hold, once again, for all measurable subspaces of $\mathbf{R}^T$.

For general infinitely divisible processes, the situation turned out to be more complicated, both because they do not have scaling properties and because they clearly do not satisfy zero–one law for *all measurable subspaces*, as the reader can easily convince him/herself by thinking of the probability that the standard Poisson random variable equals zero. Still, zero–one laws do hold for certain subspaces, which can be described in fairly simple terms using the Lévy measure of the process. Even though the statements of the results are simple, the arguments have typically been quite complicated, and they rely on continuous convolution semigroup embedding of infinitely divisible distributions and processes. We refer the reader once again to Janssen's (1984) survey for more details and references. A significant shortening and simplification of these arguments, by using certain methods of operator semigroups, has been presented recently by Byczkowski and Rajput (1993).

Neither of these methods applies to the case of Lévy chaos. There is no scaling property and the distribution structure of such processes is very complicated. Therefore our starting point will be yet another idea of proving zero–one laws for infinitely divisible processes due to Rosiński (1990a). The approach is based on *series expansions* for infinitely divisible processes [see LePage (1980) and Rosiński (1990b)]. We present it here in the language of stochastic integrals, which is the most suitable to the subject of this paper. We further restrict ourselves to the symmetric case.

Let $\{X(t),\ t \in T\}$ be a symmetric infinitely divisible stochastic process without a Gaussian component given by

$$X(t) = I_1(f(t)) = \int_S f(t;s)M(ds), \qquad t \in T. \tag{1.2}$$

Here $(S, \mathcal{S})$ is a measurable space, with a δ-ring $\mathcal{G}$ of $\mathcal{S}$-sets on it. Further, M is a symmetric infinitely divisible random measure without a Gaussian component on $(S, \mathcal{G})$; that is, $\{M(A),\ A \in \mathcal{G}\}$ is a stochastic process such that, for every $A \in \mathcal{G}$, $M(A)$ is a symmetric infinitely divisible random variable without a Gaussian component, with the following properties: for pairwise disjoint $A_1, A_2, \dots$ in $\mathcal{G}$, the random variables $M(A_1), M(A_2), \dots$ are independent ("M is independently scattered"), and if $\bigcup_{n=1}^{\infty} A_n \in \mathscr{S}$, then $M(\bigcup_{n=1}^{\infty} A_n) = \sum_{n=1}^{\infty} M(A_n)$ a.s. ("M is σ-additive").

The random measure M is commonly parametrized as follows. For an $A \in \mathscr{S}$, write the characteristic function of $M(A)$ in the form

$$\mathbb{E}\exp\{i\theta M(A)\} = \exp\left\{-2\int_0^{\infty} (1-\cos\theta x)\eta_A(dx)\right\}. \tag{1.3}$$

It turns out that there is a probability measure λ on $(S, \mathscr{S})$ and a measurable family of Lévy measures on $(0,\infty)$, $\{\rho(\cdot, s),\ s \in S\}$ such that the Lévy measure η_A in (1.3) can be represented in the form

$$2\eta_A(B) = \int_A \left[\int_0^{\infty} \mathbf{1}_B(x)\rho(dx, s)\right]\lambda(ds), \tag{1.4}$$

for every Borel subset B of $(0,\infty)$ and

$$\lambda\{s \in S: \rho((0,\infty), s) = 0\} = 0.$$

The measure λ (a control measure of M) and the family of one-dimensional Lévy measures $\{\rho(\cdot, s),\ s \in S\}$ is a parametrization of the random measure M.

Finally, for every $t \in T$, the function $f(t, \cdot)$ in (1.2) is a measurable function $S \to R$ satisfying certain integrability conditions for the integral to be well defined. We refer the reader to Rajput and Rosiński (1989) for more details.

The series expansion of the integral process (1.2) is constructed as follows. For $s \in S$, let

$$R(u, s) = \inf\{x > 0: \rho((x,\infty), s) \le u\},$$

$u > 0$. Let $\{\varepsilon_j,\ j \ge 1\}$, $\{\tau_j,\ j \ge 1\}$ and $\{\Gamma_j,\ j \ge 1\}$ be independent sequences of random variables, the first one being a sequence of i.i.d. Rademacher random variables, the second one being a sequence of i.i.d. S-valued random variables with common distribution λ, and the last one being a sequence of arrival times of a Poisson process on $(0,\infty)$ with unit rate. Then the series

$$Y(t) = \sum_{i=1}^{\infty} \varepsilon_i R(\Gamma_i, \tau_i) f(t; \tau_i) \tag{1.5}$$

converges a.s. for every $t \in T$ and, moreover, $\mathbf{X} \stackrel{d}{=} \mathbf{Y}$ in terms of equality of finite-dimensional distributions, where $\mathbf{X} = \{X(t),\ t \in T\}$ and $Y = \{Y(t),\ t \in T\}$. This fact allows one to treat $\mathbf{Y}$ as opposed to $\mathbf{X}$ for the purpose of considering zero–one laws. Since $\mathbf{Y}$ is based on an i.i.d. sequence and its terms have a high degree of independence, zero–one laws hold for $\mathbf{Y}$ and so for $\mathbf{X}$ [see (1990a)].

In the case of stochastic processes represented by *multiple* stochastic integrals with respect to infinitely divisible random measures there are, perhaps somewhat unexpectedly, zero–one laws corresponding to the above zero–one laws for infinitely divisible processes, although the arguments, quite expectedly, turned out to be more complicated than in the latter case. In the Gaussian case the problem was considered by de Acosta (1976), who established the zero–one law for quadratic forms, and by Rosiński, Samorodnitsky and Taqqu (1993), who proved the zero–one law for finite sums of multlinear forms of arbitrary order in Gaussian random variables. The zero–one law for Gaussian chaos is a simple consequence of these results.

This leaves the non-Gaussian infinitely divisible case to be considered. Let M be an infinitely divisible random measure as above, and let

$$X(t) = f_0(t) + \sum_{j=1}^{d} I_j\big(f_j(t)\big), \qquad t \in T, \tag{1.6}$$

where

$$I_j(f(t)) = \int_{S^j} f_j(t; s_1, \dots, s_j) M(ds_1) \cdots M(ds_j). \tag{1.7}$$

Here, for every $t \in T$, $f_j(t; \cdot)$: $S^j \to R$ is *symmetric* (i.e., invariant under permutations of its j arguments), and *vanishing on the diagonals* [i.e., $f_j(t; s_1, \dots, s_j) = 0$ whenever $s_i = s_k$ for some $i \neq k$). When M is a symmetric stable random measure, or, more generally, a type G random measure, a zero–one law for all measurable subspaces of $\mathbf{R}^T$ was proved once again by Rosiński, Samorodnitsky and Taqqu (1993) using their zero–one law for the Gaussian case and conditional Gaussianity of the above random measures. Clearly, this argument breaks down once conditional Gaussianity is no longer present. The next step was made by Rosiński and Samorodnitsky (1994), who proved a zero–one law for finite sums of multiple integral processes of the type (1.6) under the following assumption on the infinitely divisible random measure M. The Lévy measure η of M has a representation (1.4) such that, for λ-almost every $s \in S$,

$$\rho((0, \infty), s) = \infty. \tag{1.8}$$

Additionally, they assumed that $\rho(\cdot, s)$ is atomless. This includes, in particular, all type G random measures. The argument in the latter paper used once again the series representation of multiple stochastic integrals, introduced in the context of multiple stable integrals by Samorodnitsky and Szulga (1989) and extended to the general case by Szulga (1992). We have that $\mathbf{X} \overset{\mathrm{d}}{=} \mathbf{Y}$ in terms of equality of finite-dimensional distributions, where $\mathbf{Y}$ is defined by the series

$$\begin{aligned} Y(t) = f_0(t) + \sum_{j=1}^{d} j! \sum_{1 \le n_1 < \cdots < n_j} \left(\prod_{h=1}^{j} \varepsilon_{n_h} R(\Gamma_{n_h}, \tau_{n_h}) \right) \\ \times f_j(t; \tau_{n_1}, \dots, \tau_{n_j}), \end{aligned} \tag{1.9}$$

which converges a.s. and unconditionally for every $t \in T$.

It is the purpose of the present paper to prove a general zero–one law for all stochastic processes of type (1.6) and all measurable subspaces of $\mathbf{R}^T$, when the random measure M satisfies only the assumption (1.8). The importance of this step can be seen from the fact that this is exactly the case when the zero–one law holds for all measurable subsets of $\mathbf{R}^T$ in the case of infinitely divisible stochastic processes.

The crucial step in the argument used by Rosiński (1990a) for the proof of the zero–one law for infinitely divisible processes was an extension of a theorem of Paul Lévy. This argument does not seem to work in the case of multiple integrals. Therefore, in this paper we propose another approach, which works for both single and multiple integrals. This approach is based on a multilinear version of a concentration inequality of Kanter (1976). We prove this inequality for multiple Rademacher forms, and this is done in Section 3. The proof relies on a simple symmetrization technique for multiple Rademacher forms which is presented in Section 2. Finally, in Section 4 we prove our general zero–one law.

2. Symmetrization. In this section we derive an explicit form of a symmetrized Rademacher multilinear form. Roughly speaking, the symmetrization lowers the order of a multilinear form. To illustrate this phenomenon, consider the following simple example of a Gaussian quadratic form. Let $Q = (aG + b)^2$, where G is $N(0,1)$, and let $Q' = (aG' + b)^2$ be an independent copy of Q. Then

$$\tfrac{1}{2}(Q - Q') \overset{\mathrm{d}}{=} a^2 GG' + \sqrt{2}\, abG';$$

that is, the symmetrization of Q is conditionally a linear form in G. For Rademacher multilinear forms we have the following lemma.

LEMMA 2.1. *Let ε_j, $j \ge 1$, ε_j', $j \ge 1$, and δ_j, $j \ge 1$, be three independent sequences of i.i.d. random variables, the former two sequences being Rademacher sequences, and the latter being a Bernoulli sequence with parameter $\frac{1}{2}$. Let $\mathbf{a}^{(j)}_{i_1,\dots,i_j}$, $j = 0,1,\dots$, $i_k = 1,2,\dots$, for $k = 1,\dots,j$ be arrays of vectors in a topological vector space E. Let*

$$\mathbf{Q} = \sum_{j=0}^{d} \sum_{i_1=1}^{\infty} \cdots \sum_{i_j=1}^{\infty} \varepsilon_{i_1} \cdots \varepsilon_{i_j} \mathbf{a}^{(j)}_{i_1,\dots,i_j} \tag{2.1}$$

be a convergent multilinear form. Let $\mathbf{Q}'$ be an independent copy of $\mathbf{Q}$.

If, for each $j = 1,\dots,d$, the array of vectors $\mathbf{a}^{(j)}_{i_1,\dots,i_j}$ is invariant under permutations of the subscripts, then

$$\begin{aligned} \tfrac{1}{2}(\mathbf{Q} - \mathbf{Q}') \overset{\mathrm{d}}{=} \sum_{j=1}^{d} \sum_{k=0}^{j-1} m_k^j \sum_{i_1=1}^{\infty} \cdots \sum_{i_j=1}^{\infty} \varepsilon_{i_1} \cdots \varepsilon_{i_k} \varepsilon'_{i_{k+1}} \cdots \varepsilon'_{i_j} \delta_{i_1} \cdots \delta_{i_k} \\ \times (1 - \delta_{i_{k+1}}) \cdots (1 - \delta_{i_j}) \mathbf{a}^{(j)}_{i_1,\dots,i_j}, \end{aligned} \tag{2.2}$$

where

$$m_k^j = \begin{cases} \binom{j}{k}, & \text{if } k \text{ and } j \text{ have different parities}, \\ 0, & \text{if } k \text{ and } j \text{ have the same parity}. \end{cases} \tag{2.3}$$

The proof of this lemma will be based on the following two simple facts. The first one is somewhat parallel to the well-known equality: if G and G' are i.i.d. $N(0,1)$ random variables, then

$$\left(\frac{G+G'}{\sqrt{2}}, \frac{G-G'}{\sqrt{2}} \right) \stackrel{\mathrm{d}}{=} (G, G').$$

LEMMA 2.2. *Let ε, ε' and δ be independent random variables, the first two of which are Rademacher and the third has Bernoulli distribution with parameter $\frac{1}{2}$. Then*

$$\left(\frac{\varepsilon+\varepsilon'}{2}, \frac{\varepsilon-\varepsilon'}{2} \right) \stackrel{\mathrm{d}}{=} (\varepsilon\delta, \varepsilon'(1-\delta)). \tag{2.4}$$

PROOF. The proof is obvious. □

LEMMA 2.3. *Let $(a_1, \dots, a_d)$ and $(b_1, \dots, b_d)$ be two sequences of real numbers. Let $\alpha_i = (a_i + b_i)/2$ and $\beta_i = (a_i - b_i)/2$, $i = 1, \dots, d$. Then*

$$\begin{aligned} \frac{1}{2}(a_1 \cdots a_d - b_1 \cdots b_d) &= \sum_{I \in \mathscr{I}_1^d} \prod_{i \in I} \alpha_i \prod_{j \notin I} \beta_j, \\ \frac{1}{2}(a_1 \cdots a_d + b_1 \cdots b_d) &= \sum_{I \in \mathscr{I}_2^d} \prod_{i \in I} \alpha_i \prod_{j \notin I} \beta_j, \end{aligned} \tag{2.5}$$

where $\mathscr{I}_1^d$ is the collection of all subsets of $\{1, \dots, d\}$ whose cardinality has a different parity than d, and $\mathscr{I}_2^d$ is the collection of all subsets of $\{1, \dots, d\}$ whose cardinality has the same parity as d.

PROOF. Write $p_i = \frac{1}{2}(a_1 \cdots a_i - b_1 \cdots b_i)$ and $q_i = \frac{1}{2}(a_1 \cdots a_i + b_1 \cdots b_i)$, for $i = 1, \dots, d$. Then the vector

$$\begin{pmatrix} p_i \\ q_i \end{pmatrix} = \begin{pmatrix} \alpha_i & \beta_i \\ \beta_i & \alpha_i \end{pmatrix} \begin{pmatrix} p_{i-1} \\ q_{i-1} \end{pmatrix}. \tag{2.6}$$

Now, (2.5) is obviously true for $d = 1$, and if it is true for $d - 1$, then its truth for d follows from (2.6). This inductive argument completes the proof. □

PROOF OF LEMMA 2.1. It follows from Lemma 2.3 that

$$\begin{aligned} &\frac{1}{2}(\mathbf{Q} - \mathbf{Q}') \\ &= \sum_{j=1}^{d} \sum_{I \in \mathscr{I}_1^j} \sum_{i_1=1}^{\infty} \cdots \sum_{i_j=1}^{\infty} \prod_{i \in I} \tfrac{1}{2}(\varepsilon_i + \varepsilon_i') \prod_{l \notin I} \tfrac{1}{2}(\varepsilon_l - \varepsilon_l') \mathbf{a}_{i_1, \dots, i_j}^{(j)}. \end{aligned} \tag{2.7}$$

Now, the assumptions on the arrays $\mathbf{a}^{(j)}_{i_1,\dots,i_j}$ imply that, for each fixed $j = 1,\dots,d$, the expression being summed over $I \in \mathscr{I}^j_1$ depends only on the cardinality of the set I. Since, for each $k = 0,\dots,j$, there are $\binom{j}{k}$ subsets I of $\{1,\dots,j\}$ of cardinality k, we conclude that

$$\begin{aligned} &\tfrac{1}{2}(\mathbf{Q} - \mathbf{Q}') \\ &= \sum_{j=1}^{d} \sum_{k=0}^{j-1} m^j_k \sum_{i_1=1}^{\infty} \cdots \sum_{i_j=1}^{\infty} \prod_{n=1}^{k} \tfrac{1}{2}(\varepsilon_{i_n} + \varepsilon'_{i_n}) \prod_{n=k+1}^{j} \tfrac{1}{2}(\varepsilon_{i_n} - \varepsilon'_{i_n})\mathbf{a}^{(j)}_{i_1,\dots,i_j}. \end{aligned} \tag{2.8}$$

Finally, (2.2) follows from (2.8) and Lemma 2.2. □

3. Concentration inequality for multilinear forms. The main purpose of this section is to extend Kanter's (1976) concentration inequality to multilinear forms. Recall that in our situation (of Rademacher forms) Kanter's concentration inequality can be stated in the following way. Let $\mathbf{Q}$ be a Rademacher form (2.1) with $d = 1$. Let C be a symmetric convex measurable subset of the topological vector space E. If, for some $n \geq 1$, the cardinality of the set $\{i: \mathbf{a}^{(1)}_i \notin C\}$ is at least n, then

$$P(\mathbf{Q} \in C) \leq c_1(1 + n)^{-1/2}, \tag{3.1}$$

where $0 < c_1 < \infty$ is an absolute constant; that is, if many of $\mathbf{a}^{(1)}_i$'s are not in C, then $\mathbf{Q}$ itself is very unlikely to be in C.

Now let $\mathbf{Q}$ be a multilinear form (2.1), with $d > 1$. Can one prove a concentration-type inequality for such forms under the assumption that "many of $\mathbf{a}^{(d)}_{i_1,\dots,i_d}$'s are not in C"? One observes immediately that "numerical size alone" does not suffice for this purpose.

EXAMPLE 3.1. Let $E = R$ and $d = 2$, and let the set C be the singleton $\{0\}$. Let $a^{(2)}_{1j} = a^{(2)}_{2j} = 1$, for $j = 3,4,\dots,n+2$, and 0 for all other subscripts. Further, let $\mathbf{a}^{(0)} = 0$, and let $\mathbf{a}^{(1)}_i = 0$ for all $i \geq 1$. Then, no matter what n is, the probability $P(\mathbf{Q} \in C)$ is at least $\frac{1}{2}$.

The set of $(i_1,\dots,i_d)$ for which $\mathbf{a}^{(d)}_{i_1,\dots,i_d} \notin C$ in the previous example was "too thin." This leads us to introduce the following definition.

DEFINITION 3.1. Let $F \subset \mathbf{N}^d$ and $n \geq 1$. Define recurrently sets F_k, for $k = d, d-1,\dots,1$, as follows: $F_d \coloneqq F$ and if $k < d$,

$$F_k \coloneqq \{(i_1,\dots,i_k): \operatorname{card}\{j: (i_1,\dots,i_k,j) \in F_{k+1}\} \geq n\}.$$

We say that F *admits sections of size* n if all F_k, $1 \leq k \leq d$, are nonempty and $\operatorname{card}(F_1) \geq n$.

What kind of sets admit sections of size n for a possibly large n? Here are a few examples.

EXAMPLE 3.2. Let $I_j \subset \mathbf{N}$, for $j = 1, \dots, d$, be sets of cardinality at least n each. Then $F = I_1 \times \cdots \times I_d$ (a *box*) clearly admits sections of size n.

EXAMPLE 3.3. Let $F \subset \{1, 2, \dots, m\}^d$ and let the cardinality of F be at least pm^d, for some $0 < p < 1$. We claim that F admits sections of size $n = c(d, p)m$, where $c(d, p) = p2^{1-d}$ [so that $n \to \infty$ when $\operatorname{card}(F) \to \infty$ and p remains bounded away from zero]. The case $d = 1$ is trivial. Assuming the truth of the statement for $d - 1 \geq 1$, consider an $F \subset \{1, 2, \dots, m\}^d$ as above. It is elementary that the cardinality of the set

$$H_1 = \big\{ i_1 \in \{1, 2, \dots, m\} : (i_1, i_2, \dots, i_d) \in F \text{ for at least } pm^{d-1}/2 \text{ different } (i_2, \dots, i_d) \big\}$$

is at least $(p/(2 - p))m$. For each $i_1 \in H_1$, by the induction hypothesis, there is a set $G_{i_1} \subset \mathbf{N}^{d-1}$ admitting sections of size $(p/2)2^{2-d}m$ and $(i_1, i_2, \dots, i_d) \in F$ for each $(i_2, \dots, i_d) \in G_{i_1}$. Therefore, F admits sections of size $c(d, p)m$, with

$$c(d, p) = \min\left(\frac{p}{2 - p}, \frac{p}{2} 2^{2-d} \right).$$

EXAMPLE 3.4. The previous two examples notwithstanding, the set admitting sections of size n can be very sparse. For example, for $d = 2$ take $F = \{(i, j): i = 1, 2, \dots, n, \ j = n^{in+1}, n^{in+2}, \dots, n^{(i+1)n}\}$.

We need the following simple lemma.

LEMMA 3.1. *Let $F \subset \mathbf{N}^d$ admit sections of size $n \geq 2^d$. Then there is a subset $G \subset F$ admitting sections of size at least $[n^{1/d}]$, such that the projections of G on the axes are pairwise disjoint.*

PROOF. Choose an $m \leq n$, and take any m of the indices in F_1. For each one of these m indices i_1, choose m indices i_2 *not equal to any of the above* m indices i_1 and such that $(i_1, i_2) \in F_2$. This is possible when $m \leq n - m$.

For each resulting pair (i_1, i_2), choose m indices i_3 *not equal to any of the above* indices i_1 and i_2 and such that $(i_1, i_2, i_3) \in F_3$. This can be done as long as $m \leq n - m - m^2$.

We continue in the same way. We can obtain a set $G \subset F$ admitting sections of size m such that the projections of G on the axes are pairwise disjoint as long as

$$m \leq n - m - m^2 - \cdots - m^{d-1}.$$

It is clear that if $n \geq 2^d$, then $m = [n^{1/d}]$ satisfies the above conditions. □

DEFINITION 3.2. A Rademacher form (2.1) such that, for each $j = 1, \dots, d$, the array of vectors $\mathbf{a}^{(j)}_{i_1, \dots, i_j}$ is invariant under permutations of the subscripts is called symmetric.

Our next theorem is a concentration inequality for multilinear forms. It is the crucial tool in the proof of the zero–one law in the next section, but is also of interest of its own.

THEOREM 3.1. *Let C be a symmetric convex measurable subset of a topological vector space E. Let $\mathbf{Q}$ be a convergent E-valued symmetric Rademacher form of degree not exceeding d, given by* (2.1). *Suppose that the set*

$$F := \{(i_1,\dots,i_d) \in \mathbf{N}^d: d\mathbf{a}^{(d)}_{i_1,\dots,i_d} \notin C\} \tag{3.2}$$

admits sections of size n. Then

$$P(\mathbf{Q} \in C) \le c_d(1+n)^{-\alpha_d}, \tag{3.3}$$

where

$$\alpha_d = \{2^{3d-2}d!\}^{-1} \tag{3.4}$$

and $c_d \in (0,\infty)$ is a constant that depends only on d.

PROOF. We prove the theorem first under the following additional assumption:

(3.5) the projections of F on the axes are pairwise disjoint.

The proof is by induction on d. Observe that the truth of our statement for $d = 1$ follows from Kanter's concentration inequality (1976). Assume that our statement holds for some $d - 1 \ge 1$, and let us prove it for d.

We start with rewriting (2.1) in the form

$$\mathbf{Q} = \sum_{i_1=1}^{\infty} \cdots \sum_{i_d=1}^{\infty} \varepsilon_{i_1} \cdots \varepsilon_{i_j} \mathbf{a}_{i_1,\dots,i_d} + \mathbf{R}, \tag{3.6}$$

where $\mathbf{R}$ is a multilinear form of order not exceeding $d-1$, and where we have dropped the superscript of $\mathbf{a}^{(d)}_{i_1,\dots,i_d}$.

Let $\mathbf{Q}'$ be an independent copy of $\mathbf{Q}$. Since C is a symmetric convex set,

$$P(\mathbf{Q} \in C)^2 \le P\big(\tfrac{1}{2}(\mathbf{Q} - \mathbf{Q}') \in C\big). \tag{3.7}$$

We now apply Lemma 2.1, to conclude that

$$\begin{aligned} P(\mathbf{Q} \in C)^2 \le P\Bigg(& d \sum_{i_1=1}^{\infty} \cdots \sum_{i_{d-1}=1}^{\infty} \varepsilon_{i_1} \cdots \varepsilon_{i_{d-1}} \delta_{i_1} \cdots \delta_{i_{d-1}} \\ & \times \sum_{i_d=1}^{\infty} \varepsilon'_{i_d}(1-\delta_{i_d})\mathbf{a}_{i_1,\dots,i_d} + \mathbf{R}' \in C\Bigg), \end{aligned} \tag{3.8}$$

where ε_j, $j \ge 1$, ε'_j, $j \ge 1$, and δ_j, $j \ge 1$, are, correspondingly, two Rademacher sequences and a Bernoulli $(\frac{1}{2})$ sequence living on, say, probability spaces

$(\Omega_1, \mathscr{F}_1, P_1)$, $(\Omega_2, \mathscr{F}_2, P_2)$, and $(\Omega_3, \mathscr{F}_3, P_3)$. Furthermore, for every fixed $(\omega_2, \omega_3) \in \Omega_2 \times \Omega_3$, $\mathbf{R}'$ is a symmetric Rademacher form in ε_j, $j \geq 1$, of degree strictly less than $d - 1$.

We will use the following simple statement. There is a finite positive constant c such that, for every $n \geq 1$,

$$(3.9) \qquad P_3\left(\delta_1 + \cdots + \delta_n < \frac{n}{4}\right) \leq c \exp\left(-\frac{n}{c}\right).$$

Let $m \leq n$ be an integer to be chosen later. Choose an arbitrary set $G_1 \subset F_1$ of cardinality m. By (3.9), apart from an event of probability at most $c\exp(-m/c)$, there are $[m/4]$ indices $i_1 \in G_1$ such that $\delta_{i_1} = 1$. Call the obtained (random) set of indices H_1.

Now, for every $i_1 \in H_1$, choose m indices i_2 such that $(i_1, i_2) \in F_2$. Again, by (3.9), apart from an event of probability at most $c\exp(-m/c)$, there are $[m/4]$ indices i_2 out of the above such that $\delta_{i_2} = 1$. Call the resulting random set of pairs H_2.

Continue in this way. We conclude that, apart from an event of probability at most

$$(3.10) \qquad c\exp\left(-\frac{m}{c}\right)\left(1 + \frac{m}{4} + \cdots + \left(\frac{m}{4}\right)^{d-2}\right),$$

there is a random set $H_{d-1} \subset F_{d-1}$ that admits sections of size $[m/4]$ such that for every $(i_1, \dots, i_{d-1}) \in H_{d-1}$ we have $\delta_{i_1} \cdots \delta_{i_{d-1}} = 1$.

Now, by Kanter's inequality, apart from an event of probability at most

$$(3.11) \qquad c(1+n)^{-1/2}\left(\frac{m}{4}\right)^{d-1},$$

all the vectors $\sum_{i_d=1}^{\infty} \varepsilon'_{i_d}(1 - \delta_{i_d}) d\mathbf{a}_{i_1, \dots, i_d}$ are not in C, provided $(i_1, \dots, i_{d-1}) \in H_{d-1}$.

We are now in a position to apply the induction hypothesis [on the probability space $(\Omega_1, \mathscr{F}_1, P_1)$], to conclude that

$$P(\mathbf{Q} \in C)^2 \leq c\left(\frac{m}{4}\right)^{-\alpha_{d-1}} + c(1+n)^{-1/2}\left(\frac{m}{4}\right)^{d-1} + c\exp\left(-\frac{m}{c}\right)\left(1 + \frac{m}{4} + \cdots + \left(\frac{m}{4}\right)^{d-2}\right).$$

Choosing $m = [(1+n)^{\rho}]$ with $\rho = 1/[2(d - 1 + \alpha_{d-1})]$, we conclude that

$$P(\mathbf{Q} \in C)^2 \leq c(1+n)^{-2\alpha_d}$$

with

$$0 < \alpha_d \leq \frac{\alpha_{d-1}}{4(d - 1 + \alpha_{d-1})}.$$

It is elementary to see that we may take

$$\alpha_d = \{2^{3d-2}(d-1)!\}^{-1}. \tag{3.12}$$

Therefore, under the assumption (3.5), the statement of the theorem holds with α_d given by (3.12).

In the general case we apply Lemma 3.1 to reduce the situation to the case when (3.5) holds, but n has been replaced by $[n^{1/d}]$. Therefore, we obtain our general concentration inequality with α_d given by (3.4). This ends the proof. □

REMARK. There is no claim that the powers of n given by (3.4) are optimal. We remark here that in the case when the set (3.2) contains a *box* (Example 3.2), a simple modification of the argument yields

$$\alpha_d = \{2^{2d-1}(d-1)!\}^{-1}.$$

We conclude this section with a simple selection lemma.

LEMMA 3.2. *Let $\tau_1, \tau_2, \dots$ be i.i.d. random variables taking values in $(S, \mathscr{S})$. Let $A \in \mathscr{S}^d$, $d \ge 1$, be such that*

$$P((\tau_1, \tau_2, \dots, \tau_d) \in A) > 0.$$

Then, with probability 1, *the random set*

$$H = \{(n_1, \dots, n_d) \in \mathbf{N}^d : (\tau_{n_1}, \dots, \tau_{n_d}) \in A\}$$

contains a box of arbitrarily large size. More precisely, there exists an $\Omega_0 \subset \Omega$ with $P(\Omega_0) = 1$ such that for every $\omega \in \Omega_0$ and $n \in \mathbf{N}$ there exists $k = k(\omega, n)$ such that

$$(\tau_{n_1}(\omega), \dots, \tau_{n_d}(\omega)) \in A,$$

for all $k < n_1 \le k + n$, $k + n < n_2 \le k + 2n, \dots, k + (d-1)n < n_d \le k + dn$.

PROOF. Let

$$X_k = \prod_{n_1=k+1}^{k+n} \cdots \prod_{n_d=k+(d-1)n+1}^{k+dn} \mathbf{1}_A(\tau_{n_1}, \tau_{n_2}, \dots, \tau_{n_d});$$

$\{X_k : k \ge 0\}$ is a sequence of identically distributed Bernoulli random variables which contains an i.i.d. subsequence $\{X_{idn} : i \ge 0\}$. Therefore, it is enough to show that $P\{X_0 = 1\} > 0$; that is,

$$p_d(A) := E\left[\prod_{n_1=1}^{n} \cdots \prod_{n_d=(d-1)n+1}^{dn} \mathbf{1}_A(\tau_{n_1}, \tau_{n_2}, \dots, \tau_{n_d})\right] > 0. \tag{3.13}$$

The proof of (3.13) is by induction in d. It is obvious for $d = 1$. Suppose (3.13) is true for all sets in $\mathscr{S}^{d-1}$, $d - 1 \ge 1$, satisfying the condition of the lemma. Notice that

$$p_d(A) = \int_{S^n} p_{d-1}\left(\bigcap_{i=1}^{n} A^{y_i}\right) \mu(dy_1) \cdots \mu(dy_n),$$

where $\mu = P \circ \tau_i^{-1}$ and $A^y = \{(x_1, \dots, x_{d-1}) \colon (x_1, \dots, x_{d-1}, y) \in A\}$. Hence it is enough to show that

$$P\left((\tau_1, \dots, \tau_{d-1}) \in \bigcap_{i=1}^{n} A^{y_i}\right) > 0,$$

for $(y_1, \dots, y_n)$ from a set of positive $\mu^{\otimes n}$-measure. The last statement is true because

$$\int_{S^n} P\left((\tau_1, \dots, \tau_{d-1}) \in \bigcap_{i=1}^{n} A^{y_i}\right) \mu(dy_1) \cdots \mu(dy_n) = E\left[\mu\left(A_{\tau_1, \dots, \tau_{d-1}}\right)^n\right],$$

where

$$A_{x_1, \dots, x_{d-1}} := \{y \colon (x_1, \dots, x_{d-1}, y) \in A\},$$

and the last expectation is positive since

$$E\left[\mu\left(A_{\tau_1, \dots, \tau_{d-1}}\right)\right] = P((\tau_1, \dots, \tau_d) \in A) > 0. \qquad \square$$

4. 0–1 law. We remind the reader that $\mathbf{X} = \{X(t),\, t \in T\}$ is a stochastic process defined by (1.6) and (1.7). We will use the notation $\mathbf{f}_j(s_1, \dots, s_d) = (f(t; s_1, \dots, s_d),\, t \in T) \in \mathbf{R}^T$.

The following is the main result of this paper: the zero–one law for finite sums of multiple stochastic integrals under assumption (1.8). The crucial tool in our argument is the series representation (1.9), which will allow us to use the machinery developed in the previous sections. In terms of the series representation (1.9), our assumption (1.8) means $R(\Gamma_n, \tau_n) > 0$ a.s. for all n.

THEOREM 4.1. *Let $\mathbf{X} = \{X(t),\, t \in T\}$ be given in the form* (1.6), *with the random measure M satisfying assumption* (1.8). *Let V be a measurable linear subspace of $\mathbf{R}^T$. Then*

$$P(\mathbf{X} \in V) = 0 \text{ or } 1. \tag{4.1}$$

Moreover, if the probability in (4.1) *equals* 1, *then the following processes obtained by a reduction of $\mathbf{X}$ to sums of lower-order iterated integrals must belong to V with probability* 1: *for every $i = 1, \dots, d$ and for $\lambda^{\otimes i}$-almost all $(u_1, \dots, u_i) \in S^i$,*

$$P\left(\mathbf{X}_{i; u_1, \dots, u_i} \in V\right) = 1, \tag{4.2}$$

where

$$\begin{aligned} \mathbf{X}_{i; u_1, \dots, u_i} &= \left\{X_{i; u_1, \dots, u_i}(t),\, t \in T\right\} \\ &= \sum_{j=i}^{d} \frac{j!}{(j-i)!} I_{j-i}\left(\mathbf{f}_{j; u_1, \dots, u_i}\right), \end{aligned} \tag{4.3}$$

and where $f_{j; u_1, \dots, u_i} \colon S^{j-i} \to \mathbf{R}^T$ is given by

$$f_{j; u_1, \dots, u_i}(t; s_1, \dots, s_{j-i}) = f_j(t; u_1, \dots, u_i, s_1, \dots, s_{j-i}).$$

PROOF. The proof is by induction in d. If $d = 0$, then the theorem holds trivially. Assume that it holds for all $l \le d-1$, and we will show it for $l = d$. We will use the series representation $\mathbf{Y}$ of $\mathbf{X}$ [see (1.9)]. If $P(\mathbf{Y} \in V) = 0$, then there is nothing to prove. Suppose that $P(\mathbf{Y} \in V) > 0$. Define, for $1 \le i \le d-1$ and $k > 0$,

$$\begin{aligned}\mathbf{Y}_{i,k}(u_1,\dots,u_i) &:= i!\mathbf{f}_i(u_1,\dots,u_i)\\ &\quad + \sum_{j=i+1}^{d} j! \sum_{k<n_{i+1}<\cdots<n_j} \left(\prod_{h=i+1}^{j} \varepsilon_{n_h} R(\Gamma_{n_h}, \tau_{n_h})\right)\\ &\qquad \times \mathbf{f}_j(u_1,\dots,u_i,\tau_{n_{i+1}},\dots,\tau_{n_j})\end{aligned}$$

and

$$\mathbf{Y}_{d,k}(u_1,\dots,u_d) := \mathbf{f}_d(u_1,\dots,u_d).$$

Notice that $\mathbf{Y}_{i,0}(u_1,\dots,u_i) \stackrel{\mathrm{d}}{=} \mathbf{X}_{i;u_1,\dots,u_i}$. First we will show that, for every $1 \le i \le d$ and $k \ge 0$,

$$P\{\mathbf{Y}_{i,k}(u_1,\dots,u_i) \in V\} = 1 \quad \text{for } \lambda^{\otimes i}\text{-a.a. } (u_1,\dots,u_i) \in S^i. \tag{4.4}$$

We have the following identities: for every $1 \le i \le d$,

$$\mathbf{Y} = \sum_{1 \le n_1 < \cdots < n_i} \left(\prod_{l=1}^{i} \varepsilon_{n_l} R(\Gamma_{n_l}, \tau_{n_l})\right) \mathbf{Y}_{i,n_i}(\tau_{n_1},\dots,\tau_{n_i}) + \mathbf{Q}_i, \tag{4.5}$$

where $\mathbf{Q}_i$ is a multilinear form in $\varepsilon_1, \varepsilon_2, \dots$ of order less than i, and, for every $1 \le i \le d-1$ and $0 \le k < k'$,

$$\begin{aligned}&\mathbf{Y}_{i,k}(u_1,\dots,u_i) - \mathbf{Y}_{i,k'}(u_1,\dots,u_i)\\ &\quad = \sum_{n=k+1}^{k'} \varepsilon_n R(\Gamma_n, \tau_n) \mathbf{Y}_{i+1,n}(u_1,\dots,u_i,\tau_n).\end{aligned} \tag{4.6}$$

We will prove (4.4) by decreasing induction in $i = d, \dots, 1$. If $i = d$, then we need to show that

$$P(\mathbf{f}_d(\tau_1,\dots,\tau_d) \in V) = 1.$$

Suppose that this is not the case. Then, by the selection Lemma 3.2, the set

$$H = \{(n_1,\dots,n_d) : \mathbf{f}_d(\tau_{n_1},\dots,\tau_{n_d}) \notin V\}$$

contains an arbitrarily large box with probability 1. By the concentration inequality (3.1),

$$P(\mathbf{Y} \in V \mid \{\tau_n\}, \{\Gamma_n\}) = 0,$$

contradicting our assumption that $P\{\mathbf{Y} \in V\} > 0$.

Now we assume that (4.4) holds for $i+1 \le d$ and we will prove it for i. By (4.6) we get, for every $0 \le k < k'$ and for $\lambda^{\otimes i}$-a.a. $(u_1,\dots,u_i) \in S^i$,

$$\mathbf{Y}_{i,k}(u_1,\dots,u_i) - \mathbf{Y}_{i,k'}(u_1,\dots,u_i) \in V \quad \text{a.s.} \tag{4.7}$$

Let $k > d$ be an arbitrary integer. By (4.5) we have

$$\begin{aligned}\mathbf{Y} = {} & \sum_{1\le n_1<\cdots<n_i\le k}\left(\prod_{l=1}^{i}\varepsilon_{n_l}R(\Gamma_{n_l},\tau_{n_l})\right)\mathbf{Y}_{i,k}(\tau_{n_1},\dots,\tau_{n_i})\\ & +\sum_{1\le n_1<\cdots<n_i\le k}\left(\prod_{l=1}^{i}\varepsilon_{n_l}R(\Gamma_{n_l},\tau_{n_l})\right)\\ & \times\left\{\mathbf{Y}_{i,n_i}(\tau_{n_1},\dots,\tau_{n_i})-\mathbf{Y}_{i,k}(\tau_{n_1},\dots,\tau_{n_i})\right\}\\ & +\mathbf{W}_{i,k}+\mathbf{Q}_i,\end{aligned}\tag{4.8}$$

where $\mathbf{W}_{i,k}$ is a multlinear form in $\varepsilon_1,\dots,\varepsilon_k$ of order less than i. The second term on the right-hand side belongs to V a.s. by (4.7). The first term is of order i in $\varepsilon_1,\dots,\varepsilon_k$, and $Y_{i,k}$ is independent of $\varepsilon_1,\dots,\varepsilon_k$.

Consider the set

$$A = \left\{(u_1,\dots,u_i)\in S^i\colon P\{\mathbf{Y}_{i,0}(u_1,\dots,u_i)\notin V\}=1\right\}.$$

From (4.7) we get $P\{\mathbf{Y}_{i,k}(u_1,\dots,u_i)\notin V\}=1$ for $\lambda^{\otimes i}$-a.a. $(u_1,\dots,u_i)\in A$ and all $k\ge 0$.

Suppose that $P\{(\tau_1,\dots,\tau_i)\in A\}>0$. By Lemma 3.2, for every $n\ge 1$ there exists an integer-valued random variable K such that, with probability 1, $(\tau_1,\dots,\tau_i)\in A$ for every $K<n_1\le K+n,\dots,K+(i-1)n<n_i\le K+in$. Choose $k\in\mathbf{N}$ such that $P\{K\le k-in\}\ge 1-\varepsilon$, where $\varepsilon>0$ is arbitrary. Applying the concentration inequality (3.3) conditionally to (4.8) we get, on the set $\{K\le k-in\}$,

$$P\left(\mathbf{Y}\in V\mid\{\tau_j\},\{\Gamma_j\},\{\varepsilon_j\}_{j=k+1}^{\infty}\right)\le c_i(1+n)^{-\alpha_i}\quad\text{a.s.}$$

Since n and ε are arbitrary, we infer that $P\{\mathbf{Y}\in V\}=0$, which is a contradiction. Hence $P\{(\tau_1,\dots,\tau_i)\in A\}=0$, that is, $P\{\mathbf{Y}_{i,0}(u_1,\dots,u_i)\in V\}>0$ for $\lambda^{\otimes i}$-a.a. $(u_1,\dots,u_i)$. Since $\mathbf{Y}_{i,0}(u_1,\dots,u_i)\overset{\mathrm{d}}{=}\mathbf{X}_{i;u_1,\dots,u_i}$ is a sum of multiple integrals of order no greater than $d-i\le d-1$, $P\{\mathbf{Y}_{i,0}(u_1,\dots,u_i)\in V\}=1$ for $\lambda^{\otimes i}$-a.a. $(u_1,\dots,u_i)$. This in conjunction with (4.7) establishes (4.4) and the second part of the theorem.

Now we will prove that $P\{\mathbf{Y}\in V\}=1$. By (4.5) we have

$$\mathbf{Y}=\sum_{n=1}^{\infty}\varepsilon_n R(\Gamma_n,\tau_n)\mathbf{Y}_{1,n}(\tau_n)+\mathbf{f}_0,$$

and, by (4.4), $\mathbf{Y}_{1,n}(\tau_n)\in V$ a.s. Hence, for every $k\ge 0$,

$$\{\mathbf{Y}\in V\}=\left\{\sum_{n=k+1}^{\infty}\varepsilon_n R(\Gamma_n,\tau_n)\mathbf{Y}_{1,n}(\tau_n)+\mathbf{f}_0\in V\right\}$$

modulo P. Since $\mathbf{Y}$ is a function of the i.i.d. sequence $\{\varepsilon_n,\Gamma_n-\Gamma_{n-1},\tau_n\}_{n\ge 1}$, the Hewitt–Savage zero–one law yields $P\{\mathbf{Y}\in V\}=0$ or 1, and under our assumption this probability must be 1. □

We conclude this paper with a short discussion of implications of Theorem 4.1 to *zero laws*. First we observe that the second part of the theorem can be reformulated to say that

$$P(\mathbf{X} \in V) = 0 \tag{4.9}$$

if one of conditions (4.2) fails.

We can be more explicit in particular situations. Many measurable linear subspaces V have *property* (C).

DEFINITION 4.1. We say that a Borel subset B of $\mathbf{R}^T$ has property (C) if for any $\mathbf{x} \notin B$, any probability measure μ on $\mathbf{R}^T$ with $\mu(B) = 1$ and any $\varepsilon > 0$ there exists a convex symmetric Borel set $D \subset B$ such that $\mathbf{x} \notin \overline{D}$ and $\mu(D) > 1 - \varepsilon$.

Examples of subspaces V with property (C) include the following: (i) separable complete locally convex spaces V of real functions on T such that the natural embedding of V into $\mathbf{R}^T$ is continuous; if, for example, T is countable, then also (ii) $l^\infty(T)$; (iii) generalized Orlicz spaces of real functions on T [which include such nonlocally convex spaces as $l^p(T)$, $0 < p < 1$, etc.]. We refer the reader to Rosiński and Samorodnitsky (1994) for more details. An immediate application of the above zero law and of Rosiński and Samorodnitsky [(1994), Theorem 5.1] establishes the following zero law.

COROLLARY 4.1. *Under the assumption of Theorem* 4.1 *suppose that the subspace V has property* (C). *If, for some* $j = 0, 1, \dots, d$,

$$P\big(\mathbf{f}_j(\tau_1, \dots, \tau_j) \notin V\big) > 0, \tag{4.10}$$

then the zero law (4.9) *holds.*

Alternatively, we can obtain a conclusion similar to that of Corollary 4.1 without assuming property (C) but assuming instead a special structure on the random measure M in (1.7). Recall that a symmetric infinitely divisible random measure M is called (symmetric) r-semistable index α [or r-SS(α)], $0 < r < 1$, $0 < \alpha < 2$, if, for every $n \geq 1$,

$$M^{*r^n} \overset{\mathrm{d}}{=} r^{n/\alpha} M, \tag{4.11}$$

where $\{M^{*\gamma}(A),\ A \in \mathscr{S}\}$ is an infinitely divisible random measure such that, for every $A \in \mathscr{S}$ and $\theta \in \mathbf{R}$, $E \exp[i\theta M^{*\gamma}(A)] = (E \exp[i\theta M(A)])^\gamma$. We refer the reader to Chung, Rajput and Tortrat (1982) for more information on semistable measures. The following zero law is an immediate consequence of Theorem 4.1 and the argument of Proposition 5.2 of Rosiński and Samorodnitsky (1994). In fact, it generalizes the latter from stable to the semistable case.

COROLLARY 4.2. *Under the assumption of Theorem* 4.1 *suppose that the random measure M is r-SS(α), $0 < r < 1$, $0 < \alpha < 2$. If* (4.10) *holds for some* $j = 0, 1, \dots,$ *then the zero law* (4.9) *holds.*

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