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ON KURZWEIL-HENSTOCK-PETTIS AND KURZWEIL-HENSTOCK INTEGRALS OF BANACH SPACE-VALUED FUNCTIONS

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Abstract. In this paper we discuss the relationship between the Kurzweil-Henstock-Pettis and Kurzweil-Henstock integrals in Banach spaces. We prove that in Schur spaces the Kurzweil-Henstock-Pettis and Kurzweil-Henstock integrability for measurable functions satisfying the condition (C) are equivalent. In particular, in Schur spaces the Kurzweil-Henstock-Dunford, Kurzweil-Henstock-Pettis and Kurzweil-Henstock integrability for measurable functions satisfying the condition (C) are equivalent.

1. Introduction

The Kurzweil-Henstock-Pettis and Kurzweil-Henstock integrals are generalization of the Pettis integral and the McShane integral of the Banach-space-valued functions. They were discussed by several authors in [2, 3, 5-11], and some interesting results were obtained. In [11], the author proved that the Kurzweil-Henstock-Pettis integrability and Kurzweil-Henstock-Dunford integrability for measurable functions are equivalent if and only if the Banach space is weakly sequentially complete. In particular, in Schur spaces the equivalence results on the Kurzweil-Henstock-Dunford and Kurzweil-Henstock-Pettis integrability for measurable functions are given. This paper is a continuation of [11]. We further study the Kurzweil-Henstock-Pettis and Kurzweil-Henstock integrability for strongly measurable functions mapping a compact interval into a Banach space.

It can be seen from the corresponding definitions that a Kurzweil-Henstock integrable function on I_0 is Kurzweil-Henstock-Pettis integrable on I_0. But an example shows that the converse is not necessarily true. It is an interesting question to discuss

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the relationship between the Kurzweil-Henstock-Pettis integral and the Kurzweil-Henstock integral of the Banach-space-valued functions. In a general Banach space, we obtain some sufficient conditions of Kurzweil-Henstock-Pettis integrable function being Kurzweil-Henstock integrable on I_0. In Schur spaces, we prove that the Kurzweil-Henstock-Pettis and Kurzweil-Henstock integrability for measurable functions are equivalent when the condition (C) is satisfied. Especially, in Schur spaces the Kurzweil-Henstock-Dunford, Kurzweil-Henstock-Pettis and Kurzweil-Henstock integrability for measurable functions are equivalent when the condition (C) is satisfied.

2. Basic Definitions and Preliminaries

Throughout this paper X denotes a real Banach space with the norm $\|\cdot\|$ and X^* its dual. $B(X^*) = \{x^* \in X^*;\ \|x^*\| \leq 1\}$ is the closed unit ball in X^*. Let $I_0 = [a, b]$ be a compact interval in R^1 and $E \subset R^1$ a measurable subset of I_0. $\mu(E)$ stands for the Lebesgue measure. The Lebesgue integral of a function f over a set E will be denoted by $(L)\int_E f$. We firstly extend the notion of partition of an interval.

We say that the intervals I and J are non-overlapping if int $(I) \cap \text{int}(J) = \emptyset$. By $\text{int}(J)$ the interior of J is denoted.

A *partial M-partition* D in I_0 is a finite collection of interval-point pairs (I, ξ) with non-overlapping intervals $I \subset I_0$, $\xi \in I_0$ being the associated point of I. Requiring $\xi \in I$ for the associated point of I we get the concept of a *partial K-partition* D in I_0. We write $D = \{(I, \xi)\}$.

A partial M-partition $D = \{(I, \xi)\}$ in I_0 is a *M-partition* of I_0 if the union of all the intervals I equals I_0 and similarly for a *K-partition* .

Let δ be a positive function defined on the interval I_0. A partial M-partition (K-partition) $D = \{(I, \xi)\}$ is said to be *δ-fine* if for each interval-point pair $(I, \xi) \in D$ we have $I \subset B(\xi, \delta(\xi))$ where $B(\xi, \delta(\xi)) = (\xi - \delta(\xi), \xi + \delta(\xi))$.

Given a partition $D = \{(I, \xi)\}$ of I_0 we write

$$f(D) = \sum_D f(\xi)\mu(I)$$

for integral sums over D, whenever $f : I_0 \to X$.

Definition 2.1. An X-valued function f is said to be *McShane integrable* on I_0 if there exists an $S_f \in X$ such that for every $\varepsilon > 0$ there is a gauge δ such that for every δ-fine M-partition $D = \{(I, \xi)\}$ of I_0, we have

$$\|f(D) - S_f\| < \varepsilon.$$

We write $(M) \int_{I_0} f = S_f$ and S_f is the *McShane integral* of f over I_0.

f is McShane integrable on a set $E \subset I_0$ if the function $f \cdot \chi_E$ is McShane integrable on I_0. We write $(M) \int_E f = (M) \int_{I_0} f\chi_E = F(E)$ for the McShane integral of f on E.

Replacing the term "M-partition" by "K-partition" in the definition above we obtain *Kurzweil-Henstock integrability* and the definition of the *Kurzweil-Henstock integral* $(KH) \int_{I_0} f$.

It is clear that if $f : I_0 \mapsto X$ is McShane integrable, then it is also Kurzweil-Henstock integrable because every K-partition is an M-partition.

It is known that linearity, integrability on subintervals, additivity of intervals of McShane and Kurzweil-Henstock integrals hold. For the details, see [3]-[4], [9]-[11].

Definition 2.2. (a) A function $f : I_0 \to X$ is *Kurzweil-Henstock-Dunford integrable* if for each x^* in X^* the function x^*f is Kurzweil-Henstock integrable on I_0 and for each interval I in I_0 there exists a vector x_I^{**} in X^{**} such that $x_I^{**}(x^*) = \int_I x^*f$ for all x^* in X^*. We write $x_{I_0}^{**} = (KHD) \int_{I_0} f = F(I_0)$ and F is the primitive of f on I_0.

(b) A function $f : I_0 \to X$ is *Kurzweil-Henstock-Pettis integrable* on I_0 if f is Kurzweil-Henstock-Dunford integrable on I_0 and $x_I^{**} \in X$ for every interval I in I_0. We write $x_{I_0}^{**} = (KHP) \int_{I_0} f = F(I_0)$ and F is the primitive of f on I_0.

Definition 2.3. A family $\{f_\alpha\}$ of Kurzweil-Henstock integrable functions is *KH-equiintegrable* on I_0 if for every $\varepsilon > 0$ there exists $\delta(\xi) > 0$ such that for every δ-fine K-partition $D = \{(I, \xi)\}$ of I_0, we have

$$\| \sum_D f_\alpha(\xi)\mu(I) - \int_{I_0} f_\alpha \| < \varepsilon$$

for all α.

For simplicity, the letters P, M, KH, KHD and KHP stand for Pettis, McShane, Kurzweil-Henstock, Kurzweil-Henstock-Dunford, Kurzweil-Henstock-Pettis and denote the sets of all P, M, KH, KHD and KHP integrable functions $f : I_0 \mapsto X$ by $\mathcal{P}$, $\mathcal{M}$, $\mathcal{KH}$, $\mathcal{KHD}$, $\mathcal{KHP}$, respectively.

From the corresponding definitions of different integrals we have

$$\mathcal{M} \subset \mathcal{P}, \quad \mathcal{M} \subset \mathcal{KH} \subset \mathcal{KHP} \subset \mathcal{KHD}.$$

The relation of the Pettis and McShane integrals have been discussed in [5]-[9] and the McShane and Kurzweil-Henstock integrals in [10]. Further, the relation of the Kurzweil-Henstock-Dunford and Kurzweil-Henstock-Pettis integrals was discussed in [11] and we obtained that the Kurzweil-Henstock-Pettis and Kurzweil-Henstock-Dunford integrability for measurable functions are equivalent if and only if the Banach space is weakly sequentially complete. For the details of the above relations of the different integrals, see the corresponding references. Here we study further the relation of the Kurzweil-Henstock-Pettis and Kurzweil-Henstock integrals of the Banach-space-valued functions.

The following two results are easy to obtain. For completeness, we provide the proofs.

Theorem 2.4. *Suppose that $f : I_0 \to X$ is Kurzweil-Henstock-Pettis integrable and $\{x^*f : x^* \in B(X^*)\}$ is KH-equiintegrable on I_0. Then f is Kurzweil-Henstock integrable on I_0 and*

$$(KH)\int_{I_0} f = (KHP)\int_{I_0} f.$$

Proof. Assume that F is the primitive of f. Since $\{x^*f : x^* \in B(X^*)\}$ is KH- equiintegrability on I_0, for every $\varepsilon > 0$ there exists $\delta(\xi) > 0$ such that for every δ-fine K-partition $D = \{(I, \xi)\}$ of I_0, we have

$$|\sum_D x^*f(\xi)\mu(I) - x^*F(I_0)| < \varepsilon, \quad for \;\; all \;\; x^* \in B(X^*).$$

That is,

$$x^*(\|\sum_D f(\xi)\mu(I) - F(I_0)\|) < \varepsilon.$$

By the Hahn-Banach theorem,

$$\|\sum_D f(\xi)\mu(I) - F(I_0)\| < \varepsilon.$$

Hence, f is Kurzweil-Henstock integrable on I_0 and $(KH)\int_{I_0} f = (KHP)\int_{I_0} f$.

Theorem 2.5. *Suppose that $D^* \subset B(X^*)$ and D^* is dense in $B(X^*)$. Assume that $f : I_0 \to X$ is Kurzweil-Henstock-Pettis integrable and $\{x^*f : x^* \in D^*\}$ is KH-equiintegrable on I_0. Then f is Kurzweil-Henstock integrable on I_0 and*

$$(KH)\int_{I_0} f = (KHP)\int_{I_0} f.$$

Proof. Assume that F is the primitive of f. Since $\{x^*f : x^* \in D^*\}$ is KH-equiintegrable on I_0, for every $\varepsilon > 0$ there exists $\delta(\xi) > 0$ such that for every δ-fine

K-partition $D = \{(I, \xi)\}$ of I_0, we have

$$|\sum_D x^* f(\xi)\mu(I) - x^* F(I_0)| < \varepsilon, \ for \ \ x^* \in D^*. \tag{2.1}$$

Note that D^* is dense in $B(X^*)$, for every $x^* \in B(X^*)$ there is $\{x_m^*\} \subset D^*$ such that

$$\|x_m^* - x^*\| \to 0 \quad (m \to \infty), \tag{2.2}$$

and

$$\begin{aligned} &\left|\sum_D x^* f(\xi)\mu(I) - x^* F(I_0)\right| \\ \le &\left|\sum_D x_m^* f(\xi)\mu(I) - x_m^* F(I_0)\right| + |(x_m^* - x^*)(\sum_D f(\xi)\mu(I) - F(I_0))| \\ \le &\left|\sum_D x_m^* f(\xi)\mu(I) - x_m^* F(I_0)\right| + \|x_m^* - x^*\| \| \sum_D f(\xi)\mu(I) - F(I_0))\|. \end{aligned} \tag{2.3}$$

(2.1), (2.2) and (2.3) imply

$$|\sum_D x^* f(\xi)\mu(I) - x^* F(I_0)| \le \varepsilon, \quad for \, \forall \, x^* \in B(X^*).$$

It follows from Theorem **??** that f is Kurzweil-Henstock integrable on I_0 and $(KH) \int_{I_0} f = (KHP) \int_{I_0} f$.

3. An Example

In this section we present an example of a function which is to say that there is a function that is Kurzweil-Henstock-Pettis integrable but not Kurzweil-Henstock integrable.

Example 3.6. Let us consider a function on $[0, 1]$ from Example in [5] (P1184) (or from 3C Example in [6], p. 143) given by
$f_0 : [0, 1] \to l_\infty(\omega_1)$ by

$$f_0(t)(\alpha) = \begin{cases} 1 & t \in N_\alpha \setminus C_\alpha, \\ 0 & \text{otherwise}. \end{cases}$$

The statement of the notations N_α, C_α and others is a bit too involved to be given. Here we need only the following three properties of the function $f_0 : [0, 1] \to l_\infty(\omega_1)$ and not prolix statement of notations. So we do not present an explanation of the above notations. For the details, see [5] (Example, p. 1184).

We have known that f_0 has the following properties (1)-(3).

(1) f_0 is scalarly negligible;

(2) f_0 is Pettis integrable;

(3) f_0 is not McShane integrable.

Now we prove that f_0 is Kurzweil-Henstock-Pettis integrable and not Kurzweil-Henstock integrable.

In fact, by (2) f_0 is Pettis integrable, then for each $x^* \in X^*$ $x^* f_0$ is McShane (=Lebesgue) integrable on $[0,1]$ and therefore the real function $x^* f_0$ is Kurzweil-Henstock integrable on $[0,1]$. For each subinterval $J \subset [0,1]$, by Pettis integrability of f_0, $(P)\int_J f \in X$ and $(KH)\int_I x^* f_0 = x^*(P)\int_I f$ for every $x^* \in X^*$. It follows from Definition 2.2 that f_0 is Kurzweil-Henstock-Pettis integrable on $[0,1]$.

Suppose now that f_0 is Kurzweil-Henstock integrable on $[0,1]$. Theorem in [7] (P475) guarantees that Kurzweil-Henstock integrable and Pettis integrable function is McShane integrable. Combining it with the property (2) above, we obtain that f_0 is McShane integrable on $[0,1]$. This contradicts with the property (3) above. Hence, f_0 is not Kurzweil-Henstock integrable on $[0,1]$.

Remark. In [9] (Theorem 6.2.1, P173), the authors proved that if Banach-space-valued function f is measurable and Pettis integrable then f is McShane integrable, but the above example also shows that for the case of the Kurzweil-Henstock-Pettis and Kurzweil-Henstock integrals we do not have a similar result. This makes us to have to think under which conditions the Kurzweil-Henstock-Pettis integrable function is Kurzweil-Henstock integrable? We will concentrate on discussing this question in next section.

4. The Main Results

In this section, we study conditions under which Kurzweil-Henstock-Pettis integrable function is Kurzweil-Henstock integrable. The main results are Theorem 4.1, Theorem 4.2 and Theorem 4.5.

Suppose that f is measurable and Kurzweil-Henstock-Pettis integrable on I_0. Then by the measurability of f and Proposition 1.1.9 in [9], there exists a bounded measurable $g : I_0 \to X$ and a measurable $h : I_0 \to X$ with

$$h(t) = \sum_{k=1}^{\infty} x_k \chi_{E_k}(t), x_k \in X, k \in \mathbb{N}, t \in I_0,$$

where $E_k \subset I_0, k \in \mathbb{N}$ are pairwise disjoint measurable sets and $\chi_{E_k}(t)$ is the characteristic function of E_k such that $f = g + h$.

We can see that g is Bochner integrable and therefore g is Kurzweil-Henstock integrable. Denote the primitive of g by G. Then $G(I_0) = (B)\int_{I_0} g = (KH)\int_{I_0} g$.

Let

$$h_n(t) = \sum_{k=1}^{n} x_k \chi_{E_k}(t), \ \ k \in \mathbb{N}, \ \ t \in I_0.$$

Then each simple function h_n is McShane integrable on I_0 and $(M)\int_{I_0} h_n = \sum_{k=1}^{n}(M)\int_{E_k} h_k = \sum_{k=1}^{n} x_k\mu(E_k)$.

So, h_n is Kurzweil-Henstock integrable on I_0 and $(KH)\int_{I_0} h_n = (M)\int_{I_0} h_n = \sum_{k=1}^{n} x_k\mu(E_k)$.

Denote the primitive of h_n by H_n. Then

$$H_n(I) = (KH)\int_I h_n = \sum_{k=1}^{n} x_k\mu(E_k \cap I) \ \ for \ \ each \ \ subinterval \ \ I \subset I_0.$$

Since f is Kurzweil-Henstock-Pettis integrable on I_0, $h = f - g$ is Kurzweil-Henstock-Pettis integrable on I_0. That is, for each $x^* \in X^*$,

$$x^*h = \sum_{k=1}^{\infty} x^*(x_k)\chi_{E_n}$$

is Kurzweil-Henstock integrable on I_0. Further, we state the condition (C) for f:

$$\text{(4.1)} \quad x^*(KHP)\int_I h = (KH)\int_I x^*h = \sum_{k=1}^{\infty} x^*(x_k)\mu(E_k \cap I) = \lim_{n\to\infty} x^*H_n(I)$$

holds for each subinterval $I \subset I_0$ and $\{H_n\}$ is uniformly $AC^{\nabla}(E_k)$ for $k \in \mathbb{N}$, that is for every $\epsilon > 0$ there exists $\delta : E_k \to (0,\infty)$ and $\eta > 0$ such that

$$\text{(4.2)} \qquad \|\sum_{i=1}^{p} H_n(I_i)\| = \|\sum_{i=1}^{p}\sum_{j=1}^{n} x_j\mu(E_j \cap I_i)\| < \epsilon$$

for any δ-fine partial K-partition $\{(t_i, I_i)\}_{i=1}^{p}$ with $t_i \in E_k$, $\sum_{i=1}^{p} |I_i| < \eta$ and all $k \in \mathbb{N}$.

For uniformly $AC^{\nabla}(E_k)$ for $k \in \mathbb{N}$, the reader can see Definition 7.6.5 in [9].

Recall that a Banach space X is a Schur space if weakly convergent sequences in X are norm convergent.

We now come to the following main result.

Theorem 4.1. *Suppose that X is a Schur space and $f : I_0 \to X$ is measurable. Further, f is Kurzweil-Henstock-Pettis integrable and the condition (C) is satisfied, then f is Kurzweil-Henstock integrable and $(KH)\int_{I_0} f = (KHP)\int_{I_0} f$.*

Proof. By (4.1), $x^*H_n(I) = \sum_{k=1}^{n} x^*(x_k)\mu(E_k \cap I)$ is convergent to $x^*(KHP)\int_I h = \sum_{k=1}^{\infty} x^*(x_k)\mu(E_k \cap I)$ for each subinterval $I \subset I_0$ and $x^* \in X^*$. Since X

is a Schur space, it follows that $H_n(I)=\sum_{k=1}^{n} x_k\mu(E_k\cap I)$ is strongly convergent to $(KHP)\int_I h=\sum_{k=1}^{\infty} x_k\mu(E_k\cap I)$.

Set

$$H(I)=(KHP)\int_I h=\sum_{k=1}^{\infty} x_k\mu(E_k\cap I)=\lim_{n\to\infty}\sum_{k=1}^{n} x_k\mu(E_k\cap I)=\lim_{n\to\infty} H_n(I).$$

Then

$$H(I)=F(I)-G(I)=(KHP)\int_I f-(KH)\int_I g$$

is an interval additive function.

Note that $\lim_{n\to\infty} h_n(t)=h(t)$ pointwise, combining the condition (C) and Theorem 7.6.14 in [9], h is Kurzweil-Henstock integrable and

$$(KH)\int_{I_0} h=(KHP)\int_{I_0} h=\sum_{k=1}^{\infty} x_k\mu(E_k).$$

Thus, $f=g+h$ is Kurzweil-Henstock integrable on I_0 and

$$(KH)\int_{I_0} f=(KH)\int_{I_0} g+(KH)\int_{I_0} h=(KHP)\int_{I_0} f. \qquad \blacksquare$$

Theorem 4.2. *Suppose that X is a separable Banach space and X^* is a Schur space. If f is Kurzweil-Henstock-Pettis integrable on I_0, then f is Kurzweil-Henstock integrable on I_0.*

Proof. Suppose that f is not Kurzweil-Henstock integrable on I_0. Then there is an $\varepsilon_0>0$ such that for every gauge $\delta_k\downarrow 0$ there are δ_k-fine partition $D_k=\{(I,\xi)\}$ of I_0 such that

$$\|f(D_k)-(KHP)\int_{I_0} f\|\geq\varepsilon_0.$$

By the Hahn-Banach theorem, there are $x_k^*\in B(X^*)$ such that

$$x_k^*(f(D_k)-(KHP)\int_{I_0} f)=\|f(D_k)-(KHP)\int_{I_0} f\|\geq\varepsilon_0. \tag{4.3}$$

Since f is Kurzweil-Henstock-Pettis integrable on I_0, for each $x^*\in X^*$ x^*f is Kurzweil-Henstock integrable. Hence, for each $x^*\in X^*$, $|x^*(f(D_k){-}(KHP)\int_{I_0} f)|$ are bounded for all $k\in\mathbb{N}$. By the Banach-Stainhous theorem, $\|f(D_k){-}(KHP)\int_{I_0} f\|$ is uniformly bounded and M is its bound.

Since X is a separable Banach space, $B(X^*)$ is $weak^*$ sequentially compact. Thus, there is an $x_0^*\in B(X^*)$ and a subsequence $x_{k_j}^*$ of $x_k^*\in B(X^*)$ such that

$$w^* - \lim_{j\to\infty} x^*_{k_j} = x^*_0.$$

Note that X^* is a Schur space, then $\lim_{j\to\infty} \|x^*_{k_j} - x^*_0\| = 0$.

By the Kurzweil-Henstock integrability of $x^*_0 f$ on I_0, we have

$$\lim_{j\to\infty} x^*_0 f(D_{k_j}) - (KH)\int_{I_0} x^*_0 f = \lim_{j\to\infty} x^*_0(f(D_{k_j}) - (KHP)\int_{I_0} f) = 0. \tag{4.4}$$

By (4.3) and (4.4), we obtain

$$\begin{aligned} \varepsilon_0 &\leq \left\| f(D_{k_j}) - (KHP)\int_{I_0} f \right\| \\ &= x^*_{k_j}\left(f\left(D_{k_j}\right) - (KHP)\int_{I_0} f \right) \\ &\leq \left\| x^*_{k_j} - x^*_0 \right\| \left\| f(D_{k_j}) - (KHP)\int_{I_0} f \right\| + \left| x^*_0(f(D_{k_j}) - (KHP)\int_{I_0} f) \right| \\ &\leq M \left\| x^*_{k_j} - x^*_0 \right\| + \left| x^*_0(f(D_{k_j}) - (KHP)\int_{I_0} f) \right| \end{aligned} \tag{4.5}$$

Let $j \to \infty$. Then right hand of (4.5) goes 0. So

$$\varepsilon_0 \leq 0.$$

This leads to a contradiction. Thus, f is Kurzweil-Henstock integrable on I_0. ■

Since X^* is separable, X is separable. By Theorem 4.2, we obtain immediately

Corollary 4.3. Suppose that X^* is a separable Schur space. If f is Kurzweil-Henstock-Pettis integrable on I_0, then f is Kurzweil-Henstock integrable on I_0.

Since X^* is a Schur space if and if X has the Dunford-Pettis property and does not contain a copy of l_1 (see [12]). So the following corollary follows easily.

Corollary 4.4. Suppose that X is separable, has the Dunford-Pettis property and does not contain a copy of l_1. If f is Kurzweil-Henstock-Pettis integrable on I_0, then f is Kurzweil-Henstock integrable on I_0.

We have known from Theorem 3.9 in [11] that if X is a Schur space and f is Kurzweil-Henstock-Dunford integrable on I_0, then f is Kurzweil-Henstock-Pettis integrable on I_0. Combining this and Theorem 4.5, we obtain

Theorem 4.5. *Suppose that X is a Schur space, $f : I_0 \to X$ is measurable and the condition (C) is satisfied. Then*

$$\mathcal{KH} = \mathcal{KHP} = \mathcal{KHD}.$$

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