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Erratum: Nonlinear filtering for reflecting diffusions in random environments via nonparametric estimation*

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Abstract

This is an erratum to EJP paper number 18, volume 9, Nonlinear filtering for reflecting diffusions in random environments via nonparametric estimation.

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Equation (2.2) in [4] is incorrect, which puts the proof of Theorem 2 [4, Appendix] in doubt. The theorem is true as stated. In the following, we will revise the places in [4, Appendix] where equation (2.2) is used.

As in [4], we let $p^0(t,x,y)$ be the transition density function of X_t^0. Theorem 3.1, Theorem 3.4 and Lemma 4.3 in [2] imply that

$$p^0(t,x,y) \le c_1 t^{-d/2} \exp(-|x-y|^2/c_2 t), \quad \forall t>0, x,y \in \overline{D}, \tag{0.1}$$

and

$$p^0(t,x,y) \ge c_3 t^{-d/2}, \quad \forall t>0, x,y \in \overline{D} \text{ such that } |x-y| \le \varepsilon\sqrt{t}, \tag{0.2}$$

where $c_1, c_2, c_3, \varepsilon > 0$ are constants independent of x, y, t.

We denote by $p(t,x,y)$ the transition density function of X_t and $\mathcal{E}$. It is known that (0.1) and (0.2) are quasi-isometry stable (cf. the remark before Section 1.2 and the remark after Theorem 1.2 in [3]). For any $M > 0$, there exist constants $c_1(M), c_2(M), c_3(M), \varepsilon(M) > 0$ independent of x, y, t, such that if $\|W\|_\infty \le M$ then

$$p(t,x,y) \le c_1(M) t^{-d/2} \exp(-|x-y|^2/c_2(M) t), \quad \forall t>0, x,y \in \overline{D}, \tag{0.3}$$

and

$$p(t,x,y) \ge c_3(M) t^{-d/2}, \quad \forall t>0, x,y \in \overline{D} \text{ such that } |x-y| \le \varepsilon(M)\sqrt{t}. \tag{0.4}$$

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It is known that (0.3) and (0.4) imply that (cf. [1, Corollary 4.2]) for any $M > 0$, there exist constants $c_4(M), 0 < \alpha(M) < 1$ independent of x, x', t, such that if $\|W\|_\infty \leq M$ and $f \in B_b(\overline{D})$ satisfy $\|f\|_\infty \leq M$ then

$$\left| \int_D p(t,x,y) f(y) \mu(dy) - \int_D p(t,x',y) f(y) \mu(dy) \right| \leq c_4(M)|x - x'|^{\alpha(M)}, \ \forall t > 0, x, x' \in \overline{D}. \tag{0.5}$$

Hence X_t is a strong Feller diffusion.

We define on $L^2(\overline{D}; dx)$ the symmetric bilinear form

$$\begin{cases} \mathcal{A}^W(u,v) = \frac{1}{2} \int_D \sum_{i,j=1}^d a_{ij}(x) \frac{\partial (ue^{W/2})}{\partial x_i}(x) \frac{\partial (ve^{W/2})}{\partial x_j}(x) e^{-W(x)} dx, \quad u, v \in D(\mathcal{A}^W), \\ D(\mathcal{A}^W) = \{u \in L^2(D; dx) : ue^{W/2} \in H^{1,2}(D)\}. \end{cases}$$

Let $W_n \in B_b(\overline{D})$, $n \in \mathbb{N}$, satisfy $\lim_{n\to\infty} \|W_n - W\|_\infty = 0$. Similar to [5, Lemma, page 864], we can show that the form $\mathcal{A}^{W_n}$ is Mosco-convergent to the form $\mathcal{A}^W$ on $L^2(\overline{D}; dx)$, equivalently, $(s_t^{W_n})_{t>0}$ converges to $(s_t^W)_{t>0}$ strongly on $L^2(\overline{D}; dx)$, where $(s_t^{W_n})_{t>0}$ and $(s_t^W)_{t>0}$ denote the semigroups of $\mathcal{A}^{W_n}$ and $\mathcal{A}^W$, respectively. Note that for $f \in B_b(\overline{D})$, we have

$$p_t f = e^{W/2} s_t^W (e^{-W/2} f), \quad \forall t > 0.$$

Denote by $(p_t^n)_{t>0}$ the semigroup associated with X^n. Then, we obtain by Theorem 1 in [4] that $p_t^n f$ converges to $p_t f$ on $L^2(\overline{D}; dx)$ for any $f \in B_b(\overline{D})$ and $t > 0$. Therefore, we obtain by (0.5) that for any sequence $\{\nu^n\}$ of probability measures on $\overline{D}$ converging weakly to some probability measure ν on $\overline{D}$, $(X_0^n, X_{t_1}^n)$ with the initial distribution ν^n converges weakly to (X_0, X_{t_1}) with the initial distribution ν.

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