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N-JORDAN $*$-HOMOMORPHISMS IN C^*-ALGEBRAS

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Abstract. In this paper, we investigate n-Jordan $*$-homomorphisms in C^*-algebras associated with the following functional inequality

$$\left\| f\left(\frac{b-a}{3}\right) + f\left(\frac{a-3c}{3}\right) + f\left(\frac{3a+3c-b}{3}\right) \right\| \le \|f(a)\|.$$

We moreover prove the superstability and the Hyers-Ulam stability of n-Jordan $*$-homomorphisms in C^*-algebras associated with the following functional equation

$$f\left(\frac{b-a}{3}\right) + f\left(\frac{a-3c}{3}\right) + f\left(\frac{3a+3c-b}{3}\right) = f(a).$$

1. Introduction and Preliminaries

Let A,B be complex algebras. A $\mathbb{C}$-linear mapping $h : A \to B$ is called an n-Jordan homomorphism if $h(a^n) = h(a)^n$ for all $a \in A$. The concept of n-Jordan homomorphisms was studied for complex algebras by Eshaghi Gordji et al. [3] (see also [4, 9]).

In this paper, assume that n is an integer greater than 1.

Definition 1.1. ([10]). Let A, B be complex algebras. A $\mathbb{C}$-linear mapping $h : A \to B$ is called an n-Jordan homomorphism if

$$h(a^n) = h(a)^n$$

for all $a \in A$.

Definition 1.2. Let A, B be C^*-algebras. An n-Jordan homomorphism $h : A \to B$ is called an n-Jordan $*$-homomorphism if

$$h(a^*) = h(a)^*$$

for all $a \in A$.

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The stability of functional equations was first introduced by Ulam [22] in 1940. More precisely, he proposed the following problem: Given a group G_1, a metric group (G_2, d) and $\epsilon > 0$, does there exist a $\delta > 0$ such that if a mapping $f : G_1 \to G_2$ satisfies the inequality $d(f(xy), f(x)f(y)) < \delta$ for all $x, y \in G_1$, then there exists a homomorphism $T : G_1 \to G_2$ such that $d(f(x), T(x)) < \epsilon$ for all $x \in G_1$? As mentioned above, when this problem has a solution, we say that the homomorphisms from G_1 to G_2 are stable. In 1941, Hyers [11] gave a partial solution of $Ulam's$ problem for the case of approximate additive mappings under the assumption that G_1 and G_2 are Banach spaces. In 1950, Aoki [1] generalized the Hyers' theorem for approximately additive mappings. In 1978, Th.M. Rassias [21] generalized the theorem of Hyers by considering the stability problem with unbounded Cauchy differences.

During the last decades several stability problems of functional equations have been investigated by many mathematicians (see [2, 5, 6, 7, 12, 13, 14, 16, 15, 18, 19, 20]).

Miura et al. [17] proved the Hyers-Ulam stability of Jordan homomorphisms.

In this paper, we investigate the Hyers-Ulam stability of n-Jordan $*$-homomorphisms in C^*-algebras associated with the following functional inequality

$$\left\| f\left(\frac{b-a}{3}\right) + f\left(\frac{a-3c}{3}\right) + f\left(\frac{3a+3c-b}{3}\right) \right\| \leq \|f(a)\|.$$

We moreover prove the the Hyers-Ulam stability of n-Jordan $*$-homomorphisms in C^*-algebras associated with the following functional equation

$$f\left(\frac{b-a}{3}\right) + f\left(\frac{a-3c}{3}\right) + f\left(\frac{3a+3c-b}{3}\right) = f(a).$$

2. Main Results

Lemma 2.1. ([8]). *Let A, B be C^*-algebras, and let $f : A \to B$ be a mapping such that*

$$\left\| f\left(\frac{b-a}{3}\right) + f\left(\frac{a-3c}{3}\right) + f\left(\frac{3a+3c-b}{3}\right) \right\|_B \leq \|f(a)\|_B$$

for all $a, b, c \in A$. Then f is additive.

We prove the superstability of n-Jordan $*$-homomorphisms.

Theorem 2.2. *Let A, B be C^*-algebras, and let $p < 1$ and θ be nonnegative real numbers. Let $f : A \to B$ be a mapping such that*

$$\left\| f\left(\frac{b-a}{3}\mu\right) + f\left(\frac{a-3c}{3}\mu\right) + \mu f\left(\frac{3a+3c-b}{3}\right) \right\|_B \leq \|f(a)\|_B, \tag{2.1}$$

$$\|f(a^n) - f(a)^n\|_B \le \theta \|a\|_A^{np}, \tag{2.2}$$

$$\|f(a^*) - f(a)^*\|_B \le \theta \|a^*\|_A^p \tag{2.3}$$

for all $\mu \in \mathbb{T}^1 := \{\lambda \in \mathbb{C} : |\lambda| = 1\}$ *and all* $a, b, c \in A$. *Then the mapping* $f : A \to B$ *is an* n*-Jordan* $*$*-homomorphism.*

Proof. Let $\mu = 1$ in (2.1). By Lemma 2.1, the mapping $f : A \to B$ is additive. Letting $a = b = 0$ in (2.1), we get

$$\|f(-\mu c) + \mu f(c)\|_B \le \|f(0)\|_B = 0$$

for all $c \in A$ and all $\mu \in \mathbb{T}^1$. So

$$-f(\mu c) + \mu f(c) = f(-\mu c) + \mu f(c) = 0$$

for all $c \in A$ and all $\mu \in \mathbb{T}^1$. Hence $f(\mu c) = \mu f(c)$ for all $c \in A$ and all $\mu \in \mathbb{T}^1$. By [18, Theorem 2.1], the mapping $f : A \to B$ is $\mathbb{C}$-linear.

It follows from (2.2) that

$$\begin{aligned} \|f(a^n) - f(a)^n\|_B &= \left\| \frac{1}{m^n} f(m^n a^n) - \left(\frac{1}{m} f(ma) \right)^n \right\|_B \\ &= \frac{1}{m^n} \|f(m^n a^n) - f(ma)^n\|_B \\ &\le \frac{\theta}{m^n} m^{np} \|a\|_A^{np} \end{aligned}$$

for all $a \in A$. Since $p < 1$, by letting m tend to ∞ in the last inequality, we obtain $f(a^n) = f(a)^n$ for all $a \in A$.

It follows from (2.3) that

$$\begin{aligned} \|f(a^*) - f(a)^*\|_B &= \left\| \frac{1}{m} f(ma^*) - \frac{1}{m} f(ma)^* \right\|_B \\ &= \frac{1}{m} \|f(ma^*) - f(ma)^*\|_B \\ &\le \frac{\theta}{m} m^p \|a^*\|_A^p \end{aligned}$$

for all $a \in A$. Since $p < 1$, by letting m tend to ∞ in the last inequality, we obtain $f(a^*) = f(a)^*$ for all $a \in A$. Hence the mapping $f : A \to B$ is an n-Jordan $*$-homomorphism. ■

Theorem 2.3. *Let* A, B *be* C^**-algebras, and let* $p > 1$ *and* θ *be nonnegative real numbers. Let* $f : A \to B$ *be a mapping satisfying* (2.1), (2.2) *and* (2.3). *Then the mapping* $f : A \to B$ *is an* n*-Jordan* $*$*-homomorphism.*

Proof. The proof is similar to the proof of Theorem 2.2. ∎

Now we prove the Hyers-Ulam stability of n-Jordan homomorphisms in C^*-algebras.

Theorem 2.4. *Let A, B be C^*-algebras. Let $f : A \to B$ be an odd mapping for which there exists a function $\varphi : A \times A \times A \to \mathbb{R}^+$ such that*

$$\sum_{i=0}^{\infty} 3^{ni} \varphi\left(\frac{a}{3^i}, \frac{b}{3^i}, \frac{c}{3^i}\right) < \infty, \tag{2.4}$$

$$\|f(c^*) - f(c)^*\|_B \leq \varphi(c, c, c), \tag{2.5}$$

$$\left\| f\left(\frac{b-a}{3}\mu\right) + f\left(\frac{a-3c}{3}\mu\right) + \mu f\left(\frac{3a-b}{3} + c\right) - f(a) + f(c^n) - f(c)^n \right\|_B \leq \varphi(a, b, c) \tag{2.6}$$

for all $a, b, c \in A$ and all $\mu \in \mathbb{T}^1$. Then there exists a unique n-Jordan $*$-homomorphism $h : A \to B$ such that

$$\|h(a) - f(a)\|_B \leq \sum_{i=0}^{\infty} 3^i \varphi\left(\frac{a}{3^i}, \frac{2a}{3^i}, 0\right) \tag{2.7}$$

for all $a \in A$.

Proof. Letting $\mu = 1$, $b = 2a$ and $c = 0$ in (2.6), we get

$$\left\| 3f\left(\frac{a}{3}\right) - f(a) \right\|_B \leq \varphi(a, 2a, 0) \tag{2.8}$$

for all $a \in A$. Using the induction method, we have

$$\left\| 3^m f\left(\frac{a}{3^m}\right) - f(a) \right\|_B \leq \sum_{i=0}^{m-1} 3^i \varphi\left(\frac{a}{3^i}, \frac{2a}{3^i}, 0\right) \tag{2.9}$$

for all $a \in A$. In other to show that

$$h_m(a) = 3^m f\left(\frac{a}{3^m}\right)$$

form a convergent sequence, we use the Cauchy convergence criterion. Indeed, replacing a by $\frac{a}{3^k}$ and multiplying by 3^k in (2.9), where k is an arbitrary positive integer, we get that

$$\left\| 3^{k+m} f\left(\frac{a}{3^{k+m}}\right) - 3^k f\left(\frac{a}{3^k}\right) \right\|_B \leq \sum_{i=k}^{k+m-1} 3^i \varphi\left(\frac{a}{3^i}, \frac{2a}{3^i}, 0\right)$$

for all positive integers m. Hence by the Cauchy criterion, the limit

$$h(a) = \lim_{m\to\infty} h_m(a)$$

exists for each $a \in A$. By taking the limit as $m \to \infty$ in (2.9), we obtain that

$$\|h(a) - f(a)\| \le \sum_{i=0}^{\infty} 3^i \varphi\left(\frac{a}{3^i}, \frac{2a}{3^i}, 0\right)$$

and (2.7) holds for all $a \in A$. Letting $\mu = 1$ and $c = 0$ in (2.6), we get

$$\left\| f\left(\frac{b-a}{3}\right) + f\left(\frac{a}{3}\right) + f\left(\frac{3a-b}{3}\right) - f(a) \right\|_B \le \varphi(a, b, 0) \tag{2.10}$$

for all $a, b, c \in A$. Multiplying both sides (2.10) by 3^m and replacing a, b by $\frac{a}{3^m}$, $\frac{b}{3^m}$, respectively, we get

$$\begin{aligned}&\left\| 3^m f\left(\frac{b-a}{3^{m+1}}\right) + 3^m f\left(\frac{a}{3^{m+1}}\right) + 3^m f\left(\frac{3a-b}{3^{m+1}}\right) - 3^m f\left(\frac{a}{3^m}\right) \right\|_B \\ &\le 3^m \varphi\left(\frac{a}{3^m}, \frac{b}{3^m}, 0\right)\end{aligned}$$

for all $a, b \in A$. Taking the limit as $m \to \infty$, we obtain

$$h\left(\frac{b-a}{3}\right) + h\left(\frac{a}{3}\right) + h\left(\frac{3a-b}{3}\right) - h(a) = 0 \tag{2.11}$$

for all $a, b \in A$. Putting $b = 2a$ in (2.11), we get

$$3h\left(\frac{a}{3}\right) = h(a)$$

for all $a \in A$. Replacing a by $2a$ in (2.11), we get

$$h(b - 2a) + h(6a - b) = 2h(2a) \tag{2.12}$$

for all $a, b \in A$. Letting $b = 2a$ in (2.12), we get

$$h(4a) = 2h(2a)$$

for all $a \in A$. So

$$h(2a) = 2h(a)$$

for all $a \in A$. Letting $3a - b = s$ and $b - a = t$ in (2.11), we get

$$h\left(\frac{t}{3}\right) + h\left(\frac{s+t}{6}\right) + h\left(\frac{t}{3}\right) = h\left(\frac{s+t}{2}\right)$$

for all $s, t \in A$. Hence

$$h(s) + h(t) = h(s+t)$$

for all $s, t \in A$. So h is additive.

Letting $a = b = 0$ in (2.6) and using the above method, we have

$$h(\mu b) = \mu h(b)$$

for all $a \in A$ and all $\mu \in \mathbb{T}^1$. Hence by [18, Theorem 2.1], the mapping $f : A \to B$ is $\mathbb{C}$-linear.

Now, let $h_1 : A \to B$ be another $\mathbb{C}$-linear mapping satisfying (2.7). Then we have

$$\begin{aligned}
\|h(a) - h_1(a)\|_B &= 3^m \left\| h\left(\frac{a}{3^m}\right) - h_1\left(\frac{a}{3^m}\right) \right\|_B \\
&\le 3^m \left[\left\| h\left(\frac{a}{3^m}\right) - f\left(\frac{a}{3^m}\right) \right\|_B + \left\| h_1\left(\frac{a}{3^m}\right) - f\left(\frac{a}{3^m}\right) \right\|_B \right] \\
&\le 2 \sum_{i=m}^{\infty} 3^i \varphi\left(\frac{a}{3^i}, \frac{2a}{3^i}, 0\right) = 0
\end{aligned}$$

for all $a \in A$. So h is unique.

Letting $\mu = 1$ and $a = b = 0$ in (2.6), we get

$$\|f(-c) + f(c) + f(c^n) - f(c)^n\|_B \le \varphi(0, 0, c)$$

for all $c \in A$. So

$$\begin{aligned}
\|h(c^n) - h(c)^n\|_B &= \|h(-c) + h(c) + h(c^n) - h(c)^n\|_B \\
&= \lim_{m\to\infty} 3^{nm} \left\| f\left(\frac{-c}{3^{nm}}\right) + f\left(\frac{c}{3^{nm}}\right) + f\left(\frac{c^n}{3^{nm}}\right) - f\left(\frac{c}{3^m}\right)^n \right\|_B \\
&\le \lim_{m\to\infty} 3^{nm} \varphi\left(0, 0, \frac{c}{3^m}\right) = 0
\end{aligned}$$

for all $c \in A$. Hence

$$h(c^n) = h(c)^n$$

for all $c \in A$.

On the other hand, we have

$$\begin{aligned}
\|h(c^*) - h(c)^*\|_B &= \lim_{m\to\infty} 3^m \left\| f\left(\frac{c^*}{3^m}\right) - f\left(\frac{c}{3^m}\right)^* \right\|_B \\
&\le \lim_{m\to\infty} 3^m \varphi\left(\frac{c}{3^m}, \frac{c}{3^m}, \frac{c}{3^m}\right) = 0
\end{aligned}$$

for all $c \in A$. Hence

$$h(c^*) = h(c)^*$$

for all $c \in A$. Hence $h : A \to B$ is a unique n-Jordan $*$-homomorphism. ■

Corollary 2.5. *Let A, B be C^*-algebras, and let $f : A \to B$ be a mapping with $f(0) = 0$ for which there exist constants $\theta \geq 0$ and $p_1, p_2, p_3 > n$ such that*

$$\left\| f\left(\frac{b-a}{3}\mu\right) + f\left(\frac{a-3c}{3}\mu\right) + \mu f\left(\frac{3a-b}{3} + c\right) - f(a) + f(c^n) - f(c)^n \right\|_B \tag{2.13}$$
$$\leq \theta \left(\|a\|^{p_1} + \|b\|^{p_2} + \|c\|^{p_3}\right),$$

$$\|f(c^*) - f(c)^*\|_B \leq \theta(\|c\|^{p_1} + \|c\|^{p_2} + \|c\|^{p_3}) \tag{2.14}$$

for all $a, b, c \in A$ and all $\mu \in \mathbb{T}^1$. Then there exists a unique n-Jordan $*$-homomorphism $h : A \to B$ such that

$$\|f(a) - h(a)\|_B \leq \frac{\theta\|a\|^{p_1}}{1 - 3^{(1-p_1)}} + \frac{2^{p_2}\theta\|a\|^{p_2}}{1 - 3^{(1-p_2)}}$$

for all $a \in A$.

Proof. Letting $\varphi(a, b, c) = \theta(\|a\|^{p_1} + \|b\|^{p_2} + \|c\|^{p_3})$ in Theorem 2.4, we have

$$\|f(a) - h(a)\|_B \leq \frac{\theta\|a\|^{p_1}}{1 - 3^{(1-p_1)}} + \frac{2^{p_2}\theta\|a\|^{p_2}}{1 - 3^{(1-p_2)}}$$

for all $a \in A$, as desired. ∎

Corollary 2.6. *Let $\psi : \mathbb{R}^+ \cup \{0\} \to \mathbb{R}^+ \cup \{0\}$ be a function with $\psi(0) = 0$ such that*

$$\lim_{t \to 0} \frac{\psi(t)}{t} = 0,$$
$$\psi(st) \leq \psi(s)\psi(t),$$
$$3^n \psi\left(\frac{1}{3}\right) < 1$$

for all $s, t \in \mathbb{R}^+$. Suppose that $f : A \to B$ is a mapping satisfying $f(0) = 0$, (2.5) *and*

$$\left\| f\left(\frac{b-a}{3}\mu\right) + f\left(\frac{a-3c}{3}\mu\right) + \mu f\left(\frac{3a-b}{3} + c\right) - f(a) + f(c^n) - f(c)^n \right\|_B \tag{2.15}$$
$$\leq \theta[\psi(\|a\|) + \psi(\|b\|) + \psi(\|c\|)]$$

for all $a, b, c \in A$, where $\theta > 0$ is a constant. Then there exists a unique n-Jordan $*$-homomorphism $h : A \to B$ such that

$$\|h(a) - f(a)\|_B \leq \frac{\theta(1 + \psi(2))\psi(\|a\|)}{1 - 3\psi(\frac{1}{3})}$$

for all $a \in A$.

Proof. Letting $\varphi(a, b, c) = \theta[\psi(\|a\|) + \psi(\|b\|) + \psi(\|c\|)]$ in Theorem 2.4, we have

$$\|h(a) - f(a)\|_B \leq \frac{\theta(1 + \psi(2))\psi(\|a\|)}{1 - 3\psi(\frac{1}{3})}$$

for all $a \in A$, as desired. ■

Theorem 2.7. *Let A, B be C^*-algebras, and let $f : A \to B$ be a mapping with $f(0) = 0$ for which there exists a function $\varphi : A \times A \times A \to \mathbb{R}^+$ satisfying* (2.5)*,* (2.6) *and*

$$\sum_{i=1}^{\infty} \frac{1}{3^i} \varphi(3^i a, 3^i b, 3^i c) < \infty$$

for all $a, b, c \in A$. Then there exists a unique n-Jordan $*$-homomorphism $h : A \to B$ such that

$$\|h(a) - f(a)\|_B \leq \sum_{i=1}^{\infty} \frac{1}{3^i} \varphi(3^i a, 3^i (2a), 0) \tag{2.16}$$

for all $a \in A$.

Proof. Replacing a by $3a$ in (2.8), we get

$$\left\| \frac{1}{3} f(3a) - f(a) \right\|_B \leq \frac{1}{3} \varphi(3a, 2(3a), 0)$$

for all $a \in A$. One can apply the induction method to prove that

$$\left\| \frac{1}{3^m} f(3^m a) - f(a) \right\|_B \leq \sum_{i=1}^{m} \frac{1}{3^i} \varphi(3^i a, 2(3^i a), 0) \tag{2.17}$$

for all $a \in A$. In order to show that

$$h_m(a) = \frac{1}{3^m} f(3^m a)$$

form a convergent sequence, we use the Cauchy convergence criterion. Indeed, replacing a by $3^k a$ and multiplying by 3^{-k} in (2.17), where k is an arbitrary positive integer, we get that

$$\left\| \frac{1}{3^{k+m}} f(3^{k+m} a) - \frac{1}{3^k} f(3^k a) \right\|_B \leq \sum_{i=k+1}^{k+m} \frac{1}{3^i} \varphi(3^i a, 2(3^i a), 0)$$

for all positive integers m. Hence by the Cauchy criterion,

$$h(a) = \lim_{m\to\infty} h_m(a)$$

exists for all $a \in A$. By taking the limit as $m \to \infty$ in (2.17), we obtain that

$$\|h(a) - f(a)\|_B \le \sum_{i=1}^{\infty} \frac{1}{3^i}\varphi(3^i a, 2(3^i a), 0)$$

and (2.16) holds for all $a \in A$.

The rest of the proof is similar to the proof of Theorem 2.4. ■

Corollary 2.8. *Let A, B be C^*-algebras, and let $f : A \to B$ be a mapping with $f(0) = 0$ for which there exist constants $\theta \ge 0$ and $p_1, p_2, p_3 < 1$ satisfying* (2.13) *and* (2.14)*. Then there exists a unique n-Jordan $*$-homomorphism $h : A \to B$ such that*

$$\|f(a) - h(a)\|_B \le \frac{\theta\|a\|^{p_1}}{3^{(1-p_1)} - 1} + \frac{2^{p_2}\theta\|a\|^{p_2}}{3^{(1-p_2)} - 1}$$

for all $a \in A$.

Proof. Letting $\varphi(a, b, c) = \theta(\|a\|^{p_1} + \|b\|^{p_2} + \|c\|^{p_3})$ in Theorem 2.7, we have

$$\|f(a) - h(a)\|_B \le \frac{\theta\|a\|^{p_1}}{3^{(1-p_1)} - 1} + \frac{2^{p_2}\theta\|a\|^{p_2}}{3^{(1-p_2)} - 1}$$

for all $a \in A$. ■

Corollary 2.9. *Let $\psi : \mathbb{R}^+ \cup \{0\} \to \mathbb{R}^+ \cup \{0\}$ be a function with $\psi(0) = 0$ such that*

$$\lim_{t\to 0} \frac{\psi(t)}{t} = 0,$$
$$\psi(st) \le \psi(s)\psi(t),$$
$$\frac{1}{3}\psi(3) < 1$$

for all $s, t \in \mathbb{R}^+$. Suppose that $f : A \to B$ is a mapping satisfying $f(0) = 0$ (2.5) *and* (2.15)*. Then there exists a unique n-Jordan $*$-homomorphism $h : A \to B$ such that*

$$\|h(a) - f(a)\|_B \le \frac{\theta(1 + \psi(2))\psi(\|a\|)}{1 - \frac{1}{3}\psi(3)}$$

for all $a \in A$.

Proof. Letting $\varphi(a, b, c) = \theta[\psi(\|a\|) + \psi(\|b\|) + \psi(\|c\|)]$ in Theorem 2.7, we have

$$\|h(a) - f(a)\|_B \le \frac{\theta(1 + \psi(2))\psi(\|a\|)}{1 - \frac{1}{3}\psi(3)}$$

for all $a \in A$. ■

Corollary 2.10. *Let A, B be C^*-algebras, and let $f : A \to B$ be a mapping with $f(0) = 0$ for which there exists a constant $\theta \geq 0$ such that*

$$\left\| f\left(\frac{b-a}{3}\mu\right) + f\left(\frac{a-3c}{3}\mu\right) + \mu f\left(\frac{3a-b}{3} + c\right) - f(a) + f\left(c^n\right) - f(c)^n \right\|_B \leq \theta,$$

$$\|f(a^*) - f(a)^*\|_B \leq \theta$$

for all $a, b, c \in A$ and all $\mu \in \mathbb{T}^1$. Then there exists a unique n-Jordan $*$-homomorphism $h : A \to B$ such that

$$\|f(a) - h(a)\|_B \leq \theta$$

for all $a \in A$.

Proof. Letting $p_1 = p_2 = p_3 = 0$ in Corollary 2.8, we have

$$\|f(a) - h(a)\|_B \leq \theta$$

for all $a \in A$. ∎

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